

Please write clearly in block capitals.

Centre number

Candidate number

Surname

Forename(s)

Candidate signature

I declare this is my own work.

INTERNATIONAL A-LEVEL

FURTHER MATHEMATICS

(9665/FM03) Unit FP2 Pure Mathematics

Thursday 29 May 2025

07:00 GMT

Time allowed: 2 hours 30 minutes

Materials

- For this paper you must have the OxfordAQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 120.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
3	
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5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
TOTAL	



1 The position vectors of the points A , B and C are

Calculate the area of the triangle ABC

Give your answer in an exact form.

[4 marks]

[illegible]

Answer _____

4



2 The transformation S is represented by the matrix $\mathbf{M}$

The transformation T is represented by the matrix $\mathbf{N}$

A three-dimensional shape P is mapped onto the three-dimensional shape Q by a sequence of two transformations:

- a transformation S

followed by

- a transformation T

2 (a) Write down, in terms of $\mathbf{M}$ and $\mathbf{N}$ the matrix product that maps P onto Q

[1 mark]

Answer _____

2 (b) It is given that:

$$\det \mathbf{N} = 2$$

the volume of the three-dimensional shape P is 9 cubic units

the volume of the three-dimensional shape Q is 7 cubic units.

Find the possible values of $\det \mathbf{M}$

[3 marks]

Answer _____



3

$$z^3 + pz^2 + 65z + q = 0$$

are α , β and γ where p and q are real.

It is given that $\alpha = 4 - 3i$

3 (a)

[3 marks]

[illegible]

$$\beta = \quad \quad \quad \gamma =$$



[2 marks]

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$p =$ _____ $q =$ _____

5



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outside the
box*

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ANSWER IN THE SPACES PROVIDED**



[5 marks]

[illegible]

Answer _____

5

- 5 (a) Express $\frac{1}{2}(k+1)(4(k+1)^2 + 7(k+1) + 1)$ in the form

$$\frac{1}{2}(4k^3 + bk^2 + ck + d)$$

where b , c and d are integers.

[2 marks]

Answer _____

- 5 (b) Prove by induction that for all integers $n \geq 1$

$$2 \times 3 + 5 \times 5 + 8 \times 7 + 11 \times 9 + \dots + (3n-1) \times (2n+1) = \frac{1}{2}n(4n^2 + 7n + 1)$$

[6 marks]



[illegible]

8

6

$$x = \frac{t^3}{3} - \frac{t^2}{2} \quad \text{and} \quad y = \frac{4t^{\frac{5}{2}}}{5} \quad \text{for } 0 \leq t \leq 3$$

The point P on C is where $t = 0$

The point Q on C is where $t = \sqrt{2}$

Find the length of the arc PQ of the curve C

Give your answer in the form $\frac{a\sqrt{2}}{b} + c$ where a , b and c are integers.

[6 marks]

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This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins or other markings on the paper.

Answer _____

6

Turn over ►



[3 marks]

[illegible]

[1 mark]

[1 mark]



- 8 (a) Express $\frac{4r^2 + 8r + 2}{r(r+1)(r+2)}$ in partial fractions.

[2 marks]

Answer _____

- 8 (b) Use your answer to **part (a)** with the method of differences to show that

$$\sum_{r=1}^{2n} \frac{(-1)^r (4r^2 + 8r + 2)}{r(r+1)(r+2)} = \frac{4n+3}{(2n+1)(2n+2)} - \frac{3}{2}$$

[4 marks]



[illegible]
$$\sum_{r=1}^{\infty} \frac{(-1)^r (4r^2 + 8r + 2)}{r(r+1)(r+2)}$$

Answer

7

9 (a) (i) Find $\frac{d^6 y}{dx^6}$ in terms of x

9 (a) (ii) Hence show that the first four non-zero terms in ascending powers of x in the Maclaurin series expansion of $\cosh 2x$ are

$$1 + 2x^2 + \frac{2}{3}x^4 + \frac{4}{45}x^6$$

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$$\lim_{x \rightarrow 0} \left[\frac{\cosh 2x - \cos 2x}{(\ln(1+x))^2} \right]$$

[6 marks]

[illegible]

Answer _____

10

Turn over ►



10 (a)

$$\frac{dy}{dx} - \frac{y}{x} = \frac{x}{\sqrt{1-x^2}} \sin^{-1} x$$

Give your answer in the form $y = f(x)$

[7 marks]

[illegible]

Give your answer in the form $y = f(x)$

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$$y = \underline{\hspace{10cm}}$$

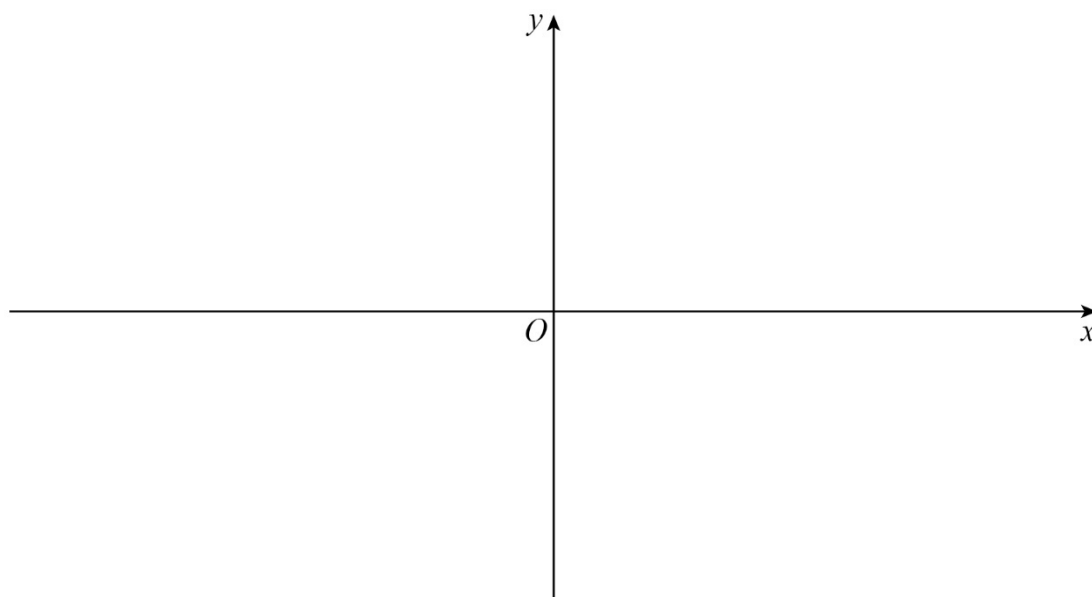
9

Turn over ►



11 (a) (i) On the axes below sketch the graph of $y = \cosh^{-1}(2x)$

[2 marks]



11 (a) (ii) By writing $y = \cosh^{-1}(2x)$ as $\cosh y = 2x$, prove that

$$\frac{dy}{dx} = \frac{2}{\sqrt{4x^2 - 1}}$$

[4 marks]



$$f(x) = 4 - x^2 + \cosh^{-1}(2x)$$

Express the x -coordinate of P in exact form.

[6 marks]

[illegible]
$$x =$$


12 (a) The points $A(1, 1, 2)$, $B(2, 3, 6)$ and $C(-1, -2, -1)$ lie in the plane Π_1

Find the equation of the plane Π_1

Give your answer in the form $\mathbf{r} = \mathbf{w} + \lambda\mathbf{p} + \mu\mathbf{q}$ where $\mathbf{w}$, $\mathbf{p}$ and $\mathbf{q}$ are column vectors.

[3 marks]

Answer _____

12 (b) The plane Π_2 has equation $2x - y - 3z = 7$

The line L is the line of intersection of planes Π_1 and Π_2

Find the equation of L

Give your answer in the form $(\mathbf{r} - \mathbf{u}) \times \mathbf{v} = \mathbf{0}$ where $\mathbf{u}$ and $\mathbf{v}$ are column vectors.

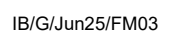
[8 marks]



[illegible]

Answer _____

Turn over ►



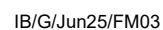
Find the solution of the differential equation

given that $y \rightarrow -1$ as $x \rightarrow \infty$ and that $y = 3$ when $x = 0$

Give your answer in the form $y = f(x)$

Clearly show any limiting processes used.

[11 marks]

[illegible]

11

14

$$\mathbf{M} = \frac{1}{5} \begin{bmatrix} a & 0 & -2 \\ 0 & 5 & 0 \\ -2 & 0 & 11 \end{bmatrix}$$

where a is a real number.

The eigenvectors of $\mathbf{M}$ are $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$

The corresponding eigenvalues of $\mathbf{M}$ are λ_1 , λ_2 and λ_3

14 (a) (i)

Find the value of a

[2 marks]

[illegible]

$$a =$$



[4 marks]

[illegible]
$$b = \qquad \qquad \qquad \lambda_2 =$$

Turn over ►



14 (a) (iii) It is given that $\mathbf{v}_3 =$

[1 mark]

$$\lambda_3 =$$

[4 marks]



14 (b) (ii) Write down the Cartesian equation of the line of invariant points of the transformation represented by **M**

[1 mark]

Answer _____

12

Turn over for the next question

Turn over ►



15

$$r^2 = \frac{64}{7 + 6\sin^2\theta + 6\sqrt{3}\sin\theta\cos\theta}$$

15 (a)

[3 marks]

[illegible]

Answer

15 (b)

Find the Cartesian equation of the curve D in an exact form.

[5 marks]



8

END OF QUESTIONS

