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(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)	$1.2\times1.8-(0.4-p)\times(0.4+p)$ $2+p^2$	**M1** **A1**	Correct expression for determinant **CAO**
		2	

Q	Answer	Marks	Comments
1(b)(i)	Shear parallel to y-axis	**B1**	Allow vertical shear, shear in direction of y-axis, shear with y-axis invariant, shear in/on/with y-axis **oe** Ignore any additional descriptions
		1	

Q	Answer	Marks	Comments
1(b)(ii)	$\begin{bmatrix}1 & 0\\ k & 1\end{bmatrix}\begin{bmatrix}3\\ 2\end{bmatrix}=\begin{bmatrix}3\\ -4\end{bmatrix}$ $\Rightarrow 3k+2=-4$ $k=-2$	**M1** **A1**	Uses correct matrix equation to obtain an equation in terms of k **CAO**
		2	

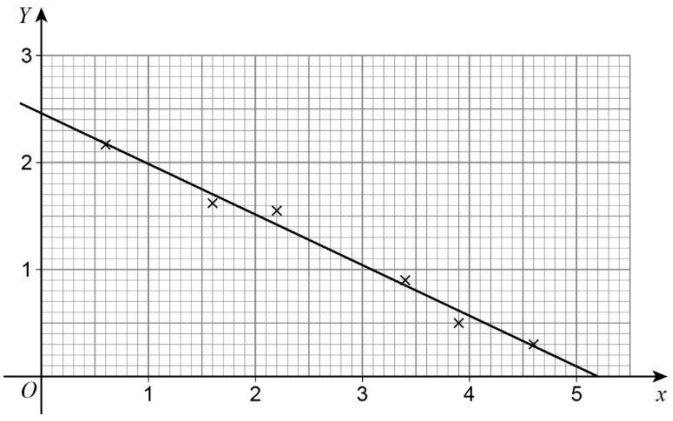
Q	Answer	Marks	Comments
1(c)	$\mathbf{C}=\begin{bmatrix}1.2 & 0.4-p\\ 0.4+p & 1.8\end{bmatrix}\begin{bmatrix}1 & 0\\ -2 & 1\end{bmatrix}$ $=\begin{bmatrix}0.4+2p & 0.4-p\\ p-3.2 & 1.8\end{bmatrix}$	**M1** **m1** **A1**	**A** × **B** with **B** in terms of k or using their k obtains a 2 × 2 matrix with at least 2 correct terms **ft B** in terms of k or using their k Terms may be unsimplified **CAO**
		3	

	Question 1 Total	**8**	

Q	Answer	Marks	Comments
2	$hf(2, 5) = 0.05 \times \frac{1}{2^2 - \sqrt{5}}$	**M1**	Correct substitution into RHS of this expression **PI**
	$= 0.02834576...$	**A1**	**AWRT** 0.028 **PI**
	$y_2 = 5 + 0.02834576... = 5.02834576...$	**M1**	5 + their value of $hf(2, 5)$ **PI**
	$y_3 = 5.02834576 + 0.05 \times \frac{1}{2^{2.05} - \sqrt{5.02834576}}$	**M1**	Correct substitution using $x_2 = 2.05$ and their y_2 into second term **PI** by sight of **AWRT** 5.054 or 5.055
	$[= 5.05468009]$		
	5.055	**A1**	**CAO**

	Question 2 Total	**5**	

Q	Answer	Marks	Comments
3(a)	$\frac{y}{x} = Ax + B$	M1	Divides through by x
	which is a linear relationship [between x and $\frac{y}{x}$]	A1	Obtains correct relationship and gives conclusion
		2	

Q	Answer	Marks	Comments
3(b)(i)		B1	Straight line of best-fit drawn following general trend of points Number of points above the line must be no more than 1 more than number of points below the line and vice versa
			
		1	

Q	Answer	Marks	Comments
3(b)(ii)	gradient = –0.47 y-intercept = 2.46	B1	Gradient or intercept seen for their straight line of best fit Half a square tolerance for identified points
	$A = -0.47$ $B = 2.46$	B1	**AWFW** $[-0.5, -0.44]$ for A **AWFW** $[2.38, 2.55]$ for B Note an award of **B0 B1** is possible
		2	

Q	Answer	Marks	Comments
3(c)	$1 = -0.47x^2 + 2.46x$	**M1**	Correct substitution of their values and $y = 1$ into original relationship
	$x = 4.7898$ or $x = 0.4442$	**m1**	**PI** Solves resulting quadratic
	$x = 4.8$ or $x = 0.44$	**A1ft**	**ft** their A and B Answers must be given to two significant figures and not rejected
		3	

	Question 3 Total	**8**	

Q	Answer	Marks	Comments
4(a)(i)	$x_1 = \dfrac{3\times -3 - 2\times 3}{-3-3} = 2.5$	**M1**	Applies linear interpolation once, must see the calculation of x_1 or equation of line $y = 6x - 15$ or evaluates both $\mathrm{f}(2) = -3$ and $\mathrm{f}(3) = 3$ which must be seen in part **(a)(i)**
	Linear interpolation also gives the mid point	**A1**	Correct conclusion **oe** eg $\lvert\mathrm{f}(2)\rvert = \lvert\mathrm{f}(3)\rvert$ so linear interpolation divides the interval in two [as does interval bisection]
		2	

Q	Answer	Marks	Comments
4(a)(ii)	$\mathrm{f}\left(\dfrac{5}{2}\right) = -2.75$ and $\mathrm{f}(3) = 3$	**M1**	Correct evaluation of $\mathrm{f}\left(\dfrac{5}{2}\right)$ and $\mathrm{f}(3)$ One or both may be seen in part **(a)(i)**
	Change of sign, so $\dfrac{5}{2} < \alpha < 3$	**A1**	Correct conclusion Must state or show that that there is a change of sign and concludes that α is present in the interval Reference to $\mathrm{f}\left(\dfrac{5}{2}\right)$ and $\mathrm{f}(3)$ must seen but not necessarily evaluated in part **(a)(ii)**
		2	

Q	Answer	Marks	Comments
4(b)	$\dfrac{x_1-\dfrac{5}{2}}{3-x_1}=\dfrac{2.75}{3}$	M1	Obtains correct equation in terms of x_1 or obtains equation of linear interpolation line $y=\dfrac{23}{2}x-\dfrac{63}{2}$ or obtains $\dfrac{p}{0.5-p}=\dfrac{2.75}{3}$ **oe** Condone use of $\mathrm{f}\left(\dfrac{5}{2}\right)$ and $\mathrm{f}(3)$ instead of values **ft** their values from **(a)**
	$3\left(x_1-\dfrac{5}{2}\right)=2.75(3-x_1)$ $3x_1-\dfrac{15}{2}=\dfrac{33}{4}-2.75x_1$ $5.75x_1=\dfrac{63}{4}$	m1	Obtains correct linear equation in terms of x_1 **oe** Correct calculation of x_1 scores **M1 m1,** eg $x_1=\dfrac{5}{2}+\dfrac{1}{2}\times\dfrac{2.75}{2.75+3}$, $x_1=\dfrac{2.5\times3-3\times-2.75}{3+2.75}$, $x_1=\dfrac{5}{2}+\dfrac{11}{46}$ Condone use of $\mathrm{f}\left(\dfrac{5}{2}\right)$ and $\mathrm{f}(3)$ instead of values **ft** their values from **(a)**
	$x_1=\dfrac{63}{23}$	A1	**AG** Must be convincingly shown $\mathrm{f}\left(\dfrac{5}{2}\right)$ and $\mathrm{f}(3)$ must be evaluated Correct linear equation in terms of x_1 or correct calculation of x_1 must be seen
	$\left[\mathrm{f}(x_1)\right]=-0.778$	B1	**AWRT** –0.8 **Allow** truncated –0.7
	$\mathrm{f}(x_1)<0$ so $\dfrac{63}{23}<\alpha<3$	A1	**AG** Must be convincingly shown Note an award of **M1 m1 A0 B1 A1** is possible
		5	

	Question 4 Total	**9**	

Q	Answer	Marks	Comments
5(a)	$\mathbf{M}^2 = \frac{1}{25}\begin{bmatrix} 2k-3 & 4-k \\ 4k+4 & 3-2k \end{bmatrix}\begin{bmatrix} 2k-3 & 4-k \\ 4k+4 & 3-2k \end{bmatrix}$ $= \frac{1}{25}\begin{bmatrix} (2k-3)^2+(4-k)(4k+4) & (4-k)(2k-3)+(3-2k)(4-k) \\ (4k+4)(2k-3)+(3-2k)(4k+4) & (4-k)(4k+4)+(3-2k)^2 \end{bmatrix}$ $= \frac{1}{25}\begin{bmatrix} 25 & 0 \\ 0 & 25 \end{bmatrix}$ $= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	**M1** **M1** **A1**	For $\frac{1}{25}\times$ a 2 × 2 matrix **oe** with at least one correct algebraic entry **PI** For one correctly evaluated entry **CAO**
		3	

Q	Answer	Marks	Comments
5(b)	$\frac{1}{5}\begin{bmatrix} 2k-3 & 4-k \\ 4k+4 & 3-2k \end{bmatrix}\begin{bmatrix} x \\ 2x \end{bmatrix}$ $= \frac{1}{5}\begin{bmatrix} (2k-3)x+(4-k)\times 2x \\ (4k+4)x+(3-2k)\times 2x \end{bmatrix}$ $= \frac{1}{5}\begin{bmatrix} 5x \\ 10x \end{bmatrix}$ $= \begin{bmatrix} x \\ 2x \end{bmatrix}$ So $y = 2x$ is a line of invariant points	**M1** **A1** **A1**	Applies **M** correctly to a general point on the line $y = 2x$ Obtains $\begin{bmatrix} x \\ 2x \end{bmatrix}$ with an intermediate line and no errors seen Correct conclusion
		3	

Q	Answer	Marks	Comments
5(c)(i)	$k = 0$ gives $\mathbf{M} = \frac{1}{5}\begin{bmatrix} -3 & 4 \\ 4 & 3 \end{bmatrix}$	**M1**	Substitutes $k = 0$
	Reflection	**A1**	
	[with mirror line] $y = 2x$	**A1**	Condone $y = (\tan\theta)x$ where θ = **AWRT** 63.4° or 1.11 radians or $\tan\theta$ = **AWRT** 2.0
		3	

Q	Answer	Marks	Comments
5(c)(ii)	$a = -\frac{1}{2}$	**B1**	
		1	

	Question 5 Total	**10**	

Q	Answer	Marks	Comments
6(a)	[Discrete] uniform [distribution]	**B1**	Must see uniform
		1	

Q	Answer	Marks	Comments
6(b)	$\frac{5}{11}$	**B1**	**oe**
		1	

Q	Answer	Marks	Comments
6(c)	$\mathrm{E}(X)=\frac{n+1}{2}$ $\mathrm{E}(X)=\frac{11+1}{2}=6$	**B1**	Obtains $\mathrm{E}(X)=\frac{11+1}{2}$ **oe** **PI**
	$\mathrm{Var}(X)=\frac{n^2-1}{12}$ $\mathrm{Var}(X)=\frac{11^2-1}{12}=10$	**B1**	Obtains $\mathrm{Var}(X)=\frac{11^2-1}{12}$ **oe** **PI**
	$6a+b=13$ $10a^2=90$	**M1**	One correct equation for their mean and variance
	$a=3$	**A1**	**CAO**
	$b=-5$	**A1**	**CAO**
		5	

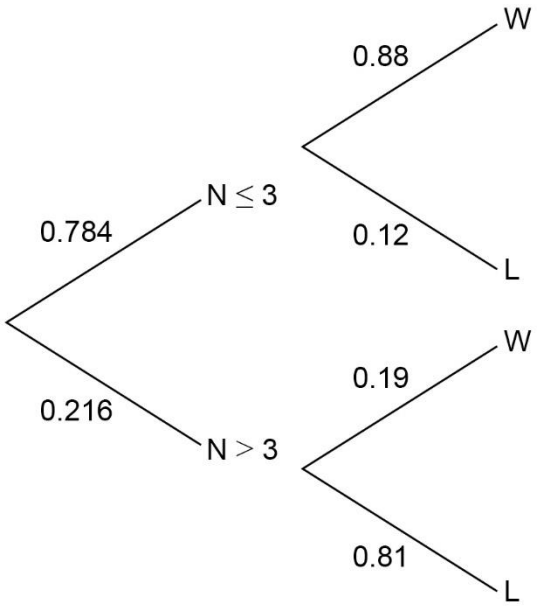
	Question 6 Total	**7**	

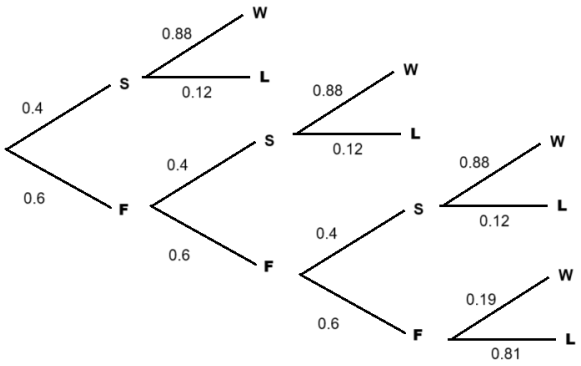
Q	Answer	Marks	Comments
7(a)	$1-0.318-0.512=0.17$	**B1**	**AG** Must be convincingly shown
		1	

Q	Answer	Marks	Comments
7(b)	$G_Y'(t)=0.512+0.17bt^{b-1}$	**M1**	Differentiates $G_Y(t)$ correctly May be in terms of a Condone $G_Y'(1)=0.512+0.17b\times1^{b-1}$ **oe**
	$1.872=0.512+0.17b$	**M1**	Forms correct equation to find b **oe** **PI**
	$b=8$	**A1**	Must see $G_Y'(t)=0.512+0.17bt^{b-1}$ **oe**
		3	

Q	Answer	Marks	Comments
7(c)	$G_Y''(t)=9.52t^6$	**M1**	Differentiates their $G_Y'(t)$ correctly May be in terms of a and b **oe**
	$\text{Var}(Y)=9.52+1.872-1.872^2$	**m1**	Applies correct variance formula with their $G_Y'(1)$ and $G_Y''(1)$
	$\text{Var}(Y)=7.887616$	**A1ft**	**AWRT** 7.89 **ft** their $0.17b(b-1)-1.632384$
		3	

	Question 7 Total	**7**	

Q	Answer	Marks	Comments
8(a)	$1-(1-0.4)^3$ or $(1-0.4)^3$	**M1**	Correct calculation of P(ball in hole ≤ 3) or P(ball in hole > 3) **PI** By sight of 0.784 or 0.216 oe
		**M1**	Correct structure **oe** eg N ≤ 3 branch split into three branches (probabilities 0.4, 0.24 and 0.144) Allow any sensible notation
		A1	Fully correct **oe**
		3	

Q	Answer	Marks	Comments
8(a) **ALT**		**M1**	0.4, 0.6, 0.88, 0.12, 0.19 and 0.81 all seen **oe**
		M1	Correct structure **oe** Allow any sensible notation
		A1	Fully correct **oe**
		3	

Q	Answer	Marks	Comments
8(b)	$\dfrac{0.784\times0.88}{0.784\times0.88+0.216\times0.19}$	M1	Correct calculation for the numerator **PI** by sight of 0.68992 **ft** their 0.784
		M1	Correct calculation for the denominator **PI** by sight of 0.73096 **ft** their 0.784 and 0.216 provided that they sum to 1
	$= 0.944$	A1	**AWRT** 0.944 or $\dfrac{8624}{9137}$ **oe**
		3	

	Question 8 Total	**6**	

Q	Answer	Marks	Comments
9(a)	MLT^{-2}	**B1**	Correct dimensions
		1	

Q	Answer	Marks	Comments
9(b)	MLT^{-1}	**B1**	Correct dimensions
		1	

Q	Answer	Marks	Comments
9(c)	[Rate of change of momentum] = $\frac{MLT^{-1}}{T}$ $= MLT^{-2}$ = [Force] So statement is dimensionally consistent	**M1** **A1**	Correct dimensions of rate of change of momentum **ft** their dimensions of momentum Condone use of units for this mark only Concludes argument with $\frac{MLT^{-1}}{T}$ or $\frac{ML}{T\times T}$ or $MLT^{-1}T^{-1}$ seen **CSO**
		2	

	Question 9 Total	**4**	

Q	Answer	Marks	Comments
10(a)	$3\times5+7\times(-2)=3v_A+7v_B$	**M1**	Uses $\pm3\times5\pm7\times2=\pm3v_A\pm7v_B$
	$1=3v_A+7v_B$	**A1**	Correct equation
	$v_A-v_B=-0.1\big(5-(-2)\big)$	**M1**	Uses $\pm v_A\pm v_B=\pm0.1\big(5\pm2\big)$
	$v_A-v_B=-0.7$	**A1**	Correct equation
	$v_A=-0.39$ Speed of A = 0.39 $\left[\text{m s}^{-1}\right]$	**A1**	Correct speed If units are seen they must be correct
		5	

Q	Answer	Marks	Comments
10(b)	$I=3\times\big(-0.39\big)-3\times5$ $=-16.17$	**M1**	Uses $3\times\big(\pm\text{their } v_A\pm5\big)$ **oe**
	Magnitude = 16.2 $\left[\text{Ns}\right]$	**A1**	Correct magnitude given to 3 s.f. If units are seen they must be correct
		2	

Q	Answer	Marks	Comments
10(c)	$16.17=F\times0.05$	**M1**	Uses $I=Ft$
	$F=323$ $\left[\text{N}\right]$	**A1**	Correct value given to 3 s.f. Allow 324 [N]
		2	

	Question 10 Total	**9**	

Q	Answer	Marks	Comments
11(a)	$\mathbf{v}_{QP} = \begin{bmatrix} V\sin\theta - 3\cos10^\circ \\ V\cos\theta + 3\sin10^\circ \end{bmatrix}$	**M1** **A1**	Finds relative velocity, with at least one correct component of $\mathbf{v}_{PQ}$ or $\mathbf{v}_{QP}$ or obtains $\begin{bmatrix} V\cos\theta + 3\sin10^\circ \\ V\sin\theta - 3\cos10^\circ \end{bmatrix}$ **oe** Condone as part of an equation to find the magnitude of relative velocity Correct relative velocity **oe** eg $\begin{bmatrix} V\sin\theta - 3\sin100^\circ \\ V\cos\theta - 3\cos100^\circ \end{bmatrix}$ **AWRT** 2.95 for $3\cos10^\circ$ **AWRT** 0.52 for $3\sin10^\circ$
		2	

Q	Answer	Marks	Comments
11(b)	$360\left(V\sin\theta - 3\cos10^\circ\right) = 0$ $360\left(V\cos\theta + 3\sin10^\circ\right) = 1200$	**M1**	Obtains one correct expression in terms of $V\sin\theta$ or $V\cos\theta$ for their $\mathbf{v}_{QP}$ in the form $\begin{bmatrix} \pm V\sin\theta \pm A \\ \pm V\cos\theta \pm B \end{bmatrix}$ or $\begin{bmatrix} \pm V\cos\theta \pm B \\ \pm V\sin\theta \pm A \end{bmatrix}$
	$V\sin\theta = 3\cos10^\circ$ $V\cos\theta = \frac{1200}{360} - 3\sin10^\circ$	**A1**	Obtains correct expressions for $V\sin\theta$ and $V\cos\theta$ **oe** **AWRT** 2.95 for $3\cos10^\circ$ **AWRT** 2.81 for $\frac{1200}{360} - 3\sin10^\circ$ **PI**
	$V = \sqrt{\left(3\cos10^\circ\right)^2 + \left(\frac{1200}{360} - 3\sin10^\circ\right)^2} = 4.1$	**m1 A1**	**m1**: Eliminates θ or V **A1**: Correct V = **AWRT** 4.1
	$\tan\theta = \frac{3\cos10^\circ}{\frac{1200}{360} - 3\sin10^\circ}$ $\theta = 46$	**A1**	Correct θ = **AWRT** 46
		5	

Q	Answer	Marks	Comments
11(b) ALT 1	$1200^2+(6\times60\times3)^2-2\times1200\times6\times60\times3\cos(180-100)$	**M1**	Applies cosine rule correctly to obtain a calculation for (distance *P* travels)^2
	Distance *P* travels = 1468[.435877]	**A1**	Obtains distance *P* travels **AWRT** 1468 **PI**
	$\frac{\sin\theta}{6\times60\times3}=\frac{\sin(180-100)}{1468.435877}$	**m1**	Applies sine rule correctly or divides distance *P* travels by 360
	$\theta=46$	**A1**	Correct θ = **AWRT** 46
	$V=4.1$	**A1**	Correct V = **AWRT** 4.1
		5	

Q	Answer	Marks	Comments
11(b) ALT 2	$V^2=3^2+\left(\frac{1200}{6\times60}\right)^2-2\times3\times\left(\frac{1200}{6\times60}\right)\cos(180-100)$	**M1**	Applies cosine rule correctly to obtain a calculation for V^2
	$V^2=16.6[3814756]$	**A1**	Obtains correct V^2 **AWRT** 16.6 **PI** by correct V
	$V=4.1$	**A1**	Correct V = **AWRT** 4.1
	$\frac{\sin\theta}{3}=\frac{\sin(180-100)}{\sqrt{16.63814756}}$	**m1**	Applies sine rule correctly
	$\theta=46$	**A1**	Correct θ = **AWRT** 46
		5	

	Question 11 Total	**7**	