



Mark Scheme (Final)

Summer 2026

Pearson Edexcel International GCSE

In Further Pure Mathematics (4PM1)

Paper 2R

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General Marking Guidance

All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.

Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.

Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.

There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.

All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.

Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.

When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.

Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

- **Types of mark**

- M marks: method marks
- A marks: accuracy marks – can only be awarded when relevant M marks have been gained
- B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**

- cao – correct answer only
- cso – correct solution only
- ft – follow through
- isw – ignore subsequent working
- SC - special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- awrt – answer which rounds to
- eeo – each error or omission

- **No working**

If no working is shown then correct answers may score full marks

If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**

If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.

If a candidate misreads a number from the question: eg. uses 252 instead of 255; follow through their working and deduct 2A marks from any gained provided the work has not been simplified. (Do not deduct any M marks gained.)

If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used

Examiners should send any instance of a suspected misread to review (but see above for simple misreads).

- **Ignoring subsequent work**

It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. incorrect cancelling of a fraction that would otherwise be correct.

It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.

- **Parts of questions**

Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c| \text{ leading to } x = \dots$$

$$(ax^2 + bx + c) = (mx + p)(nx + q) \text{ where } |pq| = |c| \text{ and } |mn| = |a| \text{ leading to } x = \dots$$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for a , b and c , leading to $x = \dots$

3. Completing the square:

$$x^2 + bx + c = 0: \left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0 \quad \text{leading to } x = \dots$$

Method marks for differentiation and integration:

1. Differentiation

$$\text{Power of at least one term decreased by 1. } (x^n \rightarrow x^{n-1})$$

2. Integration:

$$\text{Power of at least one term increased by 1. } (x^n \rightarrow x^{n+1})$$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show....")

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the rule may allow the mark to be awarded before the final answer is given.

Question number	Scheme	Marks
1(a)	$(3x-2)(x-5)$	B1 [1]
(b)	<p>Uses Factor theorem</p> $f\left(\frac{2}{3}\right) = 3 \times \left(\frac{2}{3}\right)^3 - 11 \times \left(\frac{2}{3}\right)^2 - 24 \times \left(\frac{2}{3}\right) + 20 = 0$ <p>OR</p> $f(5) = 3 \times (5)^3 - 11 \times (5)^2 - 24 \times (5) + 20 = 0$ <p>Conclusion:</p> <p>As $f\left(\frac{2}{3}\right) = 0$ <u>and</u> $f(5) = 0$ so $(x-5)$ and $(x-2/3)$ <u>are factors of</u></p> <p>$f(x)$ so $3x^2 - 17x + 10$ is <u>also a factor</u> of $f(x)$</p> <p>ALT Polynomial division</p> $\begin{array}{r} x+2 \\ 3x^2-17x+10 \overline{) 3x^3-11x^2-24x+20} \end{array}$ <p>Conclusion: There is <u>no remainder</u> so $3x^2 - 17x + 10$ <u>is a factor</u> of $f(x)$</p>	<p>M1</p> <p>A1</p> <p>A1 [3]</p> <p>[M1A1 A1]</p>
Total		4 marks

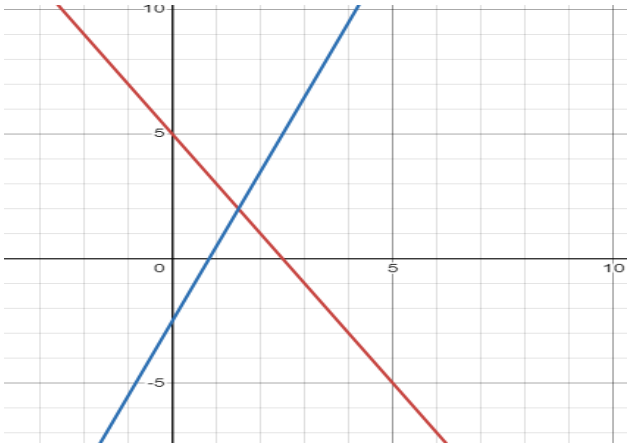
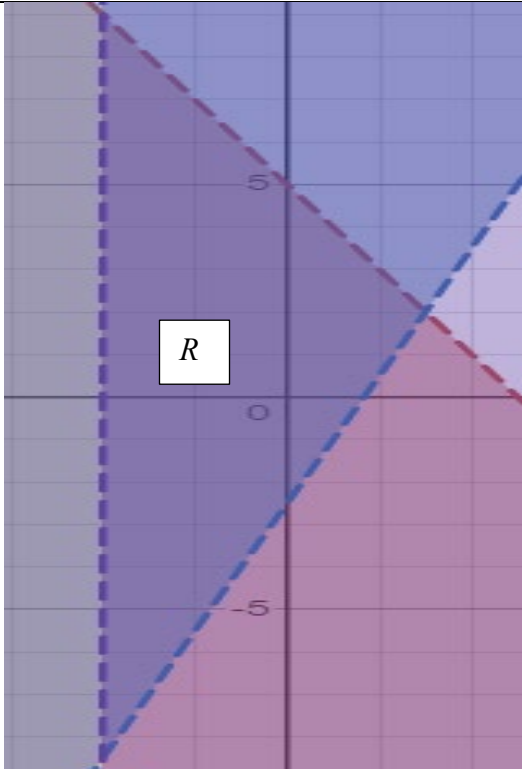
Part	Mark	Notes
1 (a)	B1	For the correct factorisation
(b)	Using factor theorem	
	M1	For the correct application of factor theorem on at least one factor
	A1	For a complete and correct use of factor theorem on both factors.
	A1	Correct conclusion stated
	Using polynomial division	
	M1	Applies polynomial division correctly to obtain at least $x \pm 2$
	A1	Fully correct division
	A1	Correct conclusion stated

Question number	Scheme	Marks
2 (a)	$y = \int (6x^2 - x - 12) dx = \frac{6x^3}{3} - \frac{x^2}{2} - 12x + c$ $\text{At } (2, -1) \quad -1 = 2 \times 2^3 - \frac{2^2}{2} - 12 \times 2 + c \Rightarrow c = \dots$ $(y =) 2x^3 - \frac{x^2}{2} - 12x + 9$	M1 M1 A1 [3]
(b)	$6x^2 - x - 12 > 0$ $\text{c.v's } (2x - 3)(3x + 4) = 0 \Rightarrow x = \frac{3}{2}, -\frac{4}{3}$ $x < -\frac{4}{3}, x > \frac{3}{2}$	M1 M1A1 [3]
Total		6 marks

Part	Mark	Notes
2 (a)	M1	For an attempt to integrate the given expression. +c not required for this mark See General Guidance for the definition of an attempt
	M1	Uses the given coordinates to find the value of c
	A1	For the expression for C written in full on one line. "y =" or "f(x) =" are not required here.
(b)	M1	Factorises the given expression or solves the 3TQ by any acceptable method. See General Guidance
	M1	Writes down the correct boundaries of the region of acceptable values
	A1	For the correct inequality stated. Accept any correct notation.

Question number	Scheme	Marks
3 (i)	$\frac{16^{(x+2)} - 4^{(2x+3)}}{2^x \times 8^{(x+1)}} = k \Rightarrow \frac{2^{(4x+8)} - 2^{(4x+6)}}{2^x \times 2^{(3x+3)}} = k$ <p>Method 1</p> $\Rightarrow \frac{2^{4x} \cdot 2^6 (2^2 - 1)}{2^{4x} \cdot 2^3} = k$ $\Rightarrow 2^3 \times 3 = k$ $\Rightarrow k = 24$ <p>Method 2</p> $\Rightarrow \frac{2^{4x} \cdot 2^3 (2^2 - 1)}{2^{4x}} = k$ $\Rightarrow 2^5 - 2^3 = k$ $\Rightarrow k = 24$	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1 [4]</p> <p>[M1</p> <p>M1</p> <p>A1]</p>
(ii)	$\frac{10}{\sqrt{12}-2} \times \frac{\sqrt{12}+2}{\sqrt{12}+2} = \frac{10\sqrt{12}+20}{8} =, \frac{20\sqrt{3}+20}{8} = \frac{5\sqrt{3}+5}{2}$ $\Rightarrow A = 5, \quad B = 2$ <p>ALT</p> $\frac{10}{\sqrt{12}-2} = \frac{A\sqrt{3}+A}{B} \Rightarrow 10B = (\sqrt{12}-2)(A\sqrt{3}+A)$ $10B = A\sqrt{12} \times 3 + A\sqrt{12} - 2A\sqrt{3} - 2A$ $= 6A - 2A = 4A$ $\Rightarrow \frac{A}{B} = \frac{10}{4} = \frac{5}{2}$ $\Rightarrow A = 5, \quad B = 2$	<p>M1,M1</p> <p>A1 [3]</p> <p>[M1</p> <p>M1</p> <p>A1]</p>
Total		7 marks

Part	Mark	Notes
3 (i)	M1	Changes 4, 8 and 16 into powers of 2
	M1	Method 1: Combines the powers and takes out the common factors/eliminates the terms involving x
	M1	Simplifies to obtain a value for k
	A1	For the correct value
	[M1]	Method 2: Combines the powers and takes out 2^3 and eliminates the terms involving x
	[M1]	Simplifies to obtain a value for k
	[A1]	For the correct value
(ii)	Method 1	
	M1	Multiplies numerator and denominator by $\sqrt{12} + 2$ - allow one slip.
	M1	Simplifies $\sqrt{12}$ to $2\sqrt{3}$ and simplifies to required form
	A1	For $A = 5$, $B = 2$ allow embedded values
	Method 2	
	M1	Multiplies through by $\sqrt{12} - 2$ - allow one slip.
	M1	Simplifies to reach an expression for A in terms of B only
	A1	For $A = 5$, $B = 2$ allow embedded values

Question number	Scheme	Marks
4(a)	 <p>Both correct lines</p>	B2 [2]
(b)	 <p>For the correct region</p>	B1 [1]
(c)	<p>Intersection points are $(-2, 9)$ $(1.5, 2)$ $(-2, -8.5)$</p> <p>$P = 3 \times -2 - 2 \times -8.5 = 11$</p> <p>$P = 3 \times -2 - 2 \times 9 = -24$</p> <p>$P = 3 \times 1.5 - 2 \times 2 = 0.5$</p> <p>Greatest value = 11</p>	M1A1 M1 A1 [4]
Total		7 marks

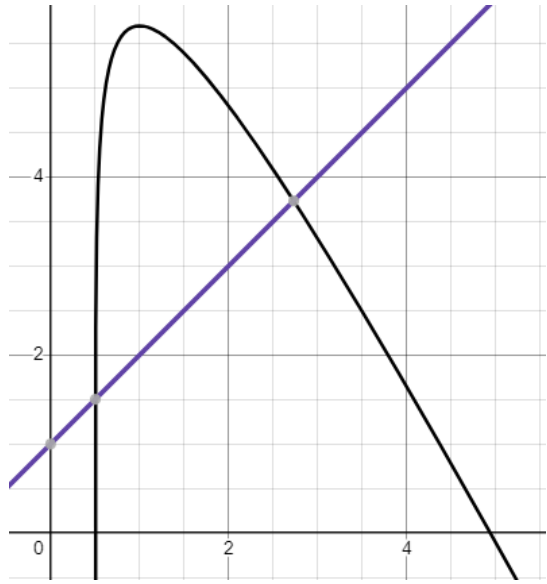
Part	Mark	Notes
4 (a)	B1	For one correct line, passing through both axes Line must pass through (-2, -8.5) and (0, -2.5). Allow one small square tolerance
	B1	For both correct lines, passing through both axes Line must pass through (-2, 9) and (0, 5). Allow one small square tolerance
(b)	B1ft	For the correct region, provided it is closed
(c)	M1	For at least one acceptable pair of coordinates
	M1	For calculating a value for P using their coordinates ACCURACY: Where students read co-ordinate values from their graph, allow a tolerance of ± 0.2 on all values. This mark appears as an A in ePen.
	A1	For acceptable value of P for one of the coordinates Acceptable range of values for top vertex is $-24.4 \leq P \leq -23.6$ Acceptable range of values for middle vertex is $0.3 \leq x \leq 0.7$ Acceptable range of values for bottom vertex is $10.6 \leq P \leq 11.4$ This mark appears as an M in ePen.
	A1	For selecting the greatest value. Allow students to use their values for co-ordinates when awarding this mark. Acceptable range of values is $10.6 \leq x \leq 11.4$

Question number	Scheme	Marks
5(a)	$6x - x^2 = -(x^2 - 6x), = -[(x-3)^2 - 9] = -(x-3)^2 + 9$ $a = -1 \quad b = -3 \quad c = 9$	M1,M1 A1 [3]
(b)	(i) 9 (ii) 3	B1ft B1ft [2]
(c)	$6x - x^2 = 3x \Rightarrow x^2 - 3x = 0 \Rightarrow x(x-3) = 0 \Rightarrow x = 0, 3$ $V = \pi \int_0^3 (6x - x^2)^2 dx - \pi \int_0^3 (3x)^2 dx = \left[\pi \int_0^3 (27x^2 - 12x^3 + x^4) dx \right]$ $V = \pi \left[\frac{27x^3}{3} - \frac{12x^4}{4} + \frac{x^5}{5} \right]_0^3$ $V = \pi \left[\left(\frac{27 \times 3^3}{3} - \frac{12 \times 3^4}{4} + \frac{3^5}{5} \right) - (0) \right]$ $V = \frac{243}{5} \pi$ <p>ALT</p> $6x - x^2 = 3x \Rightarrow x^2 - 3x = 0 \Rightarrow x(x-3) = 0 \Rightarrow x = 0, 3$ $V = \pi \int_0^3 (6x - x^2)^2 dx - \frac{1}{3} \pi \times 9^2 \times 3 = \left[\pi \int_0^3 (36x^2 - 12x^3 + x^4) dx - \frac{1}{3} \pi \times 9^2 \times 3 \right]$ $V = \pi \left[\frac{36x^3}{3} - \frac{12x^4}{4} + \frac{x^5}{5} \right]_0^3 - 81\pi$ $V = \pi \left[\left(\frac{36 \times 3^3}{3} - \frac{12 \times 3^4}{4} + \frac{3^5}{5} \right) - (0) \right] - 81\pi$ $V = \frac{243}{5} \pi$	M1A1 M1 M1 A1 [6]
Total		11 marks

Part	Mark	Notes
5 (a)	M1	Takes out – 1 as a common factor
	M1	For completing the square. See General Marking guidance on method for completing the square.
	A1	For the correct values. Accept embedded values
(b)		Accept the correct values in (b) obtained by differentiation.
	B1ft	For 9, follow through their c
	B1ft	For 3, ft their $-b$
(c)	M1	For setting the line = curve
	A1	For BOTH values of x
	M1	For setting out the correct method including their values of x and π
	M1	For a correct attempt at integration. See General guidance for the definition of an attempt. Ignore limits and absence or presence of π for this mark.
	M1	For substituting their limits into a changed expression. Ignore absence of π for this mark Substitution must be shown explicitly if the expression is not correct. Terms that have a value of 0 may be omitted.
	A1 cao	For $V = \frac{243}{5}\pi$ only.
	ALT	
	M1	For setting the line = curve
	A1	For BOTH values of x
	M1	For setting out the correct method including their values of x and π
	M1	For an attempt to integrate the curve squared. See General guidance for the definition of an attempt. Ignore limits and absence or presence of π for this mark.
	M1	For substituting their limits into their integrated expression, and evaluating the volume of the cone using their 9 and 3 correctly. Ignore absence of π for this mark
	A1 cao	For $V = \frac{243}{5}\pi$ only.

Question number	Scheme	Marks
6(a)	$S = 2\pi r^2 + 2\pi rh \Rightarrow h = \frac{S - 2\pi r^2}{2\pi r}$ $V = \pi r^2 \left(\frac{S - 2\pi r^2}{2\pi r} \right) = \frac{r}{2} (S - 2\pi r^2) \quad *$	B1 M1A1 cso [3]
(b)	$V = \frac{Sr}{2} - \pi r^3$ $\frac{dV}{dr} = \frac{900}{2} - 3\pi r^2 = 0 \Rightarrow r^2 = \frac{150}{\pi} \Rightarrow r = 6.90988... \approx 6.9$ $\frac{d^2V}{dr^2} = -6\pi r \Rightarrow \frac{d^2V}{dr^2} < 0 \Rightarrow \text{hence maximum}$ $V = \frac{6.90988}{2} (900 - 2\pi \times 6.90988^2) = 2072.96... \approx 2100$	M1M1A1 B1 M1A1 [6]
Total		9 marks

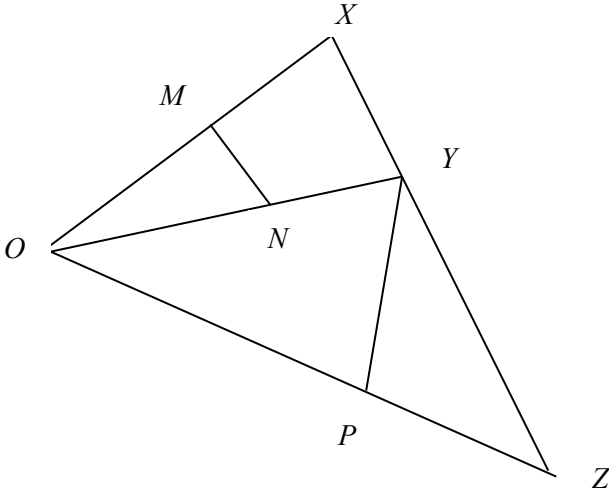
Part	Mark	Notes
6 (a)	B1	Finds a correct expression for h using a correct S
	M1	Substitutes their h into the correct expression for the volume of a cylinder.
	A1* cso	For the correct expression with no errors
(b)	M1	For differentiating the given expression. Accept as a minimum $\frac{dV}{dr} = A \pm B\pi r^2$
	M1	For setting their $\frac{dV}{dr} = 0$ and solving to find a value for r
	A1	For awrt $r = 6.9$
	B1	For applying any acceptable method to justify that the value of r is a maximum
	M1	For using the given value of S and the expression with their r to find a max value of V
	A1	For awrt 2100

Question number	Scheme	Marks
7	$3^{3x-6} + 2 = 4x$ $\Rightarrow 3^{3x-6} = 4x - 2 \Rightarrow 3x - 6 = \log_3(4x - 2)$ $\Rightarrow \log_3(4x - 2) - 2x + 7 = x + 1$ <p>or</p> $\log_3(4x - 2) - 3x - 6 = \log_3(4x - 2) - 2x + 7 - y$ $\Rightarrow y = x + 1$ <p>or</p> $y = \log_3(4x - 2) - 2x + 7 = 3x - 6 - 2x + 7 = x + 1$ <p>Draws the line $y = x + 1$ $x = 2.7$</p> 	<p>M1M1</p> <p>A1</p> <p>M1</p> <p>A1</p>
Total		5 marks

Mark	Notes
7 M1	Rearranges both sides and takes the log of both sides.
M1	Manipulates the resulting equation, using the given value for the curve, to eradicate the log term, leaving an equation for y
A1	Obtains the equation of the straight line $y = x + 1$
M1	Draws the line $y = x + 1$ on the grid Previous marks may be implied by a correct line here, if not contradicted by earlier incorrect work.
A1	For $x = 2.7$ or 2.8 – ignore values outside the given range.

Question number	Scheme	Marks
8(a)	$\frac{y-5}{1-7} = \frac{x-1}{5-1}$ $\Rightarrow 6(y-5) = -6(x-1)$ $\Rightarrow 6y+6x=36$ $\Rightarrow y+x=6 \quad *$ ALT $L_m = \frac{5-(-1)}{1-7} = -1$ $\Rightarrow y-5 = -1(x-1)$ $\Rightarrow y = -x+6$ $\Rightarrow y+x=6$	M1 A1cso [2] [M1 A1cso]
(b)	$(x-7)^2 + (y-5)^2 = 18 \Rightarrow (x-7)^2 + ([6-x]-5)^2 = 18$ $\Rightarrow x^2 - 14x + 49 + 1 - 2x + x^2 = 18 \Rightarrow x^2 - 8x + 16 = 0$ $\Rightarrow (x-4)(x-4) = 0 \text{ so one root at } x = 4$ or $b^2 - 4ac = (-8)^2 - 4 \times 1 \times 16 = 0 \text{ hence only one root}$ Conclusion: Since there is only one root, the contact with the curve must be a tangent *	M1 M1 A1cso [3]
(c)	$x = 4, \quad y = 6 - 4 = 2$ $(4, 2)$	B1, B1 [2]
(d)	Gradient of normal = $-\frac{1}{L_m} = \frac{-1}{-1} = 1$ $y - 2 = 1(x - 4) \Rightarrow y = x - 2$	B1 M1A1 [3]
(e)	$(x-7)^2 + (x-2-5)^2 = 18 \Rightarrow 2(x-7)^2 = 18, \Rightarrow x-7 = \pm 3$ $x = 10, (4)$ $y = 10 - 2 = 8$ $ AB = \sqrt{(10-4)^2 + (8-2)^2} = 6\sqrt{2}$ ALT $a^2 + b^2 = c \text{ is a circle with radius } \sqrt{c}$ $\Rightarrow (x-7)^2 + (y-5)^2 = 18 \text{ has radius } \sqrt{18} = 3\sqrt{2}$ Normal to a circle bisects it along a diameter $\Rightarrow AB = 2 \times 3\sqrt{2} = 6\sqrt{2}$	M1, M1 A1 B1 M1A1 [6] [M1M1 A1 M1 M1A1]
Total		16 marks

Part	Mark	Notes
8 (a)	M1	Uses any method to find the equation of the line using the given coordinates.
	A1* cso	For the correct equation in the required form
(b)	M1	For substituting the equation of L into S and forming a 3TQ
	M1	Solves their 3TQ
	A1* cso	Concludes that as there are repeated roots, the line L must be a tangent to S
(c)	B1	Either $x = 4$ or $y = 2$
	B1	Both $x = 4$ and $y = 2$
(d)	B1	For the gradient of the normal.
	M1	For forming an equation correctly using their coordinates of A
	A1	For the correct equation in the required form.
(e)		An answer of $6\sqrt{2}$ with minimal working is possible, and should be awarded full marks
	M1	Substitutes the equation of their normal into the equation of S
	M1	For solving their quadratic equation
	A1	For finding $x = 10$
	B1	For $y = 8$
	M1	For using a correct method to find the length AB
	A1	For the length $6\sqrt{2}$

Q No	Scheme	Marks
9(a)	$\vec{XY} = -\vec{OX} + \vec{OY} = -(4\mathbf{b} - 3\mathbf{a}) + 6\mathbf{b} = 2\mathbf{b} + 3\mathbf{a}$ $\vec{YZ} = -\vec{OY} + \vec{OZ} = -6\mathbf{b} + 12\mathbf{b} + 9\mathbf{a} = 6\mathbf{b} + 9\mathbf{a}$ $\vec{XZ} = -\vec{OX} + \vec{OZ} = -(4\mathbf{b} - 3\mathbf{a}) + 12\mathbf{b} + 9\mathbf{a} = 8\mathbf{b} + 12\mathbf{a}$ $\vec{YZ} = 3\vec{XY} \text{ or } \vec{XZ} = 4\vec{XY} \text{ and both vectors share a point, hence collinear}$	<p>B1 (one)</p> <p>B1 (any two)</p> <p>M1A1 [4]</p>
(b)	 $\text{Area}_{MNYX} = \left \frac{1}{2} \begin{bmatrix} 3 & 6 & 4 & 2 & 3 \\ 0 & 0 & -3 & -1.5 & 0 \end{bmatrix} \right $ $= \left \frac{1}{2} [(0 - 18 - 6 + 0) - (0 + 0 - 6 - 4.5)] \right = \left \frac{1}{2} \times -\frac{27}{2} \right = \frac{27}{4}$ $\text{Area}_{YPZ} = \left \frac{1}{2} \begin{bmatrix} 6 & 8 & 12 & 6 \\ 0 & 6 & 9 & 0 \end{bmatrix} \right $ $= \left \frac{1}{2} (36 + 72 + 0) - (0 + 72 + 54) \right = \left \frac{1}{2} (-18) \right = 9$ $\text{Area}_{MNYX} : \text{Area}_{YPZ} = 6.75 : 9 = (3 : 4)$ <p>ALT</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1A1 [6]</p> <p>[M1M1]</p>

$\triangle OMN = \frac{1}{2} \times \frac{1}{2} \overline{OX} \times \frac{1}{2} \overline{OY} \times \sin O = \frac{1}{4} \triangle OXY \triangle YPZ$ $\Rightarrow \square MXYN = \frac{3}{4} \triangle OXY$ $\triangle OXY = \frac{1}{2} \times \overline{OX} \times \overline{XY} \times \sin X$ $\triangle OXZ = \frac{1}{2} \times \overline{OX} \times 4 \overline{XY} \times \sin X$ $\Rightarrow \triangle OXY = \frac{1}{4} \triangle OXZ = \frac{1}{3} \triangle OYZ$ $\triangle YPZ = \frac{1}{2} \times \overline{YZ} \times \frac{1}{3} \overline{OZ} \times \sin Z = \frac{1}{3} \triangle OYZ$ $\Rightarrow \triangle YPZ = \triangle OXY$ $\Rightarrow \square MXYN : \triangle YPZ \text{ is } 3 : 4$ <p>For reference, the areas are:</p> $\triangle OXZ = k$ $\triangle OXY = \frac{1}{4} k$ $\triangle OYZ = \frac{3}{4} k$ $\triangle OMN = \frac{1}{16} k$ $\triangle YPZ = \frac{1}{4} k$ $\square MXYN = \frac{3}{16} k$	<p>A1</p> <p>M1M1</p> <p>A1] [6]</p>
Total	10 marks

Part	Mark	Notes
9 (a)	B1	For one correct vector from $\vec{XY}$ or $\vec{XZ}$ or $\vec{YZ}$
	B1	For a second correct vector from $\vec{XY}$ or $\vec{XZ}$ or $\vec{YZ}$
	M1	For establishing that their found vectors are multiples of one another.
	A1	For a correct and complete conclusion
(b)	M1	For finding the correct ratio of $OXY : OYZ$
	M1	For finding the ratio of OXY to $MXYN$
	A1	For the correct ratio
	M1	For the correct ratio of YPZ to OYZ
	M1	For using their results to obtain the required ratio
	A1	For the correct ratio of 3 : 4

23

(b)	$\frac{dy}{dx} = \frac{3(2x-1) - 2(3x+2)}{(2x-1)^2}$ <p>When $x = 0$</p> $\frac{dy}{dx} = -7$ <p>Gradient of normal $= \frac{1}{7}$</p> <p>Equation of normal $\Rightarrow y + 2 = \frac{1}{7}x$</p> <p>Intersection of C and l</p> $\frac{1}{7}x - 2 = \frac{3x+2}{2x-1} \Rightarrow 2x^2 - 50x = 0 \Rightarrow x = 25, (0)$ 	<p>M1A1</p> <p>M1</p> <p>B1ft</p> <p>M1</p> <p>dM1A1 cao [7]</p>
Total		12 marks

Part	Mark	Notes
10 (a)	B1	For the correct shape anywhere on the grid. Both branches to be shown divided by asymptotes, dividing into first and third quadrants.
	B1	For the shape in the correct position, relative to the asymptotes, with no intersection.
	B1	For the correct equation of the asymptote seen $y = 1.5$ and drawn correctly. May be seen as a solid line.
	B1	For the correct equation of the vertical asymptote seen $x = 0.5$ and drawn correctly. May be seen as a solid line.
	B1	For both coordinates of the intersections seen marked correctly. Allow for only x -coordinate or y -coordinate marked on the axes. Expect to see fractions, but use of decimal value for x is acceptable if rounding to -0.67
(b)		If no differentiation is seen here, no marks can be gained.
	M1	For an attempt at quotient rule. $\frac{a(2x-1) - b(3x+2)}{(2x-1)^2}$ where $a, b > 0$ Also accept use of the product rule applied to $(3x+2) \times (2x-1)^{-1}$
	A1	For the correct derivative. Simplification not required for this mark.
	M1	For finding the gradient of the tangent from their derivative.
	B1ft	For correctly inverting their tangent gradient (gained from their calculation of a derivative) to find the gradient of the normal.
	M1	For the equation of the normal based on their gradient of the normal, which must not be the same as the tangent gradient. This is a B1 mark on ePen.
	dM1	For equating their equation of the normal with the equation of the curve AND solving to find a value for x . Depends on previous M1 mark.
	A1 cao	$x = 25$

Question number	Scheme	Marks
11(a)	$\sin 3\theta = [\sin(2\theta + \theta)] = \sin 2\theta \cos \theta + \sin \theta \cos 2\theta$ $[\cos 2\theta = 1 - 2\sin^2 \theta] \text{ oe}$ $\sin 3\theta = 2\sin \theta \cos \theta \cos \theta + \sin \theta (1 - 2\sin^2 \theta)$ or $\sin 3\theta = 2\sin \theta \cos \theta \cos \theta + \sin \theta (\cos^2 \theta - \sin^2 \theta)$ $\sin 3\theta = 2\sin \theta (1 - \sin^2 \theta) + \sin \theta (1 - 2\sin^2 \theta)$ $\sin 3\theta = 3\sin \theta - 4\sin^3 \theta *$	M1 M1 M1 M1 A1 cso [5]
(b)	$[8\sin^3 A = 6\sin A + 1 \Rightarrow 8\sin^3 A - 6\sin A = 1]$ $8\sin^3 A - 6\sin A = 1 \Rightarrow -2\sin 3A = 1$ $\sin 3A = -\frac{1}{2} \Rightarrow 3A = 210^\circ, 330^\circ, (\text{eg. } -150^\circ, -30^\circ, 570^\circ, 690^\circ)$ $A = 70^\circ, 110^\circ$	M1 M1A1 A1 [4]
(c)	$\sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)$ $\int_0^{\frac{\pi}{3}} \sin^3 x \, dx = \frac{1}{4} \int_0^{\frac{\pi}{3}} (3\sin x - \sin 3x) \, dx$ $= \frac{1}{4} \left[-3\cos x + \frac{\cos 3x}{3} \right]_0^{\frac{\pi}{3}}$ $= \frac{1}{4} \left[\left(-3\cos\left(\frac{\pi}{3}\right) + \frac{\cos 3\left(\frac{\pi}{3}\right)}{3} \right) - \left(-3\cos(0) + \frac{\cos 3(0)}{3} \right) \right]$ $= \frac{5}{24}$	M1 M1 M1 A1 [4]
Total		13 marks

Part	Mark	Notes
11 (a)	M1	Uses the correct identity for $\sin(A + B)$ correctly
	M1	Uses a correct double-angle identity for $\cos 2\theta$ to obtain $1 - 2\sin^2 \theta$
	M1	Uses the correct identity for $\sin 2A$ correctly
	M1	Uses the Pythagorean identity for $\cos^2 \theta$
	A1* cso	For the correct answer as shown.
(b)	M1	Obtains $-2\sin 3A = 1$ oe
	M1	Finds at least one correct angle for $3A$ which may be implied by any correct value for A even if out of range.
	A1	Obtains at least one correct angle for A
	A1	Obtains both correct angles within the required range If extra angles are given within the range, this mark is not awarded. Ignore extra angles outside the range.
(c)	M1	For correct substitution for $\sin^3 x$. This step must be seen, but may be implied by acceptable integration.
	M1	For an attempt to integrate two terms, to achieve $-a \cos x + b \cos 3x$ where $a, b > 0$. This step must be explicitly seen.
	M1	For substituting BOTH given limits. Substitution must be explicitly seen in full if the integration is not fully correct. If, however, every previous step is correct, this mark can be awarded for $\frac{5}{24}$ seen without explicit substitution.
	A1	For $\frac{5}{24}$