

INTERNATIONAL A-LEVEL MATHEMATICS

MA05

(9660/MA05) Unit M2 Mechanics

Mark scheme

January 2024

Version: 1.0 Final



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Key to mark scheme abbreviations

M	Mark is for method
m	Mark is dependent on one or more M marks and is for method
A	Mark is dependent on M or m marks and is for accuracy
B	Mark is independent of M or m marks and is for method and accuracy
E	Mark is for explanation
✓ or ft	Follow through from previous incorrect result
CAO	Correct answer only
CSO	Correct solution only
AWFW	Anything which falls within
AWRT	Anything which rounds to
ACF	Any correct form
AG	Answer given
SC	Special case
oe	Or equivalent
A2, 1	2 or 1 (or 0) accuracy marks
–x EE	Deduct x marks for each error
NMS	No method shown
PI	Possibly implied
SCA	Substantially correct approach
sf	Significant figure(s)
dp	Decimal place(s)
ISW	Ignore subsequent working

Q	Answer	Marks	Comments
1(a)(i)	<p>Taking moments about B</p> <p>Anticlockwise moments</p> $= (3.60 - x) \times 800 \times 9.8$ <p>Clockwise moments</p> $= 3.60 \times 6100 \quad [= 21960]$ <p>Principle of moments</p> $3.60 \times 6100 = (3.60 - x) \times 800 \times 9.8$ $x = 3.60 - \frac{3.60 \times 6100}{800 \times 9.8}$ $x = 0.80$	<p>M1</p> <p>M1</p> <p>A1</p>	<p>oe</p> <p>oe</p> <p>AWRT 0.80 Note: unrounded answer is 0.798...</p>
		3	

Q	Answer	Marks	Comments
1(a)(ii)	<p>[Let R_B be the reaction force on the front wheels]</p> <p>Forces in equilibrium gives</p> $R_B + 6100 = 800 \times 9.8$ $R_B = 1700 \text{ [N, to 2 sf]}$	B1	AWRT 1700 [N] from correct working Note: unrounded answer is 1740 [N]
		1	

Q	Answer	Marks	Comments
1(b)(i)	$W = Fd$ $W = (0.95 \times 20^2) \times (20 \times 7.5)$ $W = 57,000 \text{ [J]}$	M1 A1	oe
		2	

Q	Answer	Marks	Comments
1(b)(ii)	$F = \frac{P}{v} - 0.95v^2$ $F = \frac{780 \times 10^3}{50} - 0.95 \times 50^2$ $F = 13,000 \text{ [N, to 2 sf]}$	M1 A1	oe Note: unrounded answer is 13,225 N
		2	

Q	Answer	Marks	Comments
1(b)(iii)	$\frac{P}{v} - 0.95v^2 = 0$ $v = \sqrt[3]{\frac{P}{0.95}}$ $v = \sqrt[3]{\frac{780 \times 10^3}{0.95}}$ $v = 94 \text{ [m s}^{-1}\text{, to 2 sf]}$	M1 A1	Note: unrounded answer is 93.639... [m s ⁻¹]
		2	

	Question 1 Total	10	
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Q	Answer	Marks	Comments
2(a)	<p>[Equilibrium perpendicular to slope]</p> $[N =] P + mg \cos(30^\circ)$ <p>[Equilibrium parallel to slope]</p> $[f =] mg \sin(30^\circ)$ $mg \sin(30^\circ) \leq \mu(P + mg \cos(30^\circ))$ $mg \left(\frac{\sin(30^\circ)}{\mu} - \cos(30^\circ) \right) \leq P$ $17 \times 9.8 \times \left(\frac{\sin(30^\circ)}{0.4} - \cos(30^\circ) \right) \leq P$ $63.97... \leq P$ <p>Least value of P is 64 [to 2 sf]</p>	<p>M1</p> <p>M1</p> <p>m1</p> <p>A1</p>	<p>N = normal reaction force</p> <p>f = friction PI by use or sight of 83.3 [N]</p> <p>Use of $f \leq \mu N$ or $f = \mu N$</p> <p>$208.25 - 144.279... \leq P$</p> <p>AWRT 64</p>
		4	

Q	Answer	Marks	Comments
2(b)(i)	The block moves in a direction that is perpendicular to the force P newtons [so no work is done on the block by the force P newtons]	E1	oe
		1	

Q	Answer	Marks	Comments
2(b)(ii)	Work is done against friction	E1	oe
		1	

Q	Answer	Marks	Comments
2(b)(iii)	$ma = mg \sin(30^\circ) - \mu(P + mg \cos(30^\circ))$ $a = 9.8 \times \sin(30^\circ) - 0.4 \times \left(\frac{40}{17} + 9.8 \times \cos(30^\circ) \right)$ $a = 0.56 \text{ [m s}^{-2}\text{]}$	<p>M1</p> <p>A1</p>	<p>Forms equation for the resultant force acting on the block, or better</p> <p>AWRT 0.56 Note: unrounded answer is 0.5640... [m s⁻²]</p>
		2	

Q	Answer	Marks	Comments
2(b)(iv)	$s = ut + \frac{1}{2}at^2$ $s = 0.5 \times 0.564... \times 6^2$ $s = 10 \text{ [m]}$	<p>M1</p> <p>A1ft</p>	<p>Use of $s = ut + \frac{1}{2}at^2$ with $u = 0$ and their $a \neq 9.8$ from (b)(ii)</p> <p>AWRT 10 ft their acceleration from (b)(iii) Note: unrounded answer is 10.15... [m]</p>
		2	

	Question 2 Total	10	
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Q	Answer	Marks	Comments
3(a)	$[\mathbf{r} =] (1.5\cos(4t) + c_1)\mathbf{i} + (1.5\sin(4t) + c_2)\mathbf{j}$ <p>When $t = 0$</p> $[\mathbf{r} =] (1.5 + c_1)\mathbf{i} + c_2\mathbf{j} = 1.5\mathbf{i}$ $\Rightarrow c_1 = 0 \text{ and } c_2 = 0$ $[\mathbf{r} =] 1.5\cos(4t)\mathbf{i} + 1.5\sin(4t)\mathbf{j}$ $[\mathbf{r} =] \sqrt{(1.5\cos(4t))^2 + (1.5\sin(4t))^2}$ $[\mathbf{r} =] 1.5 \text{ [m]}$ <p>so A is a constant distance away from O, meaning it moves on a circular path</p>	<p>M1 A1</p> <p>B1</p> <p>m1</p> <p>A1</p>	<p>M1: Uses integration to find at least one correct component A1: Finds both correct components Condone no constants of integration for M1 A1</p> <p>Finds correct position vector at time t by explicitly showing both constants of integration are zero/the constant of integration vector is zero</p> <p>Must have reference to constant distance, not just 1.5 [m]</p>
		5	

Q	Answer	Marks	Comments
3(b)(i)	$[\omega =] 4 \text{ rad s}^{-1}$	B1 B1	B1: Correct value B1: Correct units
		2	

Q	Answer	Marks	Comments
3(b)(ii)	$m\omega^2 r = 4.9 \times 4^2 \times 1.5$ or $\frac{mv^2}{r} = \frac{4.9 \times 6^2}{1.5}$ 120 [N, to 2 sf] Towards O	M1 A1 B1	Use of $m\omega^2 r$ with their ω and $r = 1.5$ or use of $\frac{mv^2}{r}$ with $r = 1.5$ or use of $\mathbf{F} = m\mathbf{a}$ and differentiation of $\mathbf{v}$ PI By correct magnitude of force CAO, AWRT 120 Exact answer is 117.6 [N] oe , such as $-\cos(4t)\mathbf{i} - \sin(4t)\mathbf{j}$
		3	

Q	Answer	Marks	Comments
3(c)	[Tension in string = 117.6 N] $117.6 = mg$ $m = 12$	M1 A1	AWRT 12 Note: $m = 12.244\dots$ if using $T = 120$ N
		2	

	Question 3 Total	12	
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Q	Answer	Marks	Comments
4(a)	$s = ut + \frac{1}{2}at^2$ $t = \sqrt{\frac{2d}{g}}$ or $t = \frac{d}{u}$ $d [= ut] = u\sqrt{\frac{2d}{g}}$ or $d = \frac{1}{2}g\left(\frac{d}{u}\right)^2$ $u = \sqrt{\frac{gd}{2}}$	<p>B1</p> <p>M1</p> <p>A1</p>	<p>Time to travel from O to A</p> <p>oe</p>
		3	

Q	Answer	Marks	Comments
4(b)	<p>[Vertical component of velocity immediately before colliding with ground at A]</p> $v = u + at$ $\left[v = 0 + g \times \sqrt{\frac{2d}{g}} \right]$ $v = \sqrt{2gd}$ <p>Speed immediately before colliding with ground at A</p> $= \sqrt{(2u)^2 + u^2}$ $= \sqrt{5} u$	<p>B1</p> <p>M1</p> <p>A1</p>	<p>or $v = 2u$</p> <p>or $= \sqrt{(2gd) + u^2}$ or</p> <p>$= \sqrt{(2gd) + \left(\frac{gd}{2}\right)}$</p>
		3	

	Question 4 Total	6	
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Q	Answer	Marks	Comments
5(a)	The logo is symmetric in the line through C , B and M	E1	Allow any mention of symmetry or that the centres of mass of the circle and the triangle lie on CBM
		1	

Q	Answer	Marks	Comments
5(b)	<p>[C.O.M. of the circle from M is]</p> $2d\sin(60^\circ) + \frac{d}{2} \left[= d\left(\sqrt{3} + \frac{1}{2}\right) \right]$ <p>[C.O.M. of the triangle from M is]</p> $\frac{1}{3} \times 2d\sin(60^\circ) \left[= d\frac{\sqrt{3}}{3} \right]$ $\rho \times \left(\frac{1}{2} \times 4d^2\sin(60^\circ) + \frac{\pi d^2}{4} \right) \bar{Y}$ $= \rho \times \frac{1}{2} \times 4d^2\sin(60^\circ) \times \frac{2d\sin(60^\circ)}{3}$ $+ \rho \times \frac{\pi d^2}{4} \times \left(2d\sin(60^\circ) + \frac{d}{2} \right)$ $\left(\sqrt{3}d^2 + \frac{\pi d^2}{4} \right) \bar{Y} = \sqrt{3}d^2 \times \frac{\sqrt{3}}{3}d$ $+ \frac{\pi d^2}{4} \times d\left(\sqrt{3} + \frac{1}{2}\right)$ $d^2\left(\sqrt{3} + \frac{\pi}{4}\right) \bar{Y} = d^3\left(1 + \frac{\pi}{4}\left(\sqrt{3} + \frac{1}{2}\right)\right)$ $\bar{Y} = \frac{4 + \pi\left(\sqrt{3} + \frac{1}{2}\right)}{4\sqrt{3} + \pi} \times d$ <p>$k = 1.09$ [3 sf]</p>	<p>B1</p> <p>B1</p> <p>M1 m1 A1</p> <p>A1</p>	<p>PI</p> <p>PI</p> <p>M1: At least one of the three terms correct m1: At least two of the three terms correct A1: Fully correct equation Condone lack of ρ</p> <p>Allow $k = \frac{4 + \pi\left(\sqrt{3} + \frac{1}{2}\right)}{4\sqrt{3} + \pi}$</p>
		6	

Q	Answer	Marks	Comments
5(c)	<p>[Let α be the angle OM makes with the vertical]</p> $\tan \alpha = \frac{1.094... \times d}{d}$ $\alpha = 48^\circ$	<p>M1</p> <p>A1</p>	<p>$\tan \alpha = \frac{\bar{Y}}{d}$ using their $\bar{Y}$</p> <p>Allow 47° as final answer if rounded answer of $1.09d$ used</p>
		2	
	Question 5 Total	9	

Q	Answer	Marks	Comments
6(a)	<p>Loss in GPE for A</p> $mg\Delta h = 3 \times 9.8 \times 5$ $= 147 \text{ [J]}$ <p>Loss in GPE = Gain in KE for A</p> $147 = \frac{1}{2}mv^2$ $v = \sqrt{\frac{2 \times 147}{3}} = 7\sqrt{2} \text{ [m s}^{-1}\text{]}$ <p>Total momentum of system before collision</p> $[p =] 3 \times 7\sqrt{2} = 21\sqrt{2} \text{ [kg m s}^{-1}\text{]}$ <p>Conservation of momentum</p> $21\sqrt{2} = 3 \times (\pm 2) + 10v_B$ <p>If A moves in the <u>same</u> direction after the collision, then speed of B is</p> $v_B = 2.37 \text{ [m s}^{-1}\text{]}$ <p>If A moves in the <u>opposite</u> direction after the collision, then speed of B is</p> $v_B = 3.57 \text{ [m s}^{-1}\text{]}$	<p>B1</p> <p>B1</p> <p>M1</p> <p>M1</p> <p>A1ft</p> <p>A1ft</p>	<p>Sight or use of 147 or 15g or use of 29.4 PI by $v = 7\sqrt{2} \text{ [m s}^{-1}\text{]}$</p> <p>oe, eg $\sqrt{10g}$ AWRT 9.9</p> <p>oe, eg $3\sqrt{10g}$ PI by sight or use of AWRT 30 Total momentum before collision ft their speed of A before the collision oe, eg $3\sqrt{10g} = 3 \times (\pm 2) + 10v_B$ Condone + or – instead of $\pm$ Total momentum after collision</p> <p>Answer given to 3 sf ft their speed of A before the collision</p> <p>Answer given to 3 sf ft their speed of A before the collision</p>
		6	

Q	Answer	Marks	Comments
6(b)	$[\Delta p_B =] 10 \times 2.37 = 23.7 \text{ [kg m s}^{-1}\text{]}$	M1	PI by correct answer
	$\left[F = \frac{\Delta p}{\Delta t} =\right] \frac{23.7}{0.20} = 120 \text{ [N, to 2 sf]}$	A1ft	AWRT 120 N
		2	

Q	Answer	Marks	Comments
6(c)	Total KE of system after collision $0.5 \times 3 \times 2^2 + 0.5 \times 10 \times 3.57^2$ $= 69.7245 \text{ [J]}$ KE lost during the collision $147 - 69.7245$ $= 77 \text{ [J, to 2 sf]}$	M1	AWRT 70 PI by correct answer
		A1	AWRT 77 CAO
		2	

	Question 6 Total	10	
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Q	Answer	Marks	Comments
7(a)	<p>Resultant force acting on the particle</p> $\begin{bmatrix} 10\cos^2 t \\ 30t \\ 50e^{-2t} \end{bmatrix} + \begin{bmatrix} 10\sin^2 t \\ 90t^2 \\ -31 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -49 \end{bmatrix} =$ $\begin{bmatrix} 10 \\ 30t + 90t^2 \\ 50e^{-2t} - 80 \end{bmatrix} \text{ [N]}$ <p>Acceleration of the particle</p> $\mathbf{a} = \begin{bmatrix} 2 \\ 6t + 18t^2 \\ 10e^{-2t} - 16 \end{bmatrix} \text{ [m s}^{-2}\text{]}$ <p>Velocity of the particle</p> $\mathbf{v} = \begin{bmatrix} 2t + c_1 \\ 3t^2 + 6t^3 + c_2 \\ -5e^{-2t} - 16t + c_3 \end{bmatrix}$ $\left[\text{When } t = 0, \mathbf{v} = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ -5 + c_3 \end{bmatrix} \right]$ $\mathbf{v} = \begin{bmatrix} 2t + 3 \\ 3t^2 + 6t^3 - 1 \\ -5e^{-2t} - 16t + 10 \end{bmatrix} \text{ [m s}^{-1}\text{]}$	<p>M1</p> <p>A1</p> <p>M1 A1</p> <p>A1</p>	<p>oe At least two components correct Condone unsimplified</p> <p>Condone unsimplified Condone -6.2 instead of -16 in k component</p> <p>M1: At least one correct component A1: All three components correct Condone no constants of integration for M1 A1</p> <p>CAO</p>
		5	

[illegible]

Q	Answer	Marks	Comments
8(a)	<p>[Time taken for the golf ball to cover the 90 m horizontal displacement]</p> $T = \frac{90}{30 \cos \theta}$ $[s =] 30 \sin \theta T - 0.5 \times 9.8 \times T^2$ $[s =] 30 \sin \theta \times \frac{90}{30 \cos \theta} - 0.5 \times 9.8 \times \left(\frac{90}{30 \cos \theta} \right)^2$ $[s =] 90 \tan \theta - \frac{44.1}{\cos^2 \theta}$ $[s =] 90 \tan \theta - 44.1 \times (1 + \tan^2 \theta)$ $-2.4 = 90 \tan \theta - 44.1 - 44.1 \tan^2 \theta$ $[44.1 \tan^2 \theta - 90 \tan \theta + 41.7 = 0]$ $\tan \theta = 1.3296..., 0.7111...$ $\theta = 53.054..., 35.417...$ $\theta = 53.1, \text{ or } \theta = 35.4$	<p>B1</p> <p>M1 A1</p> <p>m1</p> <p>m1</p> <p>B1</p> <p>m1</p> <p>A1</p> <p>A1</p>	<p>Any subject</p> <p>M1: Use of $s = ut + \frac{1}{2}at^2$ with $u = 30 \sin \theta$ and $a = \pm 9.8$ PI A1: Fully correct</p> <p>Eliminating T PI</p> <p>Correct simplification with second term in $\cos^2 \theta$</p> <p>Use of $\sec^2 \theta = 1 + \tan^2 \theta$ PI</p> <p>Use of $s = -2.4$</p> <p>PI</p> <p>At least one unrounded value of θ</p> <p>Both values of θ to 3 sf and no others</p>
		9	

Q	Answer	Marks	Comments
8(b)	$T = \frac{90}{30 \cos \theta}$ $T = \frac{90}{30 \cos(53.054...^\circ)}$ $T = 5.0 \text{ [s, to 2 sf]}$	<p>M1</p> <p>A1ft</p>	<p>or use of their largest angle with $s = ut + \frac{1}{2}at^2$, $u = 30 \sin \theta$ and $a = \pm 9.8$ PI</p> <p>ft their larger angle from part (a) Note: unrounded answer is 4.99... [s]</p>
		2	

Q	Answer	Marks	Comments
8(c)	The assumption is unlikely to be true [as the golf ball is moving quickly through the air]	E1	Allow any sensible comment
		1	

	Question 8 Total	12	
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