

Please write clearly in block capitals.

Centre number

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Candidate number

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Surname

Forename(s)

Candidate signature

I declare this is my own work.

INTERNATIONAL AS FURTHER MATHEMATICS

(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Wednesday 21 May 2025 07:00 GMT Time allowed: 1 hour 30 minutes

Materials

- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- There are three sections to this paper.
- The maximum mark for this paper is 80. There are 40 marks for **Section A**, 20 marks for **Section B** and 20 marks for **Section C**.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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7	
8	
9	
10	
11	
TOTAL	



Section A**Pure Mathematics**Answer **all** questions in the spaces provided.

1 The matrix **A** is defined by $\mathbf{A} = \begin{bmatrix} 1.2 & 0.4 - p \\ 0.4 + p & 1.8 \end{bmatrix}$ where p is a constant.

1 (a) Find in terms of p the determinant of **A**

[2 marks]

$$1.2 \times 1.8 - (0.4 - p) \times (0.4 + p)$$

$$2 + p^2$$

Answer _____

1 (b) The matrix **B** is defined by $\mathbf{B} = \begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix}$ where k is a constant.

1 (b) (i) Describe the transformation represented by the matrix **B**

[1 mark]Shear parallel to y -axis

1 (b) (ii) The point $(3, 2)$ is mapped to the point $(3, -4)$ by the transformation represented by **B**

Find the value of k **[2 marks]**

$$\begin{bmatrix} 1 & 0 \\ k & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$\Rightarrow 3k + 2 = -4$$

$$k = -2$$

Answer _____



1 (c) The matrix **C** represents the following sequence of transformations:

- the transformation represented by **B**
- followed by
- the transformation represented by **A**

Find in terms of p the matrix **C**

[3 marks]

$$\mathbf{C} = \begin{bmatrix} 1.2 & 0.4 - p \\ 0.4 + p & 1.8 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.4 + 2p & 0.4 - p \\ p - 3.2 & 1.8 \end{bmatrix}$$

Answer _____

8

Turn over for the next question

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- 2 A curve passes through the point (2, 5) and satisfies the differential equation

$$\frac{dy}{dx} = \frac{1}{2^x - \sqrt{y}}$$

Use Euler's step-by-step method with a step length of 0.05 to estimate the value of y when $x = 2.1$

Give your answer to three decimal places.

[5 marks]

$$hf(2, 5) = 0.05 \times \frac{1}{2^2 - \sqrt{5}}$$

$$= 0.02834576...$$

$$y_2 = 5 + 0.02834576... = 5.02834576...$$

$$y_3 = 5.02834576 + 0.05 \times \frac{1}{2^{2.05} - \sqrt{5.02834576}}$$

$$[= 5.05468009]$$

$$5.055$$

Answer

5

Turn over for the next question

Turn over ►



- 3** The variables x and y are related by the equation

$$y = Ax^2 + Bx$$

where A and B are constants.

- 3 (a)** Show that there is a linear relationship between x and Y where $Y = \frac{y}{x}$

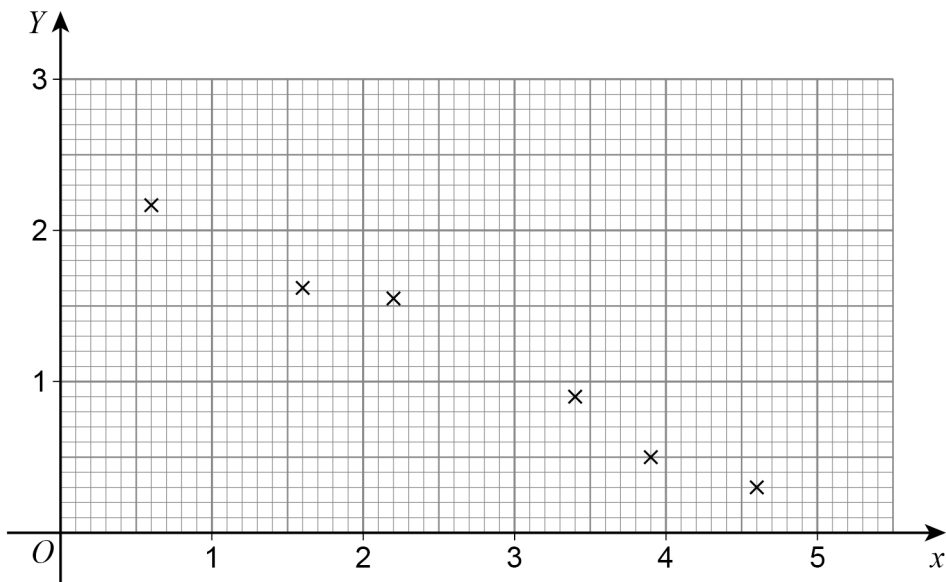
[2 marks]

$$\frac{y}{x} = Ax + B$$

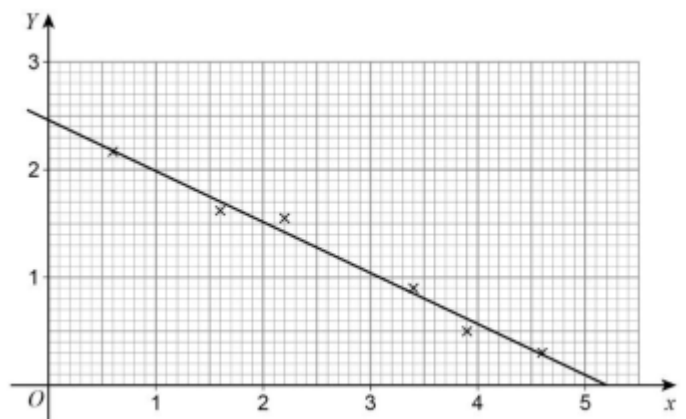
which is a linear relationship [between x
and $\frac{y}{x}$]

- 3 (b)** Six pairs of x and y values are obtained in an experiment.

The values of x and Y are plotted on the grid below.



- 3 (b) (i)** Draw a line of best fit on the grid above.



- 3 (b) (ii) Use your line of best fit to estimate the value of A and the value of B

[2 marks]

gradient = -0.47
y-intercept = 2.46

$A = -0.47$
 $B = 2.46$

B1

Gradient or intercept seen for their straight line of best fit
Half a square tolerance for identified points

B1

AWFW $[-0.5, -0.44]$ for A

AWFW $[2.38, 2.55]$ for B

Note an award of **B0 B1** is possible

$A =$ _____ $B =$ _____

- 3 (c) Use your answer to **part (b)(ii)** to estimate the values of x for which $y = 1$

Give your values to two significant figures.

[3 marks]

$$1 = -0.47x^2 + 2.46x$$

$$x = 4.7898 \text{ or } x = 0.4442$$

$$x = 4.8 \text{ or } x = 0.44$$

M1

Correct substitution of their values and $y = 1$ into original relationship

m1

PI Solves resulting quadratic

A1ft

ft their A and B

Answers must be given to two significant figures and not rejected

Turn over for the next question

Turn over ►



- 4 The equation $2x^3 - 4x^2 - 12x + 21 = 0$ has a real root α in the interval $2 < \alpha < 3$

- 4 (a) (i) Interval bisection is used once with the interval $2 < \alpha < 3$ to find an improved interval for α

Linear interpolation is also used once with the interval $2 < \alpha < 3$ to find an improved interval for α

Explain why using interval bisection and linear interpolation with the interval $2 < \alpha < 3$ result in the **same** improved interval for α

$x_1 = \frac{3 \times -3 - 2 \times 3}{-3 - 3} = 2.5$ <p>Linear interpolation also gives the mid point</p>	<p>M1</p> <p>A1</p>	<p>[2 marks]</p> <p>Applies linear interpolation once, must see the calculation of x_1 or equation of line $y = 6x - 15$ or evaluates both $f(2) = -3$ and $f(3) = 3$ which must be seen in part (a)(i)</p> <p>Correct conclusion oe</p> <p>eg $f(2) = f(3)$ so linear interpolation divides the interval in two [as does interval bisection]</p>
--	-----------------------------------	--

- 4 (a) (ii) Show that the improved interval for α is $\frac{5}{2} < \alpha < 3$

[2 marks]

$f\left(\frac{5}{2}\right) = -2.75 \text{ and } f(3) = 3$ <p>Change of sign, so $\frac{5}{2} < \alpha < 3$</p>	<p>M1</p> <p>A1</p>	<p>Correct evaluation of $f\left(\frac{5}{2}\right)$ and $f(3)$</p> <p>One or both may be seen in part (a)(i)</p> <p>Correct conclusion</p> <p>Must state or show that there is a change of sign and concludes that α is present in the interval</p> <p>Reference to $f\left(\frac{5}{2}\right)$ and $f(3)$ must be seen but not necessarily evaluated in part (a)(ii)</p>
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- 4 (b) Starting with the interval $\frac{5}{2} < \alpha < 3$ use linear interpolation once to

show that $\frac{63}{23} < \alpha < 3$

[5 marks]

$$\frac{x_1 - \frac{5}{2}}{3 - x_1} = \frac{2.75}{3}$$

$$3\left(x_1 - \frac{5}{2}\right) = 2.75(3 - x_1)$$

$$3x_1 - \frac{15}{2} = \frac{33}{4} - 2.75x_1$$

$$5.75x_1 = \frac{63}{4}$$

M1

Obtains correct equation in terms of x_1 or obtains equation of linear interpolation line

$$y = \frac{23}{2}x - \frac{63}{2} \text{ or obtains } \frac{p}{0.5 - p} = \frac{2.75}{3}$$

oe

Condone use of $f\left(\frac{5}{2}\right)$ and $f(3)$ instead of values

ft their values from (a)

Obtains correct linear equation in terms of x_1

oe

Correct calculation of x_1 scores M1 m1, eg

$$x_1 = \frac{5}{2} + \frac{1}{2} \times \frac{2.75}{2.75 + 3},$$

$$x_1 = \frac{2.5 \times 3 - 3 \times -2.75}{3 + 2.75}, \quad x_1 = \frac{5}{2} + \frac{11}{46}$$

m1

Condone use of $f\left(\frac{5}{2}\right)$ and $f(3)$ instead of values

ft their values from (a)



- 5 The matrix **M** is defined by $\mathbf{M} = \frac{1}{5} \begin{bmatrix} 2k-3 & 4-k \\ 4k+4 & 3-2k \end{bmatrix}$ where k is a constant.

- 5 (a) Find $\mathbf{M}^2$

[3 marks]

$$\begin{aligned} \mathbf{M}^2 &= \frac{1}{25} \begin{bmatrix} 2k-3 & 4-k \\ 4k+4 & 3-2k \end{bmatrix} \begin{bmatrix} 2k-3 & 4-k \\ 4k+4 & 3-2k \end{bmatrix} \\ &= \frac{1}{25} \begin{bmatrix} (2k-3)^2 + (4-k)(4k+4) & (2k-3)(4-k) + (4-k)(3-2k) \\ (4k+4)(2k-3) + (3-2k)(4k+4) & (4k+4)(4-k) + (3-2k)(3-2k) \end{bmatrix} \\ &= \frac{1}{25} \begin{bmatrix} 25 & 0 \\ 0 & 25 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

M1

For $\frac{1}{25} \times$ a 2×2 matrix **oe** with at least one correct algebraic entry

PI

M1

For one correctly evaluated entry

A1

CAO

Answer

- 5 (b) Show that, for all values of k , the line $y = 2x$ is a line of invariant points of the transformation represented by **M**

[3 marks]

$$\frac{1}{5} \begin{bmatrix} 2k-3 & 4-k \\ 4k+4 & 3-2k \end{bmatrix} \begin{bmatrix} x \\ 2x \end{bmatrix}$$

M1

Applies **M** correctly to a general point on the line $y = 2x$

$$= \frac{1}{5} \begin{bmatrix} (2k-3)x + (4-k) \times 2x \\ (4k+4)x + (3-2k) \times 2x \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 5x \\ 10x \end{bmatrix}$$

A1

Obtains $\begin{bmatrix} x \\ 2x \end{bmatrix}$ with an intermediate line and no errors seen

$$= \begin{bmatrix} x \\ 2x \end{bmatrix}$$

So $y = 2x$ is a line of invariant points

A1

Correct conclusion



5 (c) It is given that $k = 0$

5 (c) (i) Describe the single transformation represented by $\mathbf{M}$

[3 marks]

$$k = 0 \text{ gives } \mathbf{M} = \frac{1}{5} \begin{bmatrix} -3 & 4 \\ 4 & 3 \end{bmatrix}$$

Reflection

[with mirror line] $y = 2x$

M1

Substitutes $k = 0$

A1

Condones $y = (\tan \theta)x$ where

A1

$\theta = \text{AWRT } 63.4^\circ \text{ or } 1.11 \text{ radians}$

or $\tan \theta = \text{AWRT } 2.0$

5 (c) (ii) The line with equation $y = ax + 7$ is an invariant line of the transformation represented by $\mathbf{M}$

State the value of a $a = -\frac{1}{2}$

[1 mark]

2. 核心逻辑：不变线的定义 (Key Logic: Definition of Invariant Line)

不变线的核心特征是：直线上任意一点经过变换后，其像仍在这条直线上 (For any point on the line, its image after transformation still lies on the same line) 。

设直线上任意一点为 $(x, ax + 7)$ ，其变换像为 $M \cdot \begin{bmatrix} x \\ ax + 7 \end{bmatrix}$ ，记像的坐标为 (x', y') ，则需满足 $y' = ax' + 7$ 。

10

3. 详细推导 (Detailed Derivation)

(1) 计算点 $(x, ax + 7)$ 的变换像 (Calculate the image of $(x, ax + 7)$)

$$\begin{aligned} M \cdot \begin{bmatrix} x \\ ax + 7 \end{bmatrix} &= \frac{1}{5} \begin{bmatrix} -3 & 4 \\ 4 & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ ax + 7 \end{bmatrix} \\ &= \frac{1}{5} \begin{bmatrix} -3x + 4(ax + 7) \\ 4x + 3(ax + 7) \end{bmatrix} \\ &= \begin{bmatrix} \frac{(-3+4a)x+28}{5} \\ \frac{(4+3a)x+21}{5} \end{bmatrix} \end{aligned}$$

根据矩阵乘法规则，计算 $M \cdot \begin{bmatrix} x \\ ax + 7 \end{bmatrix}$ ：

因此，像的坐标为： $x' = \frac{(-3+4a)x+28}{5}$ ， $y' = \frac{(4+3a)x+21}{5}$ 。

(2) 代入不变线条件 $y' = ax' + 7$ (Substitute into the invariant line condition $y' = ax' + 7$)

将 x' 和 y' 代入 $y' = ax' + 7$ ，两边同乘5消去分母： $(4+3a)x+21 = a \cdot [(-3+4a)x+28] + 35$

展开并整理等式右边： $(4+3a)x+21 = (-3a+4a^2)x+28a+35$

(3) 比较等式两边系数 (Compare coefficients on both sides)

由于等式对**所有** x 成立，因此 x 的系数和常数项必须分别相等，得到方程组：

1. x 的系数： $4+3a = -3a+4a^2$ (Coefficient of x)

2. 常数项： $21 = 28a+35$ (Constant term)



Section B**Statistics**

Answer **all** questions in the spaces provided.

- 6** A spinner has 11 sections, numbered 1, 2, 3, ..., 11
 After the spinner is spun there is an equal probability of it landing on any one section.
 The spinner is spun once.
 The random variable X represents the number that the spinner lands on.

- 6 (a)** State the name of the distribution of X

[1 mark]

[Discrete] uniform [distribution]

B1

Must see uniform

Answer _____

- 6 (b)** Find $P(X > 6)$

[1 mark]

$$\frac{5}{11}$$

Answer _____



6 (c) It is given that

$$E(aX + b) = 13 \quad \text{and} \quad \text{Var}(aX + b) = 90$$

where a and b are constants and $a > 0$

Find the value of a and the value of b

[5 marks]

$$E(X) = \frac{n+1}{2}$$

$$E(X) = \frac{11+1}{2} = 6$$

$$\text{Var}(X) = \frac{n^2-1}{12}$$

$$\text{Var}(X) = \frac{11^2-1}{12} = 10$$

$$6a + b = 13$$

$$10a^2 = 90$$

$$a = 3$$

$$b = -5$$

$$a = \quad \quad \quad b = \quad \quad \quad$$

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Turn over for the next question

Turn over ►



- 7 The probability generating function $G_Y(t)$ of the random variable Y is given by

$$G_Y(t) = 0.318 + 0.512t + at^b$$

where a and b are constants.

- 7 (a) Show that the value of a is 0.17

For the PGF of any random variable, $G_Y(1) = 1$.

Reason: By definition, $G_Y(t) = E(t^Y) = \sum_y P(Y = y)t^y$. When $t = 1$, $G_Y(1) = \sum_y P(Y = y) = 1$ (the sum of all probabilities is 1).

- 7 (b) The mean of Y is 1.872

Use differentiation to find the value of b

[3 marks]

$G'_Y(t) = 0.512 + 0.17bt^{b-1}$	M1	Differentiates $G_Y(t)$ correctly May be in terms of a Condone $G'_Y(1) = 0.512 + 0.17b \times 1^{b-1}$ oe
$1.872 = 0.512 + 0.17b$	M1	Forms correct equation to find b oe PI
$b = 8$	A1	Must see $G'_Y(t) = 0.512 + 0.17bt^{b-1}$ oe

Answer _____



7 (c) Use differentiation to find the variance of Y

[3 marks]

$$G_Y''(t) = 9.52t^6$$

M1

Differentiates their $G_Y'(t)$ correctly
May be in terms of a and b

$$\text{Var}(Y) = 9.52 + 1.872 - 1.872^2$$

m1

oe
Applies correct variance formula with
their $G_Y'(1)$ and $G_Y''(1)$

$$\text{Var}(Y) = 7.887616$$

A1ft

AWRT 7.89
ft their $0.17b(b-1) - 1.632384$

The variance $\text{Var}(Y)$ of the random variable Y is given by: $\text{Var}(Y) = G_Y''(1) + G_Y'(1) - [G_Y'(1)]^2$

Answer _____

7

Turn over for the next question

Turn over ►



8 Akeel takes part in a contest that has two rounds.

In the first round, he makes repeated attempts to throw a ball into a hole until he is successful.

For each attempt, the probability that Akeel throws the ball into the hole is 0.4

If Akeel throws the ball into the hole within the first three attempts, he then plays a game in the second round of the contest where the probability of winning is 0.88

Otherwise, he plays a different game where the probability of winning is 0.19

8 (a) Draw a tree diagram to represent all the information for the contest.

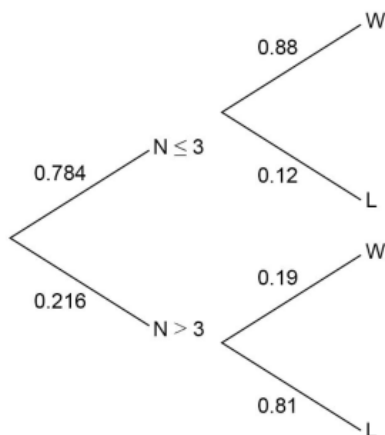
[3 marks]

Akeel 参与一场包含两轮的比赛 (contest with two rounds)：第一轮中，他反复尝试将球投入洞中，直到成功为止 (repeated attempts until success)，每次尝试成功的概率为 0.4；若他在前三次尝试内成功，第二轮将参加一场获胜概率为 0.88 的比赛；否则，第二轮参加获胜概率为 0.19 的另一场比赛。本题围绕树状图绘制与条件概率计算展开，需结合“几何分布”（首次成功概率）与“贝叶斯定理”（条件概率）求解。

$$1 - (1 - 0.4)^3$$

or

$$(1 - 0.4)^3$$



M1

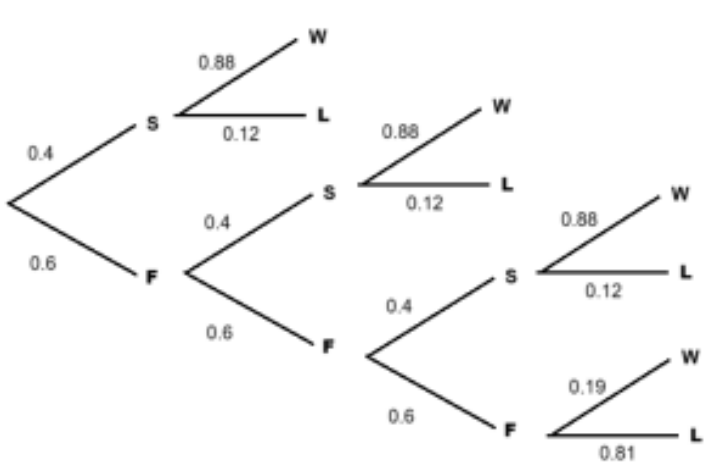
Correct calculation of
 $P(\text{ball in hole} \leq 3)$ or $P(\text{ball in hole} > 3)$
PI By sight of 0.784 or 0.216 oe

M1

Correct structure **oe**
eg $N \leq 3$ branch split into three
branches (probabilities 0.4, 0.24 and
0.144)
Allow any sensible notation

A1

Fully correct **oe**



8 (b) Akeel wins the contest if he wins the game in the second round.

Find the probability that Akeel throws the ball into the hole within three attempts given that he wins the contest.

Give your answer to three decimal places.

[3 marks]

2. 关键公式：贝叶斯定理 (Key Formula: Bayes' Theorem)

设事件 A = “前三次尝试内成功”，事件 B = “赢得整场比赛”，需计算 $P(A|B)$ (条件概率)。根据贝叶斯定理：

$$P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}$$

其中：

- $P(A)$ ：前三次尝试内成功的概率；
- $P(B|A)$ ：前三次成功的条件下，赢得比赛的概率（即第二轮高概率比赛的获胜概率）；
- $P(B)$ ：赢得整场比赛的总概率（需用“全概率公式”计算，即所有导致获胜的路径概率之和）。

(1) 计算 $P(A)$ ：前三次尝试内成功的概率 (Calculate $P(A)$: Probability of success within first three attempts)

- **英文推导：** $P(A)$ is the sum of probabilities of success on the 1st, 2nd, or 3rd attempt:

$$P(A) = P(\text{1st success}) + P(\text{2nd success}) + P(\text{3rd success}) = 0.4 + 0.24 + 0.144 = 0.784$$

- **中文推导：** $P(A)$ 为第 1、2、3 次尝试成功的概率之和：

$$P(A) = P(\text{第1次成功}) + P(\text{第2次成功}) + P(\text{第3次成功}) = 0.4 + 0.24 + 0.144 = 0.784$$

(3) 计算 $P(B)$ ：赢得整场比赛的总概率 (Calculate $P(B)$: Total probability of winning the contest)

- **英文推导：**

By the law of total probability, $P(B)$ is the sum of probabilities of "success in first three attempts and win second round" and "fail first three attempts and win second round":

$$P(B) = P(A \cap B) + P(\bar{A} \cap B) = P(B|A)P(A) + P(B|\bar{A})P(\bar{A})$$

Where:

- $P(\bar{A})$: Probability of failing first three attempts = $1 - P(A) = 1 - 0.784 = 0.216$;
- $P(B|\bar{A})$: Probability of winning given first three failures = 0.19 (low-probability game).

$$\text{Substitute values: } P(B) = 0.88 \times 0.784 + 0.19 \times 0.216 = 0.68992 + 0.04104 = 0.73096$$

- **中文推导：**

根据全概率公式， $P(B)$ 为“前三次成功且第二轮获胜”与“前三次失败且第二轮获胜”的概率之和：

$$P(B) = P(A \cap B) + P(\bar{A} \cap B) = P(B|A)P(A) + P(B|\bar{A})P(\bar{A})$$

其中：

- $P(\overline{A})$ ：前三次失败的概率 $= 1 - P(A) = 1 - 0.784 = 0.216$ ；
- $P(B|\overline{A})$ ：前三次失败时赢得比赛的概率 $= 0.19$ （低概率比赛）。

代入数值计算： $P(B) = 0.88 \times 0.784 + 0.19 \times 0.216 = 0.68992 + 0.04104 = 0.73096$

(4) 代入贝叶斯定理计算 $P(A|B)$ (Substitute into Bayes' theorem to calculate $P(A|B)$)

- 英文推导： $P(A|B) = \frac{P(B|A)P(A)}{P(B)} = \frac{0.88 \times 0.784}{0.73096} = \frac{0.68992}{0.73096} \approx 0.944$

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Section C**Mechanics**

Answer **all** questions in the spaces provided.

- 9 (a)** State the dimensions of force.

[1 mark]

length over time", so $a = \frac{\Delta v}{\Delta t} = \frac{\Delta L/\Delta t}{\Delta t} = LT^{-2}$.

Thus, dimensions of force: $F = M \times LT^{-2} = MLT^{-2}$.

Answer _____

- 9 (b)** State the dimensions of momentum.

[1 mark]

Momentum is defined as $p = mv$ (momentum = mass $\times$ velocity).

- Dimensions of mass (m): M (fundamental quantity).
- Dimensions of velocity (v): Velocity is "change in length over time", so $v = \frac{\Delta L}{\Delta t} = LT^{-1}$.

Thus, dimensions of momentum: $p = M \times LT^{-1} = MLT^{-1}$.

- 9 (c)** A student claims that:

the resultant force exerted on a body is equal to the rate of change of its momentum.

Determine if this claim is dimensionally consistent.

[2 marks]

"Rate of change of momentum" means "change in momentum over time", i.e., $\frac{\Delta p}{\Delta t}$.

- Dimensions of momentum (Δp): From part (b), $p = MLT^{-1}$, so $\Delta p = MLT^{-1}$ (change in a quantity has the same dimensions as the quantity itself).
- Dimensions of time (Δt): T (fundamental quantity).

Thus, dimensions of "rate of change of momentum": $\frac{\Delta p}{\Delta t} = \frac{MLT^{-1}}{T} = MLT^{-2}$.

Dimensions of resultant force: MLT^{-2} ;

Dimensions of rate of change of momentum: MLT^{-2} .

Since the dimensions on both sides are identical, the student's claim is dimensionally consistent.

Turn over ►



- 10** Two particles A and B are moving towards each other in a straight line on a smooth horizontal surface.

The particles then collide directly.

Particle A has mass 3 kg and particle B has mass 7 kg

Particle A is moving with a speed of 5 m s^{-1} before the collision.

Particle B is moving with a speed of 2 m s^{-1} before the collision.

The coefficient of restitution between the two particles is 0.1

- 10 (a)** Find the speed of A after the collision.

[5 marks]

(2) 牛顿碰撞定律 (Newton's Law of Restitution)

恢复系数 e 等于碰撞后分离速度 (relative speed after collision) 与碰撞前接近速度 (relative speed before collision) 的比值, 且分离速度方向与接近速度方向相反: $e = \frac{v_B - v_A}{u_A - u_B}$

The coefficient of restitution e is the ratio of the relative speed after collision to the relative speed before collision, with the relative speed after collision opposite in direction to that before collision: $e = \frac{v_B - v_A}{u_A - u_B}$

• **牛顿碰撞定律方程 (Newton's Law of Restitution Equation) :**

代入 $e = 0.1$ 、 $u_A = 5$ 、 $u_B = -2$: $0.1 = \frac{v_B - v_A}{5 - (-2)} = \frac{v_B - v_A}{7}$

两边同乘 7 : $v_B - v_A = 0.7$ (Equation 2)

$3 \times 5 + 7 \times (-2) = 3v_A + 7v_B$

$1 = 3v_A + 7v_B$

$v_A - v_B = -0.1(5 - (-2))$

$v_A - v_B = -0.7$

$v_A = -0.39$

Speed of $A = 0.39 \text{ [m s}^{-1}\text{]}$

Answer



- 10 (b)** Find the magnitude of the impulse exerted on *A* by *B* during the collision.

Give your answer to three significant figures.

[2 marks]

$$I = 3 \times (-0.39) - 3 \times 5$$

$$= -16.17$$

$$\text{Magnitude} = 16.2 \text{ [Ns]}$$

Answer _____

- 10 (c)** The particles are in contact for 0.05 seconds during the collision.

Assume that the magnitude of the force that each particle exerts on the other during the collision is constant.

Find the magnitude of the force exerted on *A* by *B* during the collision.

Give your answer to three significant figures.

[2 marks]

$$16.17 = F \times 0.05$$

$$F = 323 \text{ [N]}$$

M1

Uses $I = Ft$

A1

Correct value given to 3 s.f.
Allow 324 [N]

Answer _____

Turn over for the next question

Turn over ►



- 11 Two boats P and Q are moving on a lake.

Initially Q is 1200 metres north of P

Boat Q moves on a bearing of 100° at a constant speed of 3 m s^{-1} relative to the water.

Boat P moves on a bearing of θ° at a constant speed of $V \text{ m s}^{-1}$ relative to the water.

The boats collide 6 minutes later.

Model the boats as particles.

- 11 (a) Find the velocity of P relative to Q

Give your answer in terms of V and θ

[5 marks]

两船 P 、 Q 在湖面运动 (move on a lake)，初始时 Q 在 P 的正北方 1200 米处 (initially Q is 1200 metres north of P)。船 Q 沿方位角 (bearing) 100° 方向以恒定速度 3 m/s 运动；船 P 沿方位角 θ° 方向以恒定速度 $V \text{ m/s}$ 运动，两船在 6 分钟后碰撞 (collide 6 minutes later)。本题围绕相对运动 (Relative Motion) 与位移关系展开，需通过建立坐标系、分解速度与位移，结合碰撞条件求解 P 的速度大小 V 与运动方位角 θ 。

$$\mathbf{v}_{QP} = \begin{bmatrix} V \sin \theta - 3 \cos 10^\circ \\ V \cos \theta + 3 \sin 10^\circ \end{bmatrix}$$

Answer _____

- 11 (b) Hence find the value of V and the value of θ

[5 marks]

$$360(V \sin \theta - 3 \cos 10^\circ) = 0$$

$$360(V \cos \theta + 3 \sin 10^\circ) = 1200$$

$$V \sin \theta = 3 \cos 10^\circ$$

$$V \cos \theta = \frac{1200}{360} - 3 \sin 10^\circ$$

$$V = \sqrt{(3 \cos 10^\circ)^2 + \left(\frac{1200}{360} - 3 \sin 10^\circ\right)^2} = 4.1$$

$$\tan \theta = \frac{3 \cos 10^\circ}{\frac{1200}{360} - 3 \sin 10^\circ}$$

$$\theta = 46$$

M1

Obtains one correct expression in terms of $V \sin \theta$ or $V \cos \theta$ for their

$\mathbf{v}_{QP}$ in the form $\begin{bmatrix} \pm V \sin \theta \pm A \\ \pm V \cos \theta \pm B \end{bmatrix}$ or

$$\begin{bmatrix} \pm V \cos \theta \pm B \\ \pm V \sin \theta \pm A \end{bmatrix}$$

A1

Obtains correct expressions for $V \sin \theta$ and $V \cos \theta$ oe

AWRT 2.95 for $3 \cos 10^\circ$

AWRT 2.81 for $\frac{1200}{360} - 3 \sin 10^\circ$

PI

m1 A1

m1: Eliminates θ or V

A1: Correct $V =$ AWRT 4.1

A1

Correct $\theta =$ AWRT 46

(1) 分解船Q的速度 $\vec{v}_Q$ (Decompose $\vec{v}_Q$ into x - and y -components)

英文推导：

Boat Q moves on a bearing of 100° . The angle between $\vec{v}_Q$ and the positive y -axis (North) is 100° , so the angle with the positive x -axis (East) is $90^\circ - 100^\circ = -10^\circ$ (or 350° , equivalent for trigonometric calculations).

Velocity components of Q :

- x -component (East direction): $v_{Qx} = 3 \sin 100^\circ$ (since $\sin \theta = \text{opposite/hypotenuse}$ for x -component relative to y -axis);
- y -component (North direction): $v_{Qy} = 3 \cos 100^\circ$ (since $\cos \theta = \text{adjacent/hypotenuse}$ for y -component relative to y -axis).

(Note: $\sin 100^\circ = \sin(90^\circ + 10^\circ) = \cos 10^\circ$, $\cos 100^\circ = \cos(90^\circ + 10^\circ) = -\sin 10^\circ$, which confirms the direction: Q moves East-Northwest, so v_{Qy} is negative.)

船 Q 沿方位角 100° 运动，其速度与 y -轴正方向（正北）的夹角为 100° ，因此与 x -轴正方向（正东）的夹角为 $90^\circ - 100^\circ = -10^\circ$ （或 350° ，三角函数计算等效）。

船 Q 的速度分量：

- x -分量（正东方向）： $v_{Qx} = 3 \sin 100^\circ$ （因相对于 y -轴， x -分量为“对边”，用正弦函数）；
- y -分量（正北方向）： $v_{Qy} = 3 \cos 100^\circ$ （因相对于 y -轴， y -分量为“邻边”，用余弦函数）。

（注： $\sin 100^\circ = \sin(90^\circ + 10^\circ) = \cos 10^\circ$ ， $\cos 100^\circ = \cos(90^\circ + 10^\circ) = -\sin 10^\circ$ ，可验证方向： Q 向东南方向附近运动，故 y -分量为负）。

(2) 分解船P的速度 $\vec{v}_P$ (Decompose $\vec{v}_P$ into x - and y -components)

英文推导：

Boat P moves on a bearing of θ° . The angle between $\vec{v}_P$ and the positive y -axis is θ° , so:

Velocity components of P :

- x -component (East direction): $v_{Px} = V \sin \theta$;
- y -component (North direction): $v_{Py} = V \cos \theta$.

(3) 计算相对速度 $\vec{v}_{PQ}$ (Calculate Relative Velocity $\vec{v}_{PQ}$)

英文推导：

By $\vec{v}_{PQ} = \vec{v}_P - \vec{v}_Q$, decompose into components:

- x -component of $\vec{v}_{PQ}$: $v_{PQx} = v_{Px} - v_{Qx} = V \sin \theta - 3 \sin 100^\circ$;
- y -component of $\vec{v}_{PQ}$: $v_{PQy} = v_{Py} - v_{Qy} = V \cos \theta - 3 \cos 100^\circ$.

Thus, the velocity of P relative to Q is $\vec{v}_{PQ} = (V \sin \theta - 3 \sin 100^\circ)\vec{i} + (V \cos \theta - 3 \cos 100^\circ)\vec{j}$

(where $\vec{i}$ is unit vector East, $\vec{j}$ is unit vector North).

中文推导：

根据 $\vec{v}_{PQ} = \vec{v}_P - \vec{v}_Q$ ，分解为分量形式：

- $\vec{v}_{PQ}$ 的 x -分量： $v_{PQx} = v_{Px} - v_{Qx} = V \sin \theta - 3 \sin 100^\circ$;
- $\vec{v}_{PQ}$ 的 y -分量： $v_{PQy} = v_{Py} - v_{Qy} = V \cos \theta - 3 \cos 100^\circ$.

• 初始位置 (Initial Positions) :

- 船 P : $S_{P0} = (0, 0)$;
- 船 Q : 初始在 P 正北 1200 米 , 故 $S_{Q0} = (0, 1200)$ 。

• 360 秒后的位移 (Displacements After 360 Seconds) :

- 船 P 的位移 : $\vec{s}_P = \vec{v}_P \times t = (V \sin \theta \times 360, V \cos \theta \times 360)$;
- 船 Q 的位移 : $\vec{s}_Q = \vec{v}_Q \times t = (3 \sin 100^\circ \times 360, 3 \cos 100^\circ \times 360)$ 。

• 360 秒后的位置 (Positions After 360 Seconds) :

- 船 P 的位置 : $S_P = S_{P0} + \vec{s}_P = (360V \sin \theta, 360V \cos \theta)$;
- 船 Q 的位置 : $S_Q = S_{Q0} + \vec{s}_Q = (360 \times 3 \sin 100^\circ, 1200 + 360 \times 3 \cos 100^\circ)$ 。

(2) 建立碰撞的位移平衡方程 (Establish Displacement Balance Equations for Collision)

碰撞时 $S_P = S_Q$, 因此 x - 分量和 y - 分量分别相等 :

$$\begin{cases} 360V \sin \theta = 360 \times 3 \sin 100^\circ & \text{(Equation 1, } x\text{-component balance)} \\ 360V \cos \theta = 1200 + 360 \times 3 \cos 100^\circ & \text{(Equation 2, } y\text{-component balance)} \end{cases}$$

$$V = \quad \quad \quad \theta = \quad \quad \quad$$

END OF QUESTIONS

