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FM03

(9665/FM03) Unit FP2 Pure Mathematics

Mark scheme

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
√ **or ft**	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–*x* EE	Deduct *x* marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1	$\overrightarrow{AB} = \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix}$, $\overrightarrow{AC} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$	**B1**	Two correct directional vectors $\overrightarrow{BC} = \begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}$
	$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix} \times \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 7 \end{bmatrix}$	**M1**	Cross product of their directional vectors – not position vectors $\overrightarrow{BA} \times \overrightarrow{BC} = \begin{bmatrix} 3 \\ -1 \\ -7 \end{bmatrix}$ $\overrightarrow{CA} \times \overrightarrow{CB} = \begin{bmatrix} -3 \\ 1 \\ 7 \end{bmatrix}$
	$\text{Area} = \frac{1}{2}\left\|\begin{bmatrix} -3 \\ 1 \\ 7 \end{bmatrix}\right\|$	**M1**	Uses correct formula for area of a triangle with their cross product
	$= \frac{\sqrt{59}}{2}$	**A1**	Correct exact answer

	Question 1 Total	**4**	

Q	Answer	Marks	Comments
2(a)	**NM**	**B1**	
		1	

Q	Answer	Marks	Comments
2(b)	$\det(\mathbf{NM}) \times 9 = \pm 7$	**M1**	Use of $\det(\mathbf{NM}) \times \text{Volume}\, P = \text{Volume}\, Q$ Condone $\det(\mathbf{MN})$ and lack of $\pm$ here
	$2 \times \det(\mathbf{M}) \times 9 = \pm 7$	**M1**	Use of $\det(\mathbf{NM}) = \det(\mathbf{N}) \times \det(\mathbf{M})$ Condone use of $\det(\mathbf{MN}) = \det(\mathbf{M}) \times \det(\mathbf{N})$ and lack of $\pm$ here
	$\det(\mathbf{M}) = \pm\frac{7}{18}$	**A1**	Must have **NM** in **part (a)** or clearly shown use of **NM** in **part (b)** condone $\pm$ appearing at the end
		3	

	Question 2 Total	**4**	

Q	Answer	Marks	Comments
3(a)	$\beta = 4 + 3\mathrm{i}$	**B1**	Correctly states the value of β
	$65 = (4-3\mathrm{i})\times(4+3\mathrm{i})$ $+(4-3\mathrm{i})\gamma + \gamma(4+3\mathrm{i})$	**M1**	Correct use of $\frac{c}{a} = \alpha\beta + \beta\gamma + \gamma\alpha$ Condone $\pm\frac{c}{a} = \alpha\beta + \beta\gamma + \gamma\alpha$ or $(z^2 - 8z + 25)(z+\gamma)$ Condone $(z^2 \pm 8z \pm 25)(z+\gamma)$
	$65 = 25 + 8\gamma$ $\gamma = 5$	**A1**	Correct value of γ
		3	

Q	Answer	Marks	Comments
3(b)	$\frac{-p}{1} = 4-3\text{i}+4+3\text{i}+5$ $p = -13$		Correct use of $\alpha+\beta+\gamma = \frac{-b}{a}$ **and** $\alpha\beta\gamma = \frac{-d}{a}$ with their roots to obtain values for p and q Condone $\alpha+\beta+\gamma = \pm\frac{b}{a}$ and $\alpha\beta\gamma = \pm\frac{d}{a}$
	$\frac{-q}{1} = (4-3\text{i})\times(4+3\text{i})\times 5$ $q = -125$	**M1**	or Expands $(z^2-8z+25)(z+\gamma)$ **and** compares coefficients to obtain values for p and q Condone $(z^2\pm 8z\pm 25)(z+\gamma)$ or Substitutes a complex root into the given equation and expands to obtain two equations (one for the Real part & one for the Imaginary part)
	$p = -13$ $q = -125$	**A1**	Obtains the correct values of p and q If 0/2 scored **SC1** for writing $p = -(\alpha+\beta+\gamma)$ **and** $q = -\alpha\beta\gamma$
		2	

	Question 3 Total	**5**	

Q	Answer	Marks	Comments
4	$z^4 = 16\mathrm{i} = 16\mathrm{e}^{\frac{\pi}{2}\mathrm{i}}$	B1	Correct Eulerian form of 16i Accept mod arg form
	$z^4 = 16\mathrm{e}^{\left(\frac{\pi}{2}+2\pi k\right)\mathrm{i}}$		
	$z = 2\mathrm{e}^{\frac{\pi\left(1+4k\right)}{8}\mathrm{i}}$	M1	Divides their argument by 4
	$r = 2$	B1	Correct value of r
	$\theta = \frac{\pi}{8}\ ,\ \frac{5\pi}{8}\ ,\ \frac{9\pi}{8}\ \left(\frac{-7\pi}{8}\right)\ ,\ \frac{13\pi}{8}\ \left(\frac{-3\pi}{8}\right)$	A1	Correct angles (mod 2π)
	$[z =]2\mathrm{e}^{\frac{\pi}{8}\mathrm{i}}\ ,\ 2\mathrm{e}^{\frac{5\pi}{8}\mathrm{i}}\ ,\ 2\mathrm{e}^{\frac{9\pi}{8}\mathrm{i}}\ ,\ 2\mathrm{e}^{\frac{13\pi}{8}\mathrm{i}}$	A1	Correct with $0 \leq \theta < 2\pi$

	Question 4 Total	**5**	

Q	Answer	Marks	Comments
5(a)	$\frac{1}{2}(k+1)\left[4(k+1)^2+7(k+1)+1\right]$ $=\frac{1}{2}(k+1)\left(4k^2+8k+4+7k+7+1\right)$ $=\frac{1}{2}(k+1)\left(4k^2+15k+12\right)$	**M1**	Expands and collects to get at least one of b, c or d correct
	$=\frac{1}{2}\left(4k^3+19k^2+27k+12\right)$	**A1**	Correctly obtains the required result
		2	

Q	Answer	Marks	Comments
5(b)	LHS $=2\times3=6$ RHS $=\frac{1}{2}\times1\times\left(4\times1^2+7\times1+1\right)=6$ [True for $n=1$]	**B1**	Demonstrates result is true for $n=1$
	$n=k+1$ Assume true $n=k$ (*)		Assumes true for $n=k$ and adds $\left[3(k+1)-1\right]\times\left[2(k+1)+1\right]$ to $=\frac{1}{2}k\left(4k^2+7k+1\right)$
	$2\times3+5\times5+8\times7+11\times9+\cdots+(3k-1)\times(2k+1)$ $+\left[3(k+1)-1\right]\times\left[2(k+1)+1\right]$ $=\sum_{r=1}^{k}(3r-1)\times(2r+1)+\left[3(k+1)-1\right]\times\left[2(k+1)+1\right]$ $=\frac{1}{2}k\left(4k^2+7k+1\right)+\left[3(k+1)-1\right]\times\left[2(k+1)+1\right]$	**M1**	Condone lack of $2\times3+5\times5+8\times7+11\times9+\cdots+(3k-1)\times(2k+1)$ $+\left[3(k+1)-1\right]\times\left[2(k+1)+1\right]=$
	$=\frac{1}{2}k\left(4k^2+7k+1\right)+\left[3(k+1)-1\right]\times\left[2(k+1)+1\right]$ $=\frac{1}{2}\left(4k^3+7k^2+k\right)+\left(6k^2+13k+6\right)$ $=\frac{1}{2}\left(4k^3+19k^2+27k+12\right)$	**M1**	Expands and collects terms

		A1	Expands and collects terms correctly. Condone lack of $2\times3+5\times5+8\times7+11\times9+\cdots+(3k-1)\times(2k+1)$ $+\left[3(k+1)-1\right]\times\left[2(k+1)+1\right]=$
	$=\frac{1}{2}(k+1)\left[4(k+1)^2+7(k+1)+1\right]$ from **(a)**	**A1**	Shows result is true for $n=k+1$ Must include $2\times3+5\times5+8\times7+11\times9+\cdots+(3k-1)\times(2k+1)$ $+\left[3(k+1)-1\right]\times\left[2(k+1)+1\right]=$ or $\sum_{r=1}^{k+1}(3r-1)\times(2r+1)=$
	Hence true for $n=k+1$ (**) and since true for $n=1$ (***) By induction $2\times3+5\times5+8\times7+11\times9+\cdots+(3n-1)\times(2n+1)$ $=\frac{1}{2}n\left[4n^2+7n+1\right]$ true for all integers $n\geq1$ (****)	**E1**	Must have (*), (**), (***) present, previous 5 marks scored and a final statement (****) clearly indicating that it relates to all integers $n\geq1$ and induction Condone "formula", "statement", "equation" Do not condone "it"
		6	

	Question 5 Total	**8**	

Q	Answer	Marks	Comments
6	$\frac{\mathrm{d}x}{\mathrm{d}t} = t^2 - t$		
	$\frac{\mathrm{d}y}{\mathrm{d}t} = 2t^{\frac{3}{2}}$	**B1**	Correct expression for $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$
	$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 = t^4 - 2t^3 + t^2 + 4t^3$	**M1**	Correctly expands their $\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2$
	$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 = t^4 + 2t^3 + t^2$ $= \left(t^2 + t\right)^2$	**A1**	$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 = \left(t^2 + t\right)^2$ **oe**
	$PQ = \int_0^{\sqrt{2}} \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\ \mathrm{d}t$		
	$PQ = \int_0^{\sqrt{2}} \sqrt{\left(t^2 - t\right)^2 + \left(2t^{\frac{3}{2}}\right)^2}\ \mathrm{d}t$	**B1ft**	Their correct integrand (can be unsimplified), with $\mathrm{d}t$ and correct limits. All seen here
	$PQ = \int_0^{\sqrt{2}} \left(t^2 + t\right)\ \mathrm{d}t$		
	$PQ = \left[\frac{t^3}{3} + \frac{t^2}{2}\right]_0^{\sqrt{2}} = \frac{\left(\sqrt{2}\right)^3}{3} + \frac{\left(\sqrt{2}\right)^2}{2}$	**M1**	Correct use of limits in an expression of the form $mt^3 + nt^2$
	$PQ = \frac{2\sqrt{2}}{3} + 1$	**A1**	**CSO** – form of answer given **B1 M1 A1 B0 M1 A1** is possible

	Question 6 Total	**6**	

Q	Answer	Marks	Comments
7(a)	$0 = \begin{vmatrix} 2-k & 1 & 2 \\ 1 & 3 & -2 \\ 8 & 4 & -5k-1 \end{vmatrix}$ $= (2-k)\{3(-5k-1)+8\}$ $-(-5k-1+16)+2(4-24)$ $0 = 15k^2 - 35k + 10 + 5k - 55$ $= 15k^2 - 30k - 45$ $= k^2 - 2k - 3$ $k = -1$ or $k = 3$	**M1** **A1** **A1**	Sets determinant = 0 and attempts to expand Obtains a correct quadratic equation Obtains correct values of k
		3	

Q	Answer	Marks	Comments
7(b)(i)	$3x + y + 2z = 5 \quad (\text{I})$ $x + 3y - 2z = -1 \quad (\text{II})$ $8x + 4y + 4z = 12 \quad (\text{III})$ $(\text{I}) + (\text{II}) \Rightarrow 4x + 4y = 4$ $(\text{III}) + 2(\text{II}) \Rightarrow 10x + 10y = 10$ Infinite solutions	**B1ft** **M1** **A1**	Correct system of equations for their smallest k Eliminating one variable in order to compare two simultaneous equations From comparing correct equations
		3	

Q	Answer	Marks	Comments
7(b)(ii)	Consistent	**E1ft**	Must follow from their answer to **part (b)(i)**
		1	

Q	Answer	Marks	Comments
7(b)(iii)	Line of intersection	**E1**	**CAO**, **oe** Must come from correct working with correct equations in **part (b)(i)**
		1	

	Question 7 Total	**8**	

Q	Answer	Marks	Comments
8(a)	$\frac{A}{r}+\frac{B}{r+1}+\frac{C}{r+2}$		
	$4r^2+8r+2=A(r+1)(r+2)+Br(r+2)+Cr(r+1)$		$4r^2+8r+2=A(r+1)(r+2)$ $+Br(r+2)+Cr(r+1)$
	$r=0: 2=2A$; $r=-1: -2=-B$; $r=-2: 2=2C$	**M1**	used to form either at least two correct equations in A, B or C or two correct simultaneous equations in A, B and C
	$\frac{1}{r}+\frac{2}{r+1}+\frac{1}{r+2}$	**A1**	Correct partial fractions
		2	

Q	Answer	Marks	Comments
8(b)	$\sum_{r=1}^{2n} \frac{(-1)^r\left(4r^2+8r+2\right)}{r(r+1)(r+2)}$		
	$= -\frac{1}{1} - \frac{2}{2} - \cancel{\frac{1}{3}}$ $+ \frac{1}{2} + \cancel{\frac{2}{3}} + \cancel{\frac{1}{4}}$ $\cancel{-\frac{1}{3}} - \cancel{\frac{2}{4}} - \frac{1}{5}$ $\vdots$ $+ \frac{1}{2n-2} + \cancel{\frac{2}{2n-1}} + \cancel{\frac{1}{2n}}$ $\cancel{-\frac{1}{2n-1}} - \cancel{\frac{2}{2n}} - \frac{1}{2n+1}$ $+ \cancel{\frac{1}{2n}} + \frac{2}{2n+1} + \frac{1}{2n+2}$	**M1**	Writes at least three lines of subtracting terms, using their **part (a)**
		M1	Writes at least three consecutive lines, showing cancellation
	$= -\frac{1}{1} - \frac{1}{2} + \frac{1}{2n+1} + \frac{1}{2n+2}$	**M1**	Correctly reduces the expression to four terms
	$= -\frac{3}{2} + \frac{4n+3}{(2n+1)(2n+2)}$	**A1**	Obtains the required result This mark is only available if at least the first three lines and the last three lines are present. **AG**
		4	

Q	Answer	Marks	Comments
8(c)	$-\frac{3}{2}$	**B1**	**CAO**
		1	

	Question 8 Total	**7**	

Q	Answer	Marks	Comments
9(a)(i)	$\left[\dfrac{\mathrm{d}^6y}{\mathrm{d}x^6} =\right] 64\cosh 2x$	**B1**	**ACF**
		1	

Q	Answer	Marks	Comments
9(a)(ii)	When $x = 0$: $y = 1,\ \dfrac{\mathrm{d}y}{\mathrm{d}x} = 0,\ \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 4,\ \dfrac{\mathrm{d}^3y}{\mathrm{d}x^3} = 0,$ $\dfrac{\mathrm{d}^4y}{\mathrm{d}x^4} = 16,\ \dfrac{\mathrm{d}^5y}{\mathrm{d}x^5} = 0,\ \dfrac{\mathrm{d}^6y}{\mathrm{d}x^6} = 64$	**B1**	
	Maclaurin series: $= 1 + x\times 0 + \dfrac{x^2}{2!}\times 4 + \dfrac{x^3}{3!}\times 0$ $+ \dfrac{x^4}{4!}\times 16 + \dfrac{x^5}{5!}\times 0 + \dfrac{x^6}{6!}\times 64$	**M1**	**ft** Their derivatives
	$\left[\cosh 2x =\right] 1 + 2x^2 + \dfrac{2}{3}x^4 + \dfrac{4}{45}x^6$	**A1**	**AG** Must be convincingly shown
		3	

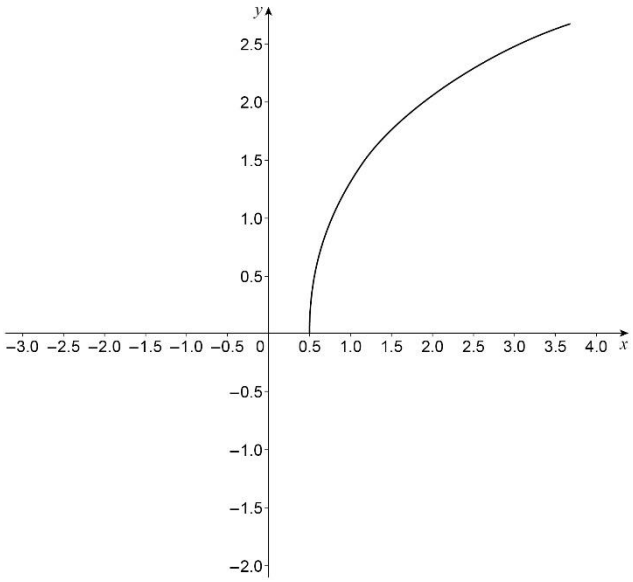
Q	Answer	Marks	Comments
9(b)	$\cos 2x = 1-2x^2+\frac{2}{3}x^4-\frac{4}{45}x^6+\ldots$	**B1**	Condone $\cos 2x = 1-\frac{4}{2!}x^2+\frac{16}{4!}x^4-\frac{64}{6!}x^6$
	$\left[\ln(1+x)\right]^2 = \left[x-\frac{x^2}{2}\right]^2 = x^2-x^3+\ldots$	**B1**	
	$\frac{\cosh 2x-\cos 2x}{\left[\ln(1+x)\right]^2} = \frac{4x^2+\frac{8}{45}x^6+\ldots}{x^2-x^3+\ldots}$	**B1**	
	$\lim_{x\to 0}\left[\frac{\cosh 2x-\cos 2x}{\left[\ln(1+x)\right]^2}\right]$		
	$= \lim_{x\to 0}\left[\frac{4x^2+\frac{8}{45}x^6+\ldots}{x^2-x^3+\ldots}\right]$		
	$= \lim_{x\to 0}\left[\frac{4+\frac{8}{45}x^4+\ldots}{1-x+\ldots}\right]$	**M1**	May be on last line May see $O\left(x^4\right)$ and/or $O\left(x\right)$
	So limit exists	**E1**	With comment
	= 4	**A1**	**B1 B1 B1 M1 E0 A1** is possible
		6	

	Question 9 Total	**10**	

Q	Answer	Marks	Comments
10(a)	I.F. is $e^{-\int \frac{1}{x}dx} = e^{-\ln(x)}$	**M1**	I.F. identified and integration attempted Condone I.F. $= +\frac{1}{x}$
	$= \frac{1}{x}$	**A1**	Correct integrating factor
	$\frac{y}{x} = \int\left(\frac{1}{\sqrt{1-x^2}}\sin^{-1}x\right)dx$	**m1**	Multiplying both sides of the Standard Form by their I.F. and integrating LHS to get $y \times$I.F.
	$u = \sin^{-1}x \qquad \frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}}$ $\frac{du}{dx} = \frac{1}{\sqrt{1-x^2}} \qquad v = \sin^{-1}x$	**M1**	Uses Integration by Parts or Uses Integration by Substitution
	$\int\left(\frac{1}{\sqrt{1-x^2}}\sin^{-1}x\right)dx$ $= \left[\left(\sin^{-1}x\right)^2\right] - \int\left(\frac{1}{\sqrt{1-x^2}}\sin^{-1}x\right)dx$	**A1**	Correct use of Integration by Parts or Correct use of Integration by Substitution
	$\int\left(\frac{1}{\sqrt{1-x^2}}\sin^{-1}x\right)dx =$ $\frac{1}{2}\left[\left(\sin^{-1}x\right)^2\right]$	**M1**	Obtains $\frac{1}{2}\left[\left(\sin^{-1}x\right)^2\right]$
	$y = x\left[\frac{1}{2}\left(\sin^{-1}x\right)^2 + c\right]$	**A1**	Correct result **ACF**, must be $y = \cdots$ **ISW**
		7	

Q	Answer	Marks	Comments
10(b)	$\frac{\pi^2}{4} = \frac{1}{2}\left(\frac{\pi}{2}\right)^2 + c$ $c = \frac{\pi^2}{8}$	**M1**	Substituting $y = \frac{\pi^2}{4}$ and $x = 1$ into their GS (with a constant of integration) to find a value of c
	$y = x\left[\frac{1}{2}\left(\sin^{-1}x\right)^2 + \frac{\pi^2}{8}\right]$	**A1**	Correct PS **ACF**, must be $y = \cdots$ **ISW**
		2	

	Question 10 Total	**9**	

Q	Answer	Marks	Comments
11(a)(i)		M1	Correct shape with x intercept clearly > 0 (does not need to be labelled). Graph in first quadrant only.
		A1	Fully correct with correct x intercept labelled (0.5)
		2	

Q	Answer	Marks	Comments
11(a)(ii)	$\cosh y = 2x$		
	$\frac{\mathrm{d}y}{\mathrm{d}x}\sinh y = 2$	B1	Correctly differentiates equation
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{\sinh y} = \frac{\pm 2}{\sqrt{\cosh^2 y - 1}}$	M1	Use of $\sinh^2 y = \cosh^2 y - 1$ in their differential equation Condone lack of ±
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\pm 2}{\sqrt{4x^2 - 1}}$	A1	Correct differential equation including ±
	$[\text{From graph}] \quad \frac{\mathrm{d}y}{\mathrm{d}x} > 0$ $\therefore \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{\sqrt{4x^2 - 1}}$	E1	Correct explanation leading to required result. **AG**
		4	

Q	Answer	Marks	Comments
11(b)	$\left[\mathrm{f}'(x)=\right]-2x+\dfrac{2}{\sqrt{4x^2-1}}$	**B1**	Correctly differentiates $\mathrm{f}(x)$
	$-2x+\dfrac{2}{\sqrt{4x^2-1}}=0$ $x=\dfrac{1}{\sqrt{4x^2-1}}$ $x\sqrt{4x^2-1}=1$ $x^2\left(4x^2-1\right)=1$		
	$4\left(x^2\right)^2-x^2-1=0$	**M1**	Obtains a quadratic equation in x^2
	$x^2=\dfrac{1-\sqrt{17}}{8}$ or $x^2=\dfrac{1+\sqrt{17}}{8}$	**A1**	Correct exact roots
	$x^2=\dfrac{1-\sqrt{17}}{8}<0$ reject as $x\in\mathbb{R}$	**E1**	Correct rejection of $x^2<0$ roots Must have correct exact roots
	$x^2=\dfrac{1+\sqrt{17}}{8}$ $x=-\sqrt{\dfrac{1+\sqrt{17}}{8}}$ reject as $x\geq\frac{1}{2}$	**E1**	Correct rejection of $x=-\sqrt{\dfrac{1+\sqrt{17}}{8}}$ root. Must reference $x\geq\frac{1}{2}$ **oe** Condone $x>\frac{1}{2}$ Must have correct roots
	$\therefore x=\sqrt{\dfrac{1+\sqrt{17}}{8}}$ Hence $\mathrm{f}(x)$ has a single stationary point	**A1**	**ACF** **A1** with correct conclusion **CSO** Must have been awarded **B1 M1 A1 E1 E1** to obtain this mark
		6	

	Question 11 Total	**12**	

Q	Answer	Marks	Comments
12(a)	$\mathbf{p} = \overrightarrow{AB} = \begin{bmatrix}1\\2\\4\end{bmatrix}$, $\mathbf{q} = \overrightarrow{AC} = \begin{bmatrix}-2\\-3\\-3\end{bmatrix}$	**M1 A1**	**M1**: clear attempt to calculate two directional vectors **A1**: two correct directional vectors
	$\mathbf{r} = \begin{bmatrix}1\\1\\2\end{bmatrix} + \lambda\begin{bmatrix}1\\2\\4\end{bmatrix} + \mu\begin{bmatrix}-2\\-3\\-3\end{bmatrix}$	**A1ft**	**oe** In correct form
		3	

Q	Answer	Marks	Comments
12(b)	$\mathbf{n}_1 = \begin{bmatrix}1\\2\\4\end{bmatrix} \times \begin{bmatrix}-2\\-3\\-3\end{bmatrix} = \begin{bmatrix}6\\-5\\1\end{bmatrix}$	**B1ft**	Calculates the cross product of their directional vectors from **part (a)**
	$\mathbf{n}_1 \times \mathbf{n}_2 = \begin{bmatrix}6\\-5\\1\end{bmatrix} \times \begin{bmatrix}2\\-1\\-3\end{bmatrix}$	**M1**	Calculates cross product of their normal vectors
	$\mathbf{v} = \alpha\begin{bmatrix}4\\5\\1\end{bmatrix}$	**A1**	Correct **v** (accept any scalar multiple)
	$d_1 = \begin{bmatrix}1\\1\\2\end{bmatrix} \bullet \begin{bmatrix}6\\-5\\1\end{bmatrix} = 3$	**M1**	Calculates the scalar product of their $\mathbf{n}_1$ and the position vector of A, B or C or Substitutes in parametric form of Π_1 into equation for Π_2
	$6x - 5y + z = 3$	**A1**	Correct Cartesian form of Π_1 **PI** Later working or Correct parametric equation

	$z = 0$ $6x - 5y = 3$ $2x - y = 7$ $x = 8, y = 9$	**M1**	Sets x, y or $z =$ constant (eg 0) and attempts to solve the resultant simultaneous equations or Substitutes parametric values to obtain values for x, y and z
	$\mathbf{u} = \begin{bmatrix} 8 \\ 9 \\ 0 \end{bmatrix}$	**A1**	A correct **u**, eg $\begin{bmatrix} 0 \\ -1 \\ -2 \end{bmatrix}$ or $\begin{bmatrix} 4/5 \\ 0 \\ -9/5 \end{bmatrix}$
	$\left(\mathbf{r} - \begin{bmatrix} 8 \\ 9 \\ 0 \end{bmatrix}\right) \times \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix} = \mathbf{0}$	**A1ft**	Correct for their **v** and their **u** Must have $= \mathbf{0}$

	Question 12 Total	**11**	

Q	Answer	Marks	Comments
13	Auxiliary equation $m^2-2m-8=0 \quad m=-2,4$	**M1**	Solves correct auxiliary equation to obtain $m=\pm 2,\pm 4$
	$\left[y_{CF}=\right] \quad Ae^{-2x}+Be^{4x}$	**A1**	Correct CF
	Particular Integral		
	$\left[y_{PI}=\right] \quad Cxe^{-2x}+D$	**B1**	Correct particular integral.
	$\left[y'_{PI}=\right] \quad Ce^{-2x}-2Cxe^{-2x}$		
	$\left[y''_{PI}=\right] \quad -4Ce^{-2x}+4Cxe^{-2x}$		
	$\left(-4Ce^{-2x}+4Cxe^{-2x}\right)$		
	$-2\left(Ce^{-2x}-2Cxe^{-2x}\right)$	**M1**	Substitutes their first and second derivatives of y_{PI} and into DE **and** attempts to compare their coefficients
	$-8\left(Cxe^{-2x}+D\right)$		
	$=12e^{-2x}+8$		
	e^{-2x}		
	$-6C=12$	**A1**	Correct coefficient of e^{-2x}
	$C=-2$		
	constant		
	$-8D=8$	**B1**	Correct constant term
	$D=-1$		
	General Solution		
	$\left[f(x)=\right]Ae^{-2x}+Be^{4x}-2xe^{-2x}-1$	**B1**	Correct General Solution with exactly two arbitrary constants
	As $x\to\infty$		
	$\lim_{x\to\infty}(y)=-1$		
	$=A\lim_{x\to\infty}\left(e^{-2x}\right)+B\lim_{x\to\infty}\left(e^{4x}\right)-2\lim_{x\to\infty}\left(xe^{-2x}\right)-1$	**M1**	Uses limits to obtain a value for B
	$=B\lim_{x\to\infty}\left(e^{4x}\right)-1$		
	$0=B\lim_{x\to\infty}\left(e^{4x}\right)$		
	$B=0$		
	As $\lim_{x\to\infty}\left(xe^{-2x}\right)=0$	**E1**	Demonstrates $\lim_{x\to\infty}\left(xe^{-2x}\right)=0$
	$x=0$		
	$y=3=A+B-1=A-1$	**A1**	Correct values for A and B
	$A=4$		
	$y=4e^{-2x}-2xe^{-2x}-1$	**A1**	Correct **ISW** **M1 A1 B1 M1 A1 B1 B1 M1 E0 A1 A1** is possible

	Question 13 Total	**11**	

Q	Answer	Marks	Comments
14(a)(i)	$\frac{1}{5}\begin{bmatrix} a & 0 & -2 \\ 0 & 5 & 0 \\ -2 & 0 & 11 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ $\frac{1}{5}\begin{bmatrix} a-4 \\ 0 \\ 20 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$ $a = 14$	**M1** **A1**	Uses $\mathbf{Mv}_1 = \lambda_1\mathbf{v}_1$ Or Characteristic equation Correct value of a
		2	

Q	Answer	Marks	Comments
14(a)(ii)	$\frac{1}{5}\begin{bmatrix} 14 & 0 & -2 \\ 0 & 5 & 0 \\ -2 & 0 & 11 \end{bmatrix}\begin{bmatrix} -2 \\ 0 \\ b \end{bmatrix} = \lambda_2\begin{bmatrix} -2 \\ 0 \\ b \end{bmatrix}$ $\frac{1}{5}\begin{bmatrix} -28-2b \\ 0 \\ 4+11b \end{bmatrix} = \lambda_2\begin{bmatrix} -2 \\ 0 \\ b \end{bmatrix}$ $-28 \quad -2b = -10\lambda_2$ $4 \quad +11b = 5b\lambda_2$ $\frac{-28-2b}{-10} = \frac{4+11b}{5b}$ $10b^2 + 30b - 40\,[= 0]$ $b = 1\ [\text{and } b = -4\ (\text{reject as } b > 0)]$ $\lambda_2 = 3$	**M1** **M1** **A1** **A1**	Obtains simultaneous equations in b and λ_2 Obtains a three-term quadratic expression in b or λ_2 $25\lambda_2^{\,2} - 125\lambda_2 + 150\,[= 0]$
		4	

Q	Answer	Marks	Comments
14(a)(iii)	$\frac{1}{5}\begin{bmatrix}14 & 0 & -2\\ 0 & 5 & 0\\ -2 & 0 & 11\end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix}=\lambda_3\begin{bmatrix}0\\1\\0\end{bmatrix}$ $\lambda_3=1$	**B1**	Obtains $\lambda_3=1$
		1	

Q	Answer	Marks	Comments
14(b)(i)	$\begin{bmatrix}1\\0\\2\end{bmatrix}\bullet\begin{bmatrix}-2\\0\\1\end{bmatrix}$	**M1**	Calculates scalar product of two eigenvectors
	$=0$ so $\mathbf{v}_1$ and $\mathbf{v}_2$ are perpendicular	**A1**	Obtains 0 **and** interprets this as **perpendicular**
	$\begin{bmatrix}1\\0\\2\end{bmatrix}\times\begin{bmatrix}-2\\0\\1\end{bmatrix}$	**M1**	Calculates the vector product of the **same** two eigenvectors or calculates the scalar products of the other two pairs of eigenvectors
	$=\begin{bmatrix}0\\-5\\0\end{bmatrix}=-5\begin{bmatrix}0\\1\\0\end{bmatrix}$ so $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$ are all perpendicular to each other	**A1**	Shows that the vector product is a multiple of the third eigenvector and **states** that the three eigenvectors are all **perpendicular** to each other or Shows that all three scalar products are zero and **states** that the three eigenvectors are all **perpendicular** to each other
		4	

Q	Answer	Marks	Comments
14(b)(ii)	$x=z=0$	**B1**	Obtains correct Cartesian equation from $\mathbf{v}_3$
		1	

	Question 14 Total	**12**	

Q	Answer	Marks	Comments
15(a)	$7r^2 + 6\left(r\sin\theta\right)^2 + 6\sqrt{3}r\sin\theta r\cos\theta = 64$		
	$7\left(x^2 + y^2\right) + 6y^2 + 6\sqrt{3}\ yx = 64$	**M1**	Correct use of $x = r\cos\theta$ **and** $y = r\sin\theta$
		M1	Correct use of $r^2 = x^2 + y^2$
	$7x^2 + 6\sqrt{3}\ xy + 13y^2 = 64$	**A1**	**ACF** Condone $64 = 7x^2 + 6\sqrt{3}\ xy + 13y^2$ **NMS** = 0/3
		3	

Q	Answer	Marks	Comments
15(b)	$\tan\theta = 1/\sqrt{3}$ so $\theta = \pi/6$ $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(2\pi/6) & \sin(2\pi/6) \\ \sin(2\pi/6) & -\cos(2\pi/6) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ $x = \dfrac{x' + \sqrt{3}\, y'}{2}$ $y = \dfrac{\sqrt{3}\, x' - y'}{2}$	**B1**	Obtains $x = \dfrac{x' + \sqrt{3}\, y'}{2}$ and $y = \dfrac{\sqrt{3}\, x' - y'}{2}$ **oe** Do not condone lack of ‘
	$64 = 7\left(\dfrac{x' + \sqrt{3}\, y'}{2}\right)^2 + 6\sqrt{3}\left(\dfrac{x' + \sqrt{3}\, y'}{2}\right)\left(\dfrac{\sqrt{3}\, x' - y'}{2}\right) + 13\left(\dfrac{\sqrt{3}\, x' - y'}{2}\right)^2$	**M1**	Uses $x = \dfrac{\pm x' \pm \sqrt{3}\, y'}{2}$ and $y = \dfrac{\pm\sqrt{3}\, x' \pm y'}{2}$ Condone lack of ‘
	So equation of curve *E* is $64 = \dfrac{1}{4}\left(7x^2 + 14\sqrt{3}\, xy + 21y^2\right) \quad (\mathrm{I})$ $+\dfrac{1}{4}\left(18x^2 + 12\sqrt{3}\, xy - 18y^2\right) (\mathrm{II})$ $+\dfrac{1}{4}\left(39x^2 - 26\sqrt{3}\, xy + 13y^2\right) (\mathrm{III})$	**m1**	Correctly expands to obtain at least one of their terms (I), (II) or (III) correct Condone presence of ‘
	$64 = 16x^2 + 4y^2$ $16 = 4x^2 + y^2$	**A1ft** **A1**	Obtains their correct equation of curve *E* as the unsimplified sum of x^2, xy and y^2 terms ie their (I), (II) and (III) correct Condone presence of ‘ **ACF** eg $1 = \dfrac{x^2}{4} + \dfrac{y^2}{16}$ Condone presence of ‘ **NMS** = 0/5 **B0 M1 m1 A1 A1** is possible
		5	

	Question 15 Total	**8**	