

INTERNATIONAL AS FURTHER MATHEMATICS FM02

(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Mark scheme

January 2025

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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
✓ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)	$hf(2, 1) = 0.1 \times (2 - \sqrt{\sin 1})$ $= 0.108268...$ $y_2 = 1 + 0.108268... = 1.108268...$ $y_3 = 1.108268 + 0.1 \times (2.1 - \sqrt{\sin(1.108268)})$ $[= 1.223667675]$ 1.224	M1 A1 A1ft M1 A1	Correct substitution into RHS of this expression PI PI AWRT 1.11 ft sin 1 evaluated in degrees to achieve AWRT 1.19 Correct substitution using their x_2 and their y_2 into second term here CAO to 3 dp
		5	

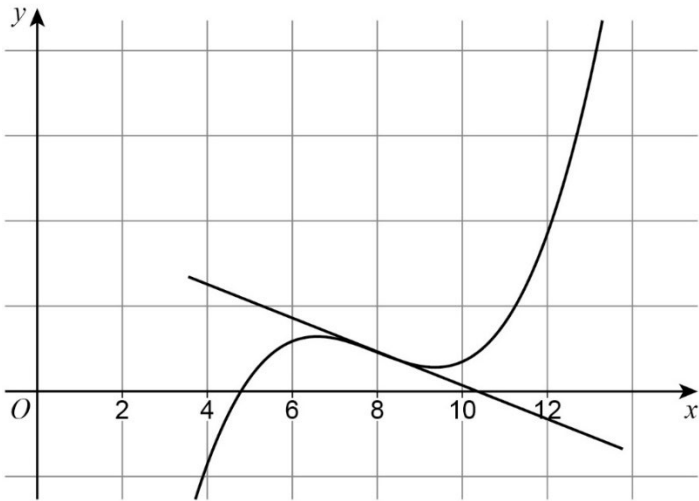
Q	Answer	Marks	Comments
1(b)	By using a smaller step length	B1	Reference to smaller step length, or suggests a smaller step Allow increase the number of steps
		1	

	Question 1 Total	6	
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Q	Answer	Marks	Comments
2(a)	$2 \times (-2) - 3p = 0$	M1	Obtains $\pm 2 \times 2 \pm 3p = 0$ PI
	$p = -\frac{4}{3}$	A1	CAO
		2	

Q	Answer	Marks	Comments
2(b)	$\mathbf{A}^2 = \begin{bmatrix} 2 & 3 \\ p & -2 \end{bmatrix} \times \begin{bmatrix} 2 & 3 \\ p & -2 \end{bmatrix}$ $= \begin{bmatrix} 4+3p & 2 \times 3 - 2 \times 3 \\ 2p-2p & 4+3p \end{bmatrix}$ $= \begin{bmatrix} 4+3p & 0 \\ 0 & 4+3p \end{bmatrix}$ $= (4+3p) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $= (4+3p)\mathbf{I}$	M1	For a 2×2 matrix with at least 3 correct entries Obtains correct result, including factoring of $4+3p$
		A1	Allow $(4+3p) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ or $(4+3p)\mathbf{I}$ CSO
		2	

Q	Answer	Marks	Comments
2(c)	$[x' =] 2x + 3(mx + c)$ $[y' =] -x - 2(mx + c)$ $-x - 2(mx + c)$ $= m(2x + 3(mx + c)) + c$ $3m^2 + 4m + 1 = 0$ $m = -1, m = -\frac{1}{3}$ $3c(m + 1) = 0$ $m = -\frac{1}{3} \Rightarrow c = 0$ $m = -1 \Rightarrow c \in \mathbb{R}$ $y = -\frac{1}{3}x, y = -x + c$	<p>M1</p> <p>Valid attempt to find x', y' Condone sign errors Condone using x for x' and y for y' Condone only considering mx Or correctly finds eigenvalues $\lambda = \pm 1$</p> <p>m1</p> <p>ft their $y' = m(\text{their } x') + c$</p> <p>m1</p> <p>Attempt to find m by comparing coefficients or setting coefficients = 0 or finding gradient associated with eigenvectors Dependent on first M mark</p> <p>A1</p> <p>Finds correct values of m</p> <p>m1</p> <p>Attempts to find c for one value of m by comparing coefficients Dependent on all previous M marks</p> <p>A1</p> <p>Correct equations, with no restrictions on c for the second</p>	
		6	
	Question 2 Total	10	

Q	Answer	Marks	Comments
3(a)	Tangent intersects the x -axis further from the root	M1 A1	Draws tangent at $x = 8$ Correct conclusion Accept x_2 is further away from α than x_1
			
		2	

Q	Answer	Marks	Comments
3(b)	$f'(x) = x^2 - 16x + 62$	M1	Differentiates expression for y PI
	$f'(5) = 7$	A1	PI
	$x_2 = 5 - \frac{\frac{1}{3} \times 5^3 - 8 \times 5^2 + 62 \times 5 - 150}{7}$	M1	Correct substitution into NR formula
	$= 5 - \frac{\left(\frac{5}{3}\right)}{7} \quad [= 4.761904762]$ $x_2 = 4.762$	A1	CAO to 3 decimal places
		4	

	Question 3 Total	6	
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Q	Answer	Marks	Comments
4(a)	$\log_{10} y = \log_{10} (a \times b^t)$ $= \log_{10} a + \log_{10} b^t$ $= \log_{10} a + t \log_{10} b$ [which is a linear relationship between x and $\log_{10} y$]	M1 A1	Takes logs of both sides and applies a correct law of logarithms to achieve $\log_{10} a + \log_{10} b^t$ Obtains correct relationship CSO
		2	

Q	Answer					Marks	Comments					
4(b)(i)	<table><tr><td>...</td><td>1.09</td><td>0.83</td><td>0.36</td><td>0.18</td></tr></table>					...	1.09	0.83	0.36	0.18	B1	At least two correct, condone more than 2 dp
	...	1.09	0.83	0.36	0.18							
					B1	All correct, condone more than 2 dp						
						2						

Q	Answer	Marks	Comments
4(b)(ii)		B1ft	Points plotted ft provided all of their points can be plotted on the grid Half a square tolerance
		B1ft	Line of best fit seen ft at least 3 points plotted
		2	

Q	Answer	Marks	Comments
4(c)	y -intercept = 1.36 gradient = -0.054 $a = 10^{1.36}$ or $b = 10^{-0.054}$ $a = 23$ $b = 0.88$	B1 M1 A1	Sight of the y -intercept or the gradient for their straight line of best fit At least one correct equation using their y -intercept or gradient for a or b A straight line must have been plotted in part (b)(ii) AWFW [19.9, 25] for a AWFW [0.87, 0.89] for b
		3	

Q	Answer	Marks	Comments
4(d)	$= 23 \times 0.88^{13}$ $= 4.4$	M1 A1ft	Substitutes values into $y = a \times b^t$ ft their exact or rounded a, b Answer must be given to at least 2 sf ft their exact or rounded a, b
		2	

	Question 4 Total	11	
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Q	Answer	Marks	Comments
5(a)	$\tan \theta = \frac{1}{2} \Rightarrow \cos 2\theta = 0.6, \sin 2\theta = 0.8$ $M = \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$	M1 A1	Uses $\tan \theta = \frac{1}{2}$ to find θ or $\cos 2\theta$ or $\sin 2\theta$ PI By correct answer CAO, oe
		2	

Q	Answer	Marks	Comments
5(b)	$NM = \begin{bmatrix} 5 & 12 \\ -13 & 13 \\ 12 & 5 \\ 13 & 13 \end{bmatrix} \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$ $= \begin{bmatrix} 33 & 56 \\ 65 & 65 \\ 56 & 33 \\ 65 & 65 \end{bmatrix} \quad \left[= \begin{bmatrix} 0.507... & -0.861... \\ 0.861... & 0.507... \end{bmatrix} \right]$ Rotation $[\phi =] \tan^{-1} \left(\frac{56}{33} \right)$ $= 59.5^\circ$, anticlockwise about the origin	M1 A1ft B1 M1 A1	Multiplication of NM in correct order, ft their M ft Their M Elements must be given to at least 3 sf Correct transformation named Attempts to calculate angle for a rotation oe eg “1.04 anticlockwise”; “5.24 (or 300.5°) clockwise”; “-2.10”; “-300.5°” condone “+1.04” or “+59.5°” CSO AWRT 59° or 59.5° or 301° or 1.0 or 5.2 for the angle
		5	

	Question 5 Total	7	
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Q	Answer	Marks	Comments
6(a)		<p>M1</p> <p>A1</p>	<p>Correct structure</p> <p>Fully correct</p>
		2	

Q	Answer	Marks	Comments
6(b)	$\frac{0.21 \times 0.75}{0.37 \times 0.32 + 0.21 \times 0.75 + 0.42 \times 0.17}$ $= \frac{1575}{3473}$	<p>M1</p> <p>M1</p> <p>A1</p>	<p>Correct calculation for the numerator PI by sight of 0.1575 oe ft their (a)</p> <p>Correct calculation for the denominator PI by sight of 0.3473 oe ft their (a)</p> <p>AWRT 0.453 or 0.4535 Allow 0.45 with a correct calculation seen</p>
		3	

	Question 6 Total	5	
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Q	Answer	Marks	Comments
7(a)	$\text{Var}(X) = \frac{1-0.3}{0.3^2} = \frac{70}{9}$	B1	oe May be unsimplified
	$\text{Var}(Y) = 20 \times 0.3 \times (1-0.3) = \frac{21}{5}$	B1	oe May be unsimplified
	$\text{Var}(T) = \frac{70}{9}a^2 + \frac{21}{5}(1-a)^2$	M1	Finds $\text{Var}(T) = a^2\text{Var}(X) + (1-a)^2\text{Var}(Y)$ for their $\text{Var}(X)$ and $\text{Var}(Y)$
	$\text{Var}(T) = \frac{539}{45}a^2 - \frac{42}{5}a + \frac{21}{5}$	M1	Expands their expression for $\text{Var}(T)$ to achieve a three-term quadratic in a
	$\frac{1078}{45}a - \frac{42}{5} = 0 \Rightarrow a = \frac{27}{77}$	A1	AG Correctly differentiates correct expression for $\text{Var}(T)$ and solves to find a with no errors seen or Completes the square to achieve $\text{Var}(T) = \frac{539}{45}\left(a - \frac{27}{77}\right)^2 + \frac{30}{11}$ and identifies that $a = \frac{27}{77}$ with no errors seen
		5	

Q	Answer	Marks	Comments
7(b)	$E(X) = \frac{1}{0.3} = \frac{10}{3}$	M1	Finds one of $E(X)$ or $E(Y)$ May be unsimplified
	$E(Y) = 20 \times 0.3 = 6$	A1	Finds both $E(X)$ and $E(Y)$ May be unsimplified
	$E(T) = \frac{27}{77} \times \frac{10}{3} + \left(1 - \frac{27}{77}\right) \times 6 = \frac{390}{77}$	A1	AWRT 5.06
		3	

	Question 7 Total	8	
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Q	Answer	Marks	Comments
8(a)	$G_{X_i}(t) = \frac{2}{5}t^{-1} + \frac{3}{5}t$ $G_Y(t) = \left(\frac{2}{5}t^{-1} + \frac{3}{5}t\right)^3$ $G_Y(t) = \left(\frac{2}{5}t^{-1} + \frac{3}{5}t\right)\left(\frac{4}{25}t^{-2} + \frac{12}{25} + \frac{9}{25}t^2\right)$ $G_Y(t) = \frac{8}{125}t^{-3} + \frac{36}{125}t^{-1} + \frac{54}{125}t + \frac{27}{125}t^3$	B1 M1 M1 A1	oe or calculates the probability of one of the possible outcomes Cubes their $G_{X_i}(t)$ or calculates the probabilities of all four possible outcomes Multiplies out to reach form $(at^{-1} + bt)(ct^{-2} + d + et^2)$ or expands using binomial expansion formulae or uses their probabilities to find $G_Y(t)$ in the required form CAO oe
		4	

Q	Answer	Marks	Comments
8(b)	$\frac{8}{125} + \frac{27}{125} = \frac{7}{25}$	B1ft	oe ft Their $A + D$
		1	

Q	Answer	Marks	Comments
8(c)	$G'_Y(t) = -\frac{24}{125}t^{-4} - \frac{36}{125}t^{-2} + \frac{54}{125} + \frac{81}{125}t^2$ <p>Mean</p> $= G'_Y(1) = -\frac{24}{125} - \frac{36}{125} + \frac{54}{125} + \frac{81}{125} = \frac{3}{5}$	M1 A1	Correct differentiation of their $G_Y(t)$ Condone a different letter being used for t oe Answer must come from $G'_Y(1)$
		2	

	Question 8 Total	7	
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Q	Answer	Marks	Comments
9(a)	$L = [x][\tan\theta]$ $L = L[\tan\theta]$ $[\tan\theta] = 1$ $L = \frac{[g][x^2]}{[2V^2][\cos^2\theta]}$ $L = \frac{LT^{-2}L^2}{L^2T^{-2}[\cos^2\theta]}$ $1 = \frac{1}{[\cos^2\theta]}$ $[\cos^2\theta] = 1$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>Recognises $[x] = L$ and $[y] = L$ and sets up equation to find $[\tan\theta]$ Condone use of units Condone missing square brackets</p> <p>AG Must be convincingly shown Units must not be used Condone missing square brackets in working but must be seen in final line</p> <p>Recognises $[g] = LT^{-2}$, $[x^2] = L^2$ and $[2V^2] = L^2T^{-2}$ and sets up equation to find $[\cos^2\theta]$ Condone use of units Condone missing square brackets</p> <p>AG Must be convincingly shown An intermediate line after $L = \frac{LT^{-2}L^2}{L^2T^{-2}[\cos^2\theta]}$ must be seen Units must not be used Condone missing square brackets in working but must be seen in final line</p> <p>Condone for all four marks eg $L = L[\tan\theta] - \frac{LT^{-2}L^2}{L^2T^{-2}[\cos^2\theta]}$ $1 = [\tan\theta] - \frac{1}{[\cos^2\theta]}$ $1 = [\tan\theta] = [\cos^2\theta]$</p>
		4	

Q	Answer	Marks	Comments
9(b)	$\tan\theta$ is dimensionless	B1	Correct statement Allow "no units" Condone dimensionless constant
		1	

	Question 9 Total	5	
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Q	Answer	Marks	Comments
10(a)	$[e =] \frac{2}{5}$	B1	Correct value
		1	

Q	Answer	Marks	Comments
10(b)	$I = 0.04 \times 2 - 0.04 \times (-5)$ $= 0.28 \text{ [N s]}$	M1 A1	Uses $\pm 0.04 \times 2 \pm 0.04 \times 5$ Correct magnitude If unit seen it must be correct
		2	

Q	Answer	Marks	Comments
10(c)(i)	$0.28 = F \times 0.2$ $F = 1.4 \text{ N}$	M1 A1ft	Uses their $I = 0.2Ft$ ft their answer to part (b) If a negative value is given in part (b), the final answer must be positive If unit seen it must be correct
		2	

Q	Answer	Marks	Comments
10(c)(ii)	$[0.28 =] \int_0^{0.2} kt(1-5t)dt$ $[0.28 =] k \left[\frac{t^2}{2} - \frac{5t^3}{3} \right]_0^{0.2}$ $0.28 = \frac{k}{150}$ $k = 42$	M1 A1 m1 A1ft	Identifies correct integral to find impulse Condone missing limits Correct integration PI By a correct equation for k Condone missing limits Forms equation to find k using their answer to part (b) ft their answer to part (b) Ignore any units given
		4	

	Question 10 Total	9	
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Q	Answer	Marks	Comments
11	<div data-bbox="172 315 654 593"> </div> <p> $v^2 = 4^2 + 3^2 - 2 \times 4 \times 3 \cos 30^\circ$ or ${}_A \mathbf{v}_B = \begin{bmatrix} 4 \sin 30^\circ \\ 4 \cos 30^\circ - 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2\sqrt{3} - 3 \end{bmatrix}$ </p> <p> $v = 2.053\dots$ or $\tan \beta = \frac{2\sqrt{3} - 3}{2}$ </p> <p> $\frac{\sin \alpha}{3} = \frac{\sin 30^\circ}{2.053\dots}$ or $\tan^{-1} \left(\frac{2\sqrt{3} - 3}{2} \right)$ </p> <p> $\alpha = 46.935\dots$ or $\tan^{-1} \left(\frac{2\sqrt{3} - 3}{2} \right) = 13.064\dots$ </p> <p> $\theta = 30 + 46.935\dots - 45 = 31.935\dots$ </p> <p> Min Distance = $1000 \times \sin 31.935\dots = 529 \text{ [m]}$ </p> <p>or</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>Uses cosine rule to find v or finds correct relative velocity</p> <p>Correct v given to 2 sf or obtains $\tan \beta = \frac{2\sqrt{3} - 3}{2}$</p> <p>Uses sine rule to find α or attempts $\tan^{-1} \left(\frac{2\sqrt{3} - 3}{2} \right)$</p> <p>Correct α given to 2 sf or $\tan^{-1} \left(\frac{2\sqrt{3} - 3}{2} \right) =$ AWRT 13</p> <p>Correct θ given to 2 sf</p> <p>Correct distance given to nearest metre</p>

	${}^A\mathbf{v}_B = \begin{bmatrix} 4\sin 30^\circ \\ 4\cos 30^\circ - 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2\sqrt{3} - 3 \end{bmatrix}$ ${}^A\mathbf{r}_B = \begin{bmatrix} 2 \\ 2\sqrt{3} - 3 \end{bmatrix} t - \begin{bmatrix} 500\sqrt{2} \\ 500\sqrt{2} \end{bmatrix}$ $s^2 = (2t - 500\sqrt{2})^2 + ((2\sqrt{3} - 3)t - 500\sqrt{2})^2$ $\left[s^2 = (2t - 707.10\dots)^2 + (0.464\dots t - 707.10\dots)^2 \right]$ $\frac{d}{dt}(s^2) = 4(2t - 500\sqrt{2}) + 2(2\sqrt{3} - 3)((2\sqrt{3} - 3)t - 500\sqrt{2})$ $0 = 8t + 2(2\sqrt{3} - 3)^2 t - 2000\sqrt{2} - 1000\sqrt{2}(2\sqrt{3} - 3)$ $t = \frac{2000\sqrt{2} + 1000\sqrt{2}(2\sqrt{3} - 3)}{8 + 2(2\sqrt{3} - 3)^2} = \frac{2000\sqrt{6} - 1000\sqrt{2}}{50 - 24\sqrt{3}} = \frac{3484.765\dots}{8.430\dots} = 413.338\dots$ $s = \sqrt{(2 \times 413.338 - 500\sqrt{2})^2 + ((2\sqrt{3} - 3) \times 413.338 - 500\sqrt{2})^2} = 529 \text{ [m]}$	<p>(M1)</p> <p>(A1)</p> <p>(M1)</p> <p>(A1)</p> <p>(A1)</p> <p>(A1)</p>	<p>Finds correct relative velocity PI</p> <p>Correct relative position or distance²</p> <p>Differentiates Condone errors if intent clear PI</p> <p>Correct equation PI</p> <p>Correct time given to 2 sf PI</p> <p>Correct distance given to nearest metre</p>
	Question 11 Total	6	