

INTERNATIONAL A-LEVEL MATHEMATICS

MA05

(9660/MA05) Unit M2 Mechanics

Mark scheme

June 2024

Version: 1.0 Final



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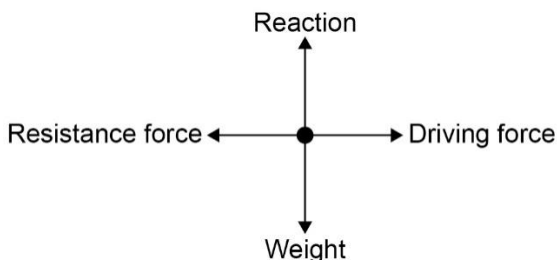
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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
✓ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)		M1 A1	<p>M1: At least two arrows/forces on the diagram with at least one vertical force, with correct names</p> <p>A1: All four arrows/forces shown on the diagram (in the correct directions), with correct names</p> <p>Allow interchange of the directions of the two horizontal forces</p>
		2	

Q	Answer	Marks	Comments
1(b)	<p>Resultant force:</p> $D - 0.91v^{\frac{5}{3}} = ma$ $D = ma + 0.91v^{\frac{5}{3}}$ $D = 280 \times 3.5 + 0.91 \times (15)^{\frac{5}{3}}$ $D = 1100 \text{ [N]}$	<p>M1</p> <p>m1</p> <p>A1</p>	<p>Forming an equation of motion Allow one error or omission PI</p> <p>Correct substitution into their equation for the driving force $280 \times 3.5 = 980$, $0.91 \times (15)^{\frac{5}{3}} = 83.02\dots$</p> <p>Correct value for the driving force Note Unrounded value is 1063.022057 [N]</p>
		3	

Q	Answer	Marks	Comments
1(c)	<p>When in equilibrium</p> $\frac{P}{v} - 0.91v^{\frac{5}{3}} = 0$ $v = \left(\frac{P}{0.91} \right)^{\frac{3}{8}}$ $v = \left(\frac{50 \times 10^3}{0.91} \right)^{\frac{3}{8}}$ $v = 60 \text{ [m s}^{-1}\text{]}$	<p>M1</p> <p>A1</p>	<p>Use of $P = Fv$ with equation of motion</p> <p>Note Unrounded answer is $59.906... \text{ [m s}^{-1}\text{]}$</p>
		2	

Q	Answer	Marks	Comments
1(d)	<p>[The maximum speed would be] lower [than 60 m s^{-1}]</p> <p>as a component of the motorcycle's weight now acts down the slope [so the resultant force becomes zero at a lower speed]</p>	<p>B1</p> <p>E1</p>	<p>oe, such as decreases</p> <p>A valid explanation</p>
		2	

	Question 1 Total	9	
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Q	Answer	Marks	Comments
2(a)(i)	$[F =] 4\cos(2t)\mathbf{i} - 8e^t\mathbf{j} + 24t\mathbf{k}$	B1	ACF
		1	

Q	Answer	Marks	Comments
2(a)(ii)	<p>Equilibrium requires the resultant force to be zero</p> <p>[so all three components of $\mathbf{F}$ would need to be simultaneously zero]</p> <p>[However] $8e^t > 0$ for $t \geq 0$</p>	<p>E1</p> <p>E1</p>	<p>States the condition for equilibrium or states the acceleration is zero</p> <p>Explains that the $\mathbf{j}$ component [of the resultant force] or e^t can never equal zero</p>
		2	

Q	Answer	Marks	Comments
2(b)(i)	$\mathbf{F}_2 = (4\cos(2t)\mathbf{i} - 8e^t\mathbf{j} + 24t\mathbf{k})$ $- (3\cos(2t)\mathbf{i} - 2e^t\mathbf{j} - 8t\mathbf{k})$	M1	their resultant force – $\mathbf{F}_1$
	$\mathbf{F}_2 = \cos(2t)\mathbf{i} - 6e^t\mathbf{j} + 32t\mathbf{k}$	A1	
		2	

Q	Answer	Marks	Comments
2(b)(ii)	$\mathbf{F}_2 = \cos\left(2 \times \frac{\pi}{4}\right)\mathbf{i} - 6e^{\frac{\pi}{4}}\mathbf{j} + \left(32 \times \frac{\pi}{4}\right)\mathbf{k}$	M1	Substitutes $t = \frac{\pi}{4}$ into their $\mathbf{F}_2$ PI Note: $6e^{\frac{\pi}{4}} = 13.159\dots$, $8\pi = 25.132\dots$ AWRT 28 ft an $\mathbf{F}_2$ of the form $\pm\cos(2t)\mathbf{i} \pm 6e^t\mathbf{j} \pm 32t\mathbf{k}$
	$\mathbf{F}_2 = -6e^{\frac{\pi}{4}}\mathbf{j} + 8\pi\mathbf{k}$		
	$ \mathbf{F}_2 = \sqrt{\left(6e^{\frac{\pi}{4}}\right)^2 + (8\pi)^2}$	m1	
	$ \mathbf{F}_2 = 28 \text{ [N]}$	A1ft	
		3	

	Question 2 Total	8	
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Q	Answer	Marks	Comments
3(a)	(3, 3)	B1	Allow any mention of symmetry of the circle
	The centre of mass of the uniform circular lamina is at its geometric centre	E1	
		2	

Q	Answer	Marks	Comments
3(b)	$(2 + 3 + 7 + 11 + 5)\bar{X}$ $= [2 \times 0 + 3 \times 3 + 7 \times 3 + 11 \times 6 + 5 \times 3]$	M1	Forming equation for x-coordinate of the centre of mass Condone one error
	$\bar{X} = \frac{111}{28}$	A1	Any correct exact form
	$(2 + 3 + 7 + 11 + 5)\bar{Y}$ $= 2 \times 3 + 3 \times 6 + 7 \times 0 + 11 \times 3 + 5 \times 3$	M1	Forming equation for y-coordinate of the centre of mass Condone one error
	$\bar{Y} = \frac{18}{7}$	A1	Any correct exact form
	$\left(\frac{111}{28}, \frac{18}{7}\right)$		If M1 A0 M1 A0 awarded, then allow SC1 for (3.96, 2.57)
		4	

Q	Answer	Marks	Comments
3(c)	$\tan \theta = \frac{\frac{111}{28} - 3}{6 - \frac{18}{7}} \left[= \frac{\left(\frac{27}{28}\right)}{\left(\frac{24}{7}\right)} = \frac{9}{32} \right]$ $\theta = \tan^{-1} \left(\frac{\frac{111}{28} - 3}{6 - \frac{18}{7}} \right)$ $\theta = 16^\circ$	<p>M1 A1ft</p> <p>A1</p>	<p>M1: At least numerator or denominator correct, ft their coordinates for the COM</p> <p>A1ft: Both numerator and denominator correct, ft their coordinates for the COM</p> <p>Note: unrounded answer is 15.7086...°</p> <p>CAO</p>
		3	
	Question 3 Total	9	

Q	Answer	Marks	Comments
4(a)(i)	$\left[\frac{1}{2} mv^2 = \right]$ $0.5 \times (80 \times 10^{-3}) \times 45^2 = 81 \text{ [J]}$	B1	AG Must be convincingly shown
		1	

Q	Answer	Marks	Comments
4(a)(ii)	At maximum height, speed of arrow is $45\cos(10^\circ)$ $\left[\frac{1}{2}mv^2=\right]$ $0.5 \times (80 \times 10^{-3}) \times (45\cos(10^\circ))^2$ $= 78.6 \text{ [J]}$	M1 A1	CAO to 3 sf AWRT 78.6 [J]
		2	

Q	Answer	Marks	Comments
4(b)	<p>Loss in KE = Gain in GPE</p> $81 - 78.6 = mg\Delta h$ $\Delta h = \frac{81 - 78.6}{(80 \times 10^{-3}) \times 9.8}$ $\Delta h = 3.06122... \text{ [m]}$ <p>Maximum height above the ground</p> $= 3.06122... + 1.6$ $= 4.7 \text{ [m]}$	<p>M1</p> <p>A1</p> <p>A1ft</p>	<p>At least LHS correct, ft their 78.6</p> <p>Allow 3.11536... [m] from unrounded value for minimum KE PI by correct final answer</p> <p>ft their Δh value + 1.6</p>
		3	

Q	Answer	Marks	Comments
4(c)(i)	<p>Time to cover the horizontal displacement of 70 metres</p> $T = \frac{70}{45\cos(10^\circ)}$ $T = 1.58 \text{ [s]}$	B1	AWRT 1.58 Note: unrounded answer is 1.57955... [s]
		1	

Q	Answer	Marks	Comments
4(c)(ii)	<p>Vertical displacement when horizontal displacement is 70 metres</p> <p>$[s =] 45\sin(10^\circ) T - 0.5 \times 9.8 \times T^2 + 1.6$</p> <p>$[s =] 45\sin(10^\circ) \times 1.57955\ldots$ $- 0.5 \times 9.8 \times (1.57955\ldots)^2 + 1.6$</p> <p>$[s =] 1.717\ldots \text{ [m]}$</p> <p>As $0.8 < 1.717\ldots < 1.8$, the arrow does hit the target</p>	<p>M1 A1</p> <p>A1</p> <p>B1</p>	<p>M1: Use of $s = ut + \frac{1}{2}at^2$ with $u = 45\sin(10^\circ)$ and $a = \pm 9.8$</p> <p>Condone +1.6 omitted (May be recovered later in solution) PI</p> <p>A1: AWFW [0.114, 0.117] or AWFW [1.714, 1.717] PI</p> <p>Note: $45\sin(10^\circ) \times 1.57955\ldots$ $- 0.5 \times 9.8 \times (1.57955\ldots)^2 = 0.117\ldots$</p> <p>Expect 1.714... if $T = 1.58$ used AWRT 1.7 from correct working</p> <p>Comparison of their vertical displacement with the height(s) of the target and correct conclusion</p>
		4	

	Question 4 Total	11	
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Q	Answer	Marks	Comments
5(a)	$ r = \sqrt{(e^{-3t}\cos(3t))^2 + (e^{-3t}\sin(3t))^2}$ $ r = \sqrt{e^{-6t}\cos^2(3t) + e^{-6t}\sin^2(3t)}$ $ r = \sqrt{e^{-6t}(\cos^2(3t) + \sin^2(3t))}$ $ r = \sqrt{e^{-6t}}$ $ r = e^{-3t}$	<p>M1</p> <p>m1</p> <p>A1</p>	<p>Use of $\cos^2(3t) + \sin^2(3t) = 1$</p> <p>AG Must be convincingly shown</p>
		3	

Q	Answer	Marks	Comments
5(b)	$\mathbf{v} = \begin{bmatrix} -3e^{-3t}(\sin(3t) + \cos(3t)) \\ -3e^{-3t}(\sin(3t) - \cos(3t)) \end{bmatrix}$	M1 A1	M1: At least one component correct Condone written in any vector form A1: Both components correct, written as a column vector
		2	

[illegible]

Q	Answer	Marks	Comments
5(c)(ii)	$\left[\text{KE} = \frac{1}{2}mv^2 \right] = 0.5 \times 6 \times 18$ $[\text{KE} =] 54 [\text{J}]$	M1 A1ft	Substitution of $m = 6$ and their b into $\frac{1}{2}mv^2$ Condone appearance of exponential term ft their b
		2	

	Question 5 Total	10	
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Q	Answer	Marks	Comments
6	<p>Kinetic energy at A</p> $\left[\frac{1}{2}mv^2 = \right] 0.5 \times 16 \times 35^2$ $= 9800 \text{ [J]}$ <p>Friction experienced by particle</p> $\mu mg \cos(28^\circ) = 0.64 \times 16 \times 9.8 \cos(28^\circ)$ $= 88.60555... \text{ [N]}$ <p>Let $AB = d$</p> <p>Change in height between A and B</p> $[\Delta h =] d \sin(28^\circ)$ <p>Conservation of Energy</p> $9800 = 88.60555... \times d + 16 \times 9.8 \times d \sin(28^\circ)$ $d = \frac{9800}{88.60555... + 16 \times 9.8 \sin(28^\circ)}$ $d = 60 \text{ [m]}$	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p>	<p>PI</p> <p>PI by correct friction force</p> <p>Seen or used</p> <p>Initial KE = 'Work Done' term (in terms of a distance) + 'Change in GPE' term (in terms of a distance) PI by correct value distance</p> <p>Unrounded answer is 60.4122... [m]</p>

Q	Answer	Marks	Comments
6 ALT	<p>Friction experienced by particle</p> $\mu mg \cos(28^\circ) = 0.64 \times 16 \times 9.8 \cos(28^\circ)$ $= 88.60555... \text{ [N]}$ <p>Resultant force experienced by particle</p> $-88.60555... - 16 \times 9.8 \times \sin(28^\circ)$ $[-162.218...]$ $-162.218... = 16a$ $a = -10.1386... \text{ [m s}^{-1}\text{]}$ <p>Let $AB = d$</p> $v^2 = u^2 + 2ad \Rightarrow d = \frac{v^2 - u^2}{2a}$ $d = \frac{0^2 - 35^2}{2 \times -10.1386..}$ $d = 60 \text{ [m]}$	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p>	<p>PI by correct friction force</p> <p>ft their acceleration</p> <p>Unrounded answer is 60.4122... [m]</p>
	Question 6 Total	5	

Q	Answer	Marks	Comments
7(a)(i)	$x = (v \cos \theta)t$ $y = (v \sin \theta)t - \frac{1}{2}gt^2$ $t = \frac{x}{v \cos \theta}$ $y = (v \sin \theta) \times \frac{x}{v \cos \theta} - \frac{1}{2}g \times \left(\frac{x}{v \cos \theta}\right)^2$ $(v \sin \theta) \times \frac{x}{v \cos \theta} = x \tan \theta$ $-\frac{1}{2}g \times \left(\frac{x}{v \cos \theta}\right)^2 = -\frac{gx^2}{2v^2} \times \frac{1}{\cos^2 \theta}$ $-\frac{gx^2}{2v^2} \times \frac{1}{\cos^2 \theta} = -\frac{gx^2}{2v^2} \sec^2 \theta$ $y = x \tan \theta - \frac{gx^2}{2v^2} \sec^2 \theta$	B1 B1 M1 A1 A1	oe M1: Eliminates t in both terms with at least one term correct A1: All correct AG Must be convincingly shown
		5	

Q	Answer	Marks	Comments
7(a)(ii)	The particle does not experience air resistance	E1	Any valid assumption
		1	

Q	Answer	Marks	Comments
7(b)	$10 = 25 \tan \alpha - \frac{9.8 \times 25^2}{2 \times 30^2} \sec^2 \alpha$ $10 = 25 \tan \alpha - \frac{9.8 \times 25^2}{2 \times 30^2} \times (1 + \tan^2 \alpha)$ $245 \tan^2 \alpha - 1800 \tan \alpha + 965 = 0$ $\tan \alpha = \frac{1800 \pm \sqrt{1800^2 - 4 \times 245 \times 965}}{2 \times 245}$ $\tan \alpha = 0.5822..., 6.7646$ $\alpha = 30, 82$	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1 A1</p>	<p>Substitution of values into the result given in part (a)(i) PI</p> <p>Use of $1 + \tan^2 \alpha = \sec^2 \alpha$ PI</p> <p>Forms a three-term quadratic equation in $\tan \alpha$</p> <p>A1: At least one correct value A1: Both values correct and no others</p> <p>Note: unrounded values are 30.210... and 81.591...</p>
		5	
	Question 7 Total	11	

Q	Answer	Marks	Comments
8(a)	The length of the string does not increase	E1	Allow '[length] does not change'
		1	

Q	Answer	Marks	Comments
8(b)	$T \sin \theta = m \omega^2 r$ $r = l \sin \theta$ $T \sin \theta = m \omega^2 l \sin \theta$ $T = m \omega^2 l$	M1 A1	
		2	

Q	Answer	Marks	Comments
8(c)	$T \cos \theta = mg$ $\frac{mg}{\cos \theta} = m \omega^2 l$ $\frac{g}{l \cos \theta} = \omega^2$ $\sqrt{\frac{g}{l \cos \theta}} = \frac{2\pi}{t}$ $[t =] 2\pi \sqrt{\frac{l \cos \theta}{g}}$	B1 M1 A1	PI Use of $\omega = \frac{2\pi}{t}$, where t is the time period ACF
		3	

[illegible]

	Question 8 Total	9	
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Q	Answer	Marks	Comments
9(a)	<p>Taking moments about B</p> $12g\cos(75^\circ) \times 3.5 = 15\sin(75^\circ) \times 7 + \mu_1 \times 15\cos(75^\circ) \times 7$ $106.529... = 101.422... + 27.175...\mu_1$ $\mu_1 = 0.188$	<p>M1 A1</p> <p>A1</p>	<p>M1: At least one side correct PI A1: Both sides fully correct PI</p> <p>Note Unrounded value is $\mu_1 = 0.187949...$</p>
		3	

Q	Answer	Marks	Comments
9(b)	<p>Forces in equilibrium (vertical direction)</p> $12g = \mu_1 \times 15 + R_G$ $R_G = 12 \times 9.8 - 0.187949... \times 15$ $R_G = 114.78... \text{ [N]}$ <p>Forces in equilibrium (horizontal direction)</p> $\mu_2 R_G = 15$ $\mu_2 = \frac{15}{114.78...}$ $\mu_2 = 0.131$	<p>M1 m1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>R_G = normal reaction on rod from ground</p> <p>M1: At least one side correct PI m1: Both sides fully correct PI ft their μ_1</p> <p>AWRT 115 [N] PI</p> <p>CAO to 3 sf Note Unrounded value is $\mu_2 = 0.13068...$</p>
		5	

	Question 9 Total	8	
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