

AP[®] Statistics Practice Exam

From the 2015 Administration

This Practice Exam is provided by the College Board for AP Exam preparation. Teachers are permitted to download the materials and make copies to use with their students in a classroom setting only. To maintain the security of this exam, teachers should collect all materials after their administration and keep them in a secure location. **Exams may not be posted on school or personal websites, nor electronically redistributed for any reason.** Further distribution of these materials outside of the secure College Board site disadvantages teachers who rely on uncirculated questions for classroom testing. Any additional distribution is in violation of the College Board's copyright policies and may result in the termination of Practice Exam access for your school as well as the removal of access to other online services such as the AP Teacher Community and Online Score Reports.

PLACE SEAL HERE

AP[®] Statistics Exam

SECTION I: Multiple Choice

2015

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time

1 hour, 30 minutes

Number of Questions

40

Percent of Total Score

50%

Writing Instrument

Pencil required

Electronic Device

Graphing calculator
expected

Instructions

Section I of this exam contains 40 multiple-choice questions. Fill in only the circles for numbers 1 through 40 on your answer sheet.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding circle on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question Sample Answer

Chicago is a (A) ● (C) (D) (E)
(A) state
(B) city
(C) country
(D) continent
(E) village

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

PLACE SEAL HERE

Form I
Form Code 4KBP6-S

90

PLACE SEAL HERE

DO NOT seal answer sheet inside

Formulas

(I) Descriptive Statistics

$$\bar{x} = \frac{\sum x_i}{n}$$

$$s_x = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2}$$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}}$$

$$\hat{y} = b_0 + b_1 x$$

$$b_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$$b_0 = \bar{y} - b_1 \bar{x}$$

$$r = \frac{1}{n-1} \sum \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right)$$

$$b_1 = r \frac{s_y}{s_x}$$

$$s_{b_1} = \frac{\sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n-2}}}{\sqrt{\sum (x_i - \bar{x})^2}}$$

(II) Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$E(X) = \mu_x = \sum x_i p_i$$

$$\text{Var}(X) = \sigma_x^2 = \sum (x_i - \mu_x)^2 p_i$$

If X has a binomial distribution with parameters n and p , then:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

$$\mu_x = np$$

$$\sigma_x = \sqrt{np(1 - p)}$$

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1 - p)}{n}}$$

If $\bar{x}$ is the mean of a random sample of size n from an infinite population with mean μ and standard deviation σ , then:

$$\mu_{\bar{x}} = \mu$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

(III) Inferential Statistics

Standardized test statistic: $\frac{\text{statistic} - \text{parameter}}{\text{standard deviation of statistic}}$

Confidence interval: $\text{statistic} \pm (\text{critical value}) \cdot (\text{standard deviation of statistic})$

Single-Sample

Statistic	Standard Deviation of Statistic
Sample Mean	$\frac{\sigma}{\sqrt{n}}$
Sample Proportion	$\sqrt{\frac{p(1-p)}{n}}$

Two-Sample

Statistic	Standard Deviation of Statistic
Difference of sample means	$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ Special case when $\sigma_1 = \sigma_2$ $\sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$
Difference of sample proportions	$\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$ Special case when $p_1 = p_2$ $\sqrt{p(1-p)} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

$$\text{Chi-square test statistic} = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

STATISTICS

SECTION I

Time—1 hour and 30 minutes

Number of questions—40

Percent of total score—50

Directions: Solve each of the following problems, using the available space for scratch work. Decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

1. Which of the following statistics is defined as the 50th percentile?
 - (A) The mean
 - (B) The median
 - (C) The mode
 - (D) The interquartile range
 - (E) The standard deviation
-
2. A researcher wanted to estimate the average amount of money spent on extracurricular activities per school in a certain region. The researcher randomly selected 20 public schools and 20 private schools in the region to use for a sample. Which of the following best describes the type of sample that was taken?
 - (A) A census
 - (B) A cluster sample
 - (C) A convenience sample
 - (D) A simple random sample
 - (E) A stratified random sample

3. Students in a large psychology class measured the time, in seconds, it took each of them to perform a certain task. The times were later converted to minutes. If a student had a standardized score of $z = 1.72$ before the conversion, what is the standardized score for the student after the conversion?
- (A) $z = 0.26$
 - (B) $z = 1.03$
 - (C) $z = 1.72$
 - (D) $z = 1.98$
 - (E) The standardized score for the student after the conversion cannot be determined.
-

4. A researcher conducting a telephone survey is concerned about possible sources of bias. Of the following, which is the best example of nonresponse bias?
- (A) The wording of the questions in the survey leads people to respond in a certain way.
 - (B) The behavior of the interviewer leads people to respond in a certain way.
 - (C) People might be uncomfortable with the survey questions and, as a result, might not always respond to those questions truthfully.
 - (D) Many of the people selected to participate in the survey who do not respond might have opinions different from those who do respond.
 - (E) People without telephones are overlooked in the sampling procedure used to determine who is surveyed.

5. At a large conference of teachers from a variety of subjects, a random sample of 50 mathematics teachers attending the conference was selected. Among the selected mathematics teachers, 28 percent had taken one or more courses in statistics. For which of the following populations is 28 percent a reasonable estimate of the percentage of those who have taken one or more courses in statistics?
- (A) All mathematics teachers
 - (B) All mathematics teachers who attended the conference
 - (C) All mathematics teachers who have taken one or more courses in statistics
 - (D) All teachers who attended the conference
 - (E) All teachers

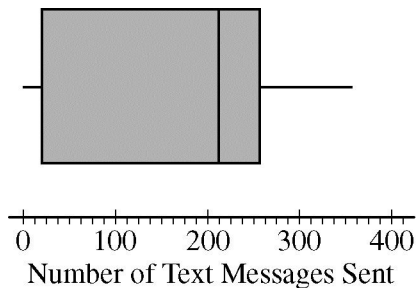
6. The prices, in thousands of dollars, of the 35 used cars at a certain car dealership are shown in the table below.

Price (in thousands)	\$7	\$8	\$9	\$10	\$11	\$12	\$13	\$14	\$15	\$16
Frequency	4	6	7	6	4	2	2	2	1	1

Which of the following best describes the shape of the distribution of used car prices at the dealership?

- (A) Skewed to the left (negatively skewed)
- (B) Skewed to the right (positively skewed)
- (C) Bimodal
- (D) Uniform
- (E) Approximately normal

7. Data were collected on the number of text messages sent by each student in a large high school for one day. A boxplot of the data is shown below.



Based on the boxplot, which of the following statements is the most reasonable conclusion?

- (A) There are more students with data values below the median than there are students with data values above the median.
- (B) There are more students with data values between the first quartile and the median than there are students with data values between the median and the third quartile.
- (C) There are fewer students with data values between the first quartile and the median than there are students with data values between the median and the third quartile.
- (D) There are approximately the same number of students with data values between the first quartile and the minimum as there are students with data values between the third quartile and the maximum.
- (E) The data are less spread out between the first quartile and the median than between the median and the third quartile.

8. On the day before an election in a large city, each person in a random sample of 1,000 likely voters is asked which candidate he or she plans to vote for. Of the people in the sample, 55 percent say they will vote for candidate Taylor. A margin of error of 3 percentage points is calculated. Which of the following statements is appropriate?
- (A) The proportion of all likely voters who plan to vote for candidate Taylor must be the same as the proportion of voters in the sample who plan to vote for candidate Taylor (55 percent), because the data were collected from a random sample.
 - (B) The sample proportion minus the margin of error is greater than 0.50, which provides evidence that more than half of all likely voters plan to vote for candidate Taylor.
 - (C) It is not possible to draw any conclusion about the proportion of all likely voters who plan to vote for candidate Taylor because the 1,000 likely voters in the sample represent only a small fraction of all likely voters in a large city.
 - (D) It is not possible to draw any conclusion about the proportion of all likely voters who plan to vote for candidate Taylor because this is not an experiment.
 - (E) It is not possible to draw any conclusion about the proportion of all likely voters who plan to vote for candidate Taylor because this is a random sample and not a census.

9. The caffeine content of 8-ounce cans of a certain cola drink is approximately normally distributed with mean 33 milligrams (mg). A randomly selected 8-ounce can containing 35 mg of caffeine is 1.2 standard deviations above the mean. Approximately what percent of 8-ounce cans of the cola have a caffeine content greater than 35 mg?
- (A) 1%
 - (B) 8%
 - (C) 12%
 - (D) 16%
 - (E) 99%

-
10. A random variable X has a mean of 120 and a standard deviation of 15. A random variable Y has a mean of 100 and a standard deviation of 9. If X and Y are independent, approximately what is the standard deviation of $X - Y$?
- (A) 24.0
 - (B) 17.5
 - (C) 12.0
 - (D) 6.0
 - (E) 4.9

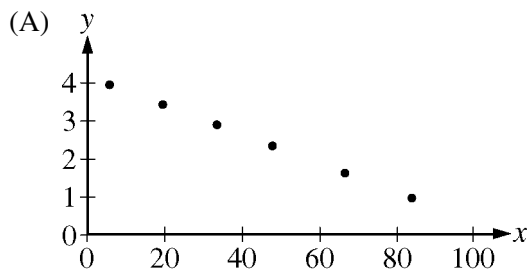
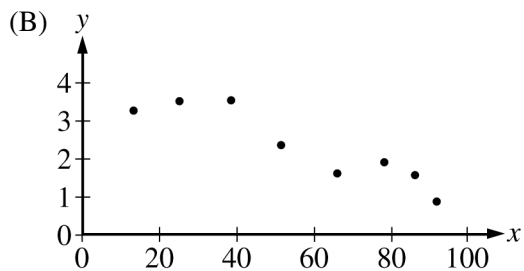
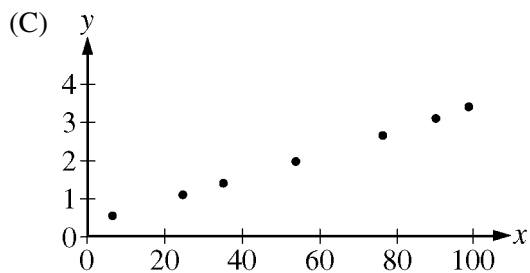
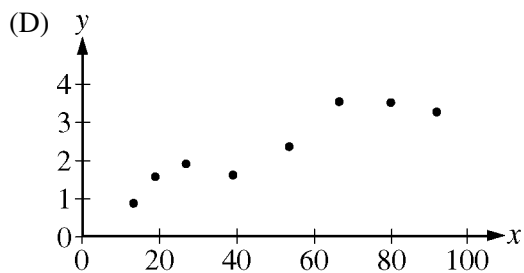
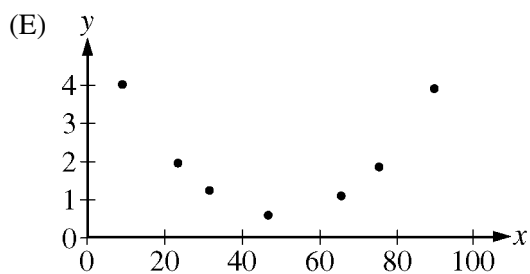
11. From a random sample of 50 people, sitting pulse rates and standing pulse rates were measured for each person. A coin was flipped to determine whether the sitting or the standing pulse rate would be measured first. Let μ_{sitting} represent the mean sitting pulse rate in the population, μ_{standing} represent the mean standing pulse rate in the population, and μ_d represent the mean of the differences between the sitting and standing (sitting – standing) pulse rates in the population. Which of the following represents an appropriate test and hypotheses to determine if there is a difference in mean pulse rates between sitting and standing in the population?
- (A) A two-sample t -test with $H_0 : \mu_{\text{sitting}} = \mu_{\text{standing}}$ and $H_a : \mu_{\text{sitting}} \neq \mu_{\text{standing}}$
- (B) A two-sample t -test with $H_0 : \mu_{\text{sitting}} = \mu_{\text{standing}}$ and $H_a : \mu_{\text{sitting}} < \mu_{\text{standing}}$
- (C) A two-sample z -test with $H_0 : \mu_{\text{sitting}} = \mu_{\text{standing}}$ and $H_a : \mu_{\text{sitting}} \neq \mu_{\text{standing}}$
- (D) A matched-pairs t -test with $H_0 : \mu_d = 0$ and $H_a : \mu_d \neq 0$
- (E) A matched-pairs t -test with $H_0 : \mu_d = 0$ and $H_a : \mu_d < 0$

12. Athletes in a particular sport are classified as either offense or defense. The distribution of weights for the athletes classified as offense is approximately normal, centered at 200 pounds, and ranges from 150 pounds to 250 pounds. The distribution of weights for the athletes classified as defense is approximately normal, centered at 300 pounds, and ranges from 250 pounds to 350 pounds. There are 1,000 athletes in each classification. Which of the following is the best description of a histogram of the weights of all 2,000 athletes?
- (A) Skewed to the right (positively skewed)
 - (B) Skewed to the left (negatively skewed)
 - (C) Approximately uniform and centered at 250 pounds
 - (D) Approximately normal and centered at 250 pounds
 - (E) Bimodal

13. Which of the following pairs of sample size n and population proportion p would produce the greatest standard deviation for the sampling distribution of a sample proportion $\hat{p}$?
- (A) $n = 1,000$ and p close to 0
 - (B) $n = 1,000$ and p close to 1
 - (C) $n = 1,000$ and p close to $\frac{1}{2}$
 - (D) $n = 100$ and p close to 0
 - (E) $n = 100$ and p close to $\frac{1}{2}$
-

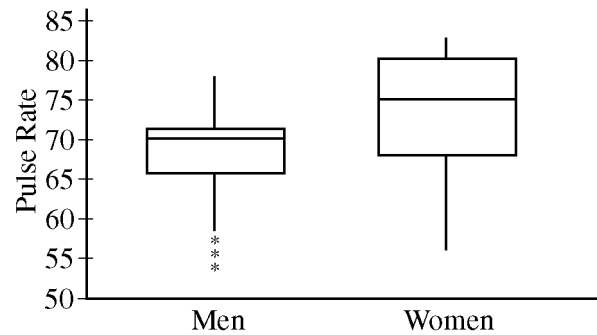
14. A 90 percent confidence interval for the slope of a regression line is determined to be $(-0.181, 1.529)$. Which of the following statements must be true?
- (A) The correlation coefficient of the data is positive.
 - (B) The sum of the residuals for the data based on the regression line is positive.
 - (C) A scatterplot of the data would show a linear pattern.
 - (D) The slope of the sample regression line is 1.348.
 - (E) The slope of the sample regression line is 0.

15. For which of the following scatterplots is the correlation between x and y closest to 0 ?



16. A matched-pairs t -test is NOT an appropriate way to analyze data consisting of which of the following?
- (A) Measurements of annual income taken both before and after a two-year training course for a random sample of 100 people who took the course
 - (B) Measurements of annual income for each twin for 100 randomly selected pairs of twins
 - (C) Measurements of annual income for both individuals in pairs formed by matching 100 people from State A and 100 people from State B based on level of education
 - (D) Measurements of annual income for both individuals in pairs formed by assigning 100 people to pairs at random
 - (E) Measurements of annual income recorded for both spouses of 100 randomly selected married couples

17. The pulse rate for each person in a sample of 20 men and 20 women was recorded. The boxplots below summarize the pulse rates for the men and the women in the sample.



Which of the following statements about the people in the sample must be true?

- (A) There are more people between the first and third quartiles for women than there are between the first and third quartiles for men.
- (B) The person with the lowest pulse rate is a woman.
- (C) At least half of the women had higher pulse rates than three-fourths of the men.
- (D) More than half of the men had lower pulse rates than three-fourths of the women.
- (E) If a man and a woman were randomly selected from the 40 people, the man would have the lower pulse rate.

18. An airline claims that the mean flight time between City X and City Y is 38 minutes. After taking many flights, a local business group believes that the claim is unrealistic and that the actual mean flight time is greater than 38 minutes. If the group conducts a study to investigate its belief, which of the following hypotheses should be tested?

(A) $H_0 : \bar{x} = 38$ versus $H_a : \bar{x} \neq 38$

(B) $H_0 : \bar{x} = 38$ versus $H_a : \bar{x} < 38$

(C) $H_0 : \bar{x} = 38$ versus $H_a : \bar{x} > 38$

(D) $H_0 : \mu = 38$ versus $H_a : \mu < 38$

(E) $H_0 : \mu = 38$ versus $H_a : \mu > 38$

19. Ali surveyed 200 students at a school and recorded the eye color and the gender of each student. Of the 80 male students who were surveyed, 60 had brown eyes. If eye color and gender are independent, how many female students surveyed would be expected to have brown eyes?
- (A) 5
 - (B) 20
 - (C) 30
 - (D) 90
 - (E) 100

20. A national health study reported that the proportion of students with elevated blood pressure is 0.15. The principal of a local high school believes that the proportion of students in the school with elevated blood pressure is greater than 0.15. If a large random sample is used, which of the following is the most appropriate test to investigate the principal's belief?
- (A) A z -test for a proportion
 - (B) A z -test for a difference between two proportions
 - (C) A chi-square test for homogeneity of proportions
 - (D) A t -test for a mean
 - (E) A matched-pairs t -test

21. Data on homes recently sold in a certain town included the area of the home, reported in square feet. The table below shows summary statistics of the reported areas, in square feet.

Mean	Minimum	Q1	Median	Q3	Maximum	Standard Deviation
1,754.14	1,656	1,704	1,758	1,806	1,843	61.0723

An auditor determined that an error was made in the reported areas and that all of the areas should have been 100 square feet greater than what was reported. The areas were corrected and new summary statistics were reported.

What are the interquartile range (IQR) and the standard deviation of the corrected areas?

- (A) IQR 102, standard deviation 61.0723
- (B) IQR 102, standard deviation 161.0723
- (C) IQR 202, standard deviation 61.0723
- (D) IQR 202, standard deviation 161.0723
- (E) IQR 187, standard deviation 61.0723

22. A two-sample t -test of the hypotheses $H_0 : \mu_1 - \mu_2 = 0$ versus $H_a : \mu_1 - \mu_2 > 0$ produces a p -value of 0.03.

Which of the following must be true?

- I. A 90 percent confidence interval for the difference in means will contain the value 0.
 - II. A 95 percent confidence interval for the difference in means will contain the value 0.
 - III. A 99 percent confidence interval for the difference in means will contain the value 0.
- (A) I only
- (B) III only
- (C) I and II only
- (D) II and III only
- (E) I, II, and III

23. A medical doctor uses a diagnostic test to determine whether a patient has arthritis. A treatment will be prescribed only if the doctor thinks the patient has arthritis. The situation is similar to using a null and an alternative hypothesis to decide whether to prescribe the treatment. The hypotheses might be stated as follows.

H_0 : The patient does not have arthritis.

H_a : The patient has arthritis.

Which of the following represents a Type II error for the hypotheses?

- (A) Diagnosing arthritis in a patient who has arthritis
- (B) Failing to diagnose arthritis in a patient who has arthritis
- (C) Diagnosing arthritis in a patient who does not have arthritis
- (D) Failing to diagnose arthritis in a patient who does not have arthritis
- (E) Prescribing treatment to a patient regardless of the diagnosis

24. A biologist wants to estimate the difference between the mean body lengths of green and brown stinkbugs. A random sample of 20 green stinkbugs has a mean body length of 16.22 millimeters (mm) and a standard deviation of 1.34 mm. A random sample of 20 brown stinkbugs has a mean body length of 13.41 mm and a standard deviation of 0.73 mm. What is the standard error of the difference (green – brown) between the sample means?

(A) $\sqrt{\frac{(1.34)^2 + (0.73)^2}{40}}$

(B) $\sqrt{\frac{(1.34)^2 - (0.73)^2}{40}}$

(C) $\sqrt{\frac{(1.34)^2 + (0.73)^2}{20}}$

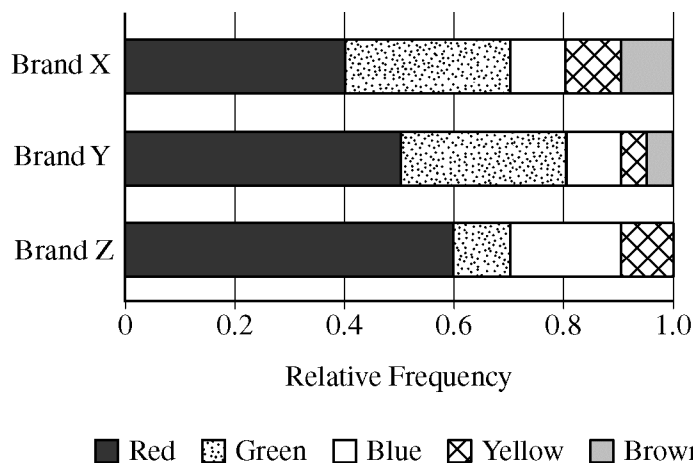
(D) $\sqrt{\frac{(1.34)^2 - (0.73)^2}{20}}$

(E) $\sqrt{\frac{(1.34) - (0.73)}{20}}$

25. A blind taste test will be conducted with 9 volunteers to determine whether people can taste a difference between bottled water and tap water. Each participant will taste the water from two different glasses and then identify which glass he or she thinks contains the tap water. Assuming that people cannot taste a difference between bottled water and tap water, what is the probability that at least 8 of the 9 participants will correctly identify the tap water?
- (A) 0.0020
(B) 0.0195
(C) 0.8889
(D) 0.9805
(E) 0.9980

26. A school administrator is interested in estimating the proportion of students in the district who participate in community service activities. From a random sample of 100 students in the district, the administrator will construct a 99 percent confidence interval for the proportion of all district students who participate in community service activities. Which of the following statements must be true?
- (A) The population proportion will be in the confidence interval.
 - (B) The probability that the confidence interval will include the population proportion is 0.99.
 - (C) The probability that the confidence interval will include the sample proportion is 0.99.
 - (D) The population proportion and the sample proportion will be equal.
 - (E) The probability that the population proportion and the sample proportion will be equal is 0.99.

27. Three brands of candy pieces—X, Y, and Z—are made in many colors. Shaela bought one bag of each brand and counted the number of pieces of each color. The graph below shows the relative frequency distribution of colors for each bag.

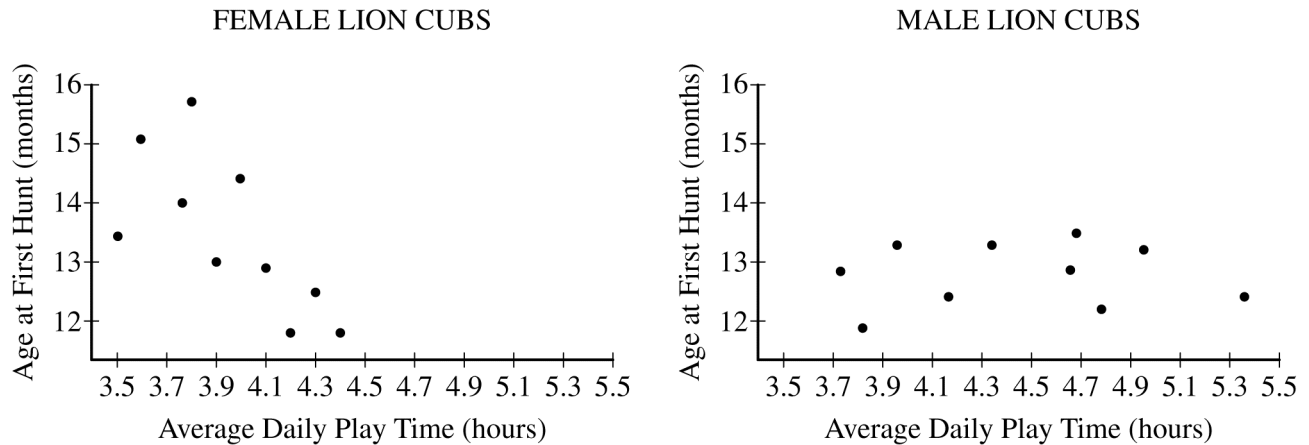


Which of the following statements must be true?

- (A) For Brand X, there were more green candy pieces than red candy pieces in the bag.
- (B) For Brand Y, there were more red candy pieces than green candy pieces in the bag.
- (C) There were more green candy pieces in the Brand X bag than were in the Brand Z bag.
- (D) There were the same number of blue candy pieces in the Brand X bag as were in the Brand Y bag.
- (E) The number of blue candy pieces in the Brand Z bag was equal to the sum of the number of blue candy pieces in the other two bags.

28. A large city newspaper periodically reports the mean cost of dinner for two people at restaurants in the city. The newspaper staff will collect data from a random sample of restaurants in the city and estimate the mean price using a 90 percent confidence interval. In past years, the standard deviation has always been very close to \$35. Assuming that the population standard deviation is \$35, which of the following is the minimum sample size needed to obtain a margin of error of no more than \$5 ?
- (A) 90
 - (B) 112
 - (C) 133
 - (D) 147
 - (E) 195

29. A field researcher who studies lions conjectured that the more time a cub spends playing, the sooner the cub will begin to hunt. Observational data were collected from 20 lion cubs. The researcher recorded how long they spent playing and the age when they began hunting. Because male and female lions have different hunting behaviors, the researcher recorded the data for males and females separately. The two scatterplots show the data for the 10 female lions and the 10 male lions.



Based on the scatterplots, for which gender does there appear to be evidence that the more time a lion cub spends playing, the sooner the cub is likely to begin hunting?

- (A) For female cubs only
- (B) For male cubs only
- (C) For both male cubs and female cubs, with equal evidence
- (D) For both male cubs and female cubs, with more evidence for female cubs than for male cubs
- (E) For neither male cubs nor female cubs

30. A 95 percent confidence interval for the mean time, in minutes, for a volunteer fire company to respond to emergency incidents is determined to be (2.8, 12.3). Which of the following is the best interpretation of the interval?
- (A) Five percent of the time, the time for response is less than 2.8 minutes or greater than 12.3 minutes.
 - (B) The probability is 0.95 that a randomly selected time for response will be between 2.8 minutes and 12.3 minutes.
 - (C) Ninety-five percent of the time the mean time for response is between 2.8 minutes and 12.3 minutes.
 - (D) We are 95% confident that the mean time for response is between 2.8 minutes and 12.3 minutes.
 - (E) We are 95% confident that a randomly selected time for response will be between 2.8 minutes and 12.3 minutes.
-

31. The height of 3-year-old boys is approximately normally distributed. Duncan and Shane are 3-year-old boys. Duncan is 32.0 inches tall and is at the 32nd percentile of the distribution. Shane is 34.0 inches tall and is at the 62nd percentile of the distribution. Which of the following is closest to the mean of the height distribution?
- (A) 32.50 inches
 - (B) 32.79 inches
 - (C) 33.00 inches
 - (D) 33.21 inches
 - (E) 36.53 inches

32. A company ships gift baskets that contain apples and pears. The distributions of weight for the apples, the pears, and the baskets are each approximately normal. The mean and standard deviation for each distribution is shown in the table below. The weights of the items are assumed to be independent.

Item	Mean	Standard Deviation
Apple	4.72 ounces	0.20 ounce
Pear	5.41 ounces	0.18 ounce
Basket	13.25 ounces	1.88 ounces

Let the random variable W represent the total weight of 4 apples, 6 pears, and 1 basket. Which of the following is closest to the standard deviation of W ?

- (A) 1.90 ounces
- (B) 1.97 ounces
- (C) 2.26 ounces
- (D) 3.76 ounces
- (E) 3.83 ounces

33. Researchers will conduct a study of the television-viewing habits of children. They will select a simple random sample of children and record the number of hours of television the children watch per week. The researchers will report the sample mean as a point estimate for the population mean. Which of the following statements is correct for the sample mean as a point estimator?
- (A) A sample of size 25 will produce more variability of the estimator than a sample of size 50.
 - (B) A sample of size 25 will produce less variability of the estimator than a sample of size 50.
 - (C) A sample of size 25 will produce a biased estimator, but a sample size of 50 will produce an unbiased estimator.
 - (D) A sample of size 25 will produce a more biased estimator than a sample of size 50.
 - (E) A sample of size 25 will produce a less biased estimator than a sample of size 50.

34. A research study indicated a negative linear relationship between two variables: the number of hours per week spent exercising (exercise time) and the number of seconds it takes to run one lap around a track (running time). Computer output from the study is shown below.

Variable	N	Mean	SE Mean	StDev
Running time	11	74.81	2.21	7.33

Predictor	Coef	SE Coef
Constant	88.01	0.49
Exercise time	-2.20	0.07

S = 0.76

R-Sq = 99.0%

Assuming that all conditions for inference are met, which of the following is an appropriate test statistic for testing the null hypothesis that the slope of the population regression line equals 0 ?

(A) $t = \frac{88.01}{0.49}$

(B) $t = \frac{74.81}{7.33}$

(C) $t = \frac{74.81}{2.21}$

(D) $t = \frac{-2.20}{0.07}$

(E) $t = \frac{-2.20}{\frac{0.07}{\sqrt{11}}}$

35. The table below shows historical data for the distribution of the number of customers, in half-hour time periods, who visit the electronics department of a retail store. For example, in 25 percent of the time periods for which data were collected, no customers were observed in the electronics department of the store.

Number of customers	0	1	2	3 or more
Proportion of time periods	0.25	0.20	0.30	0.25

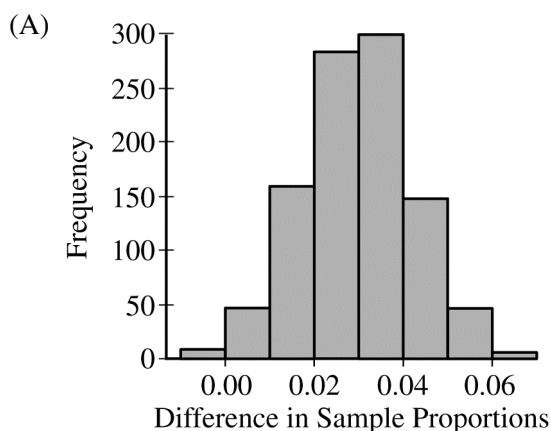
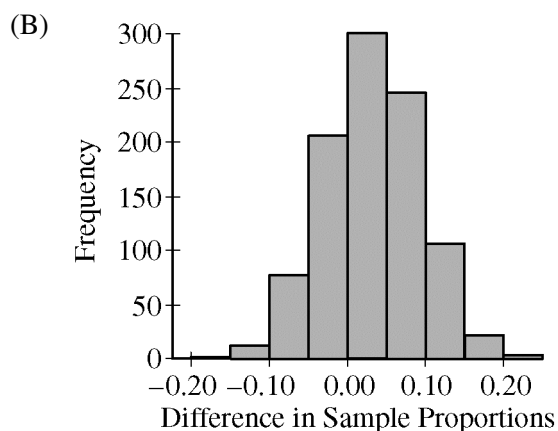
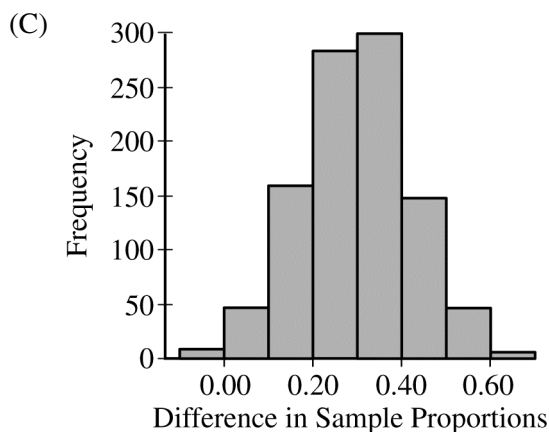
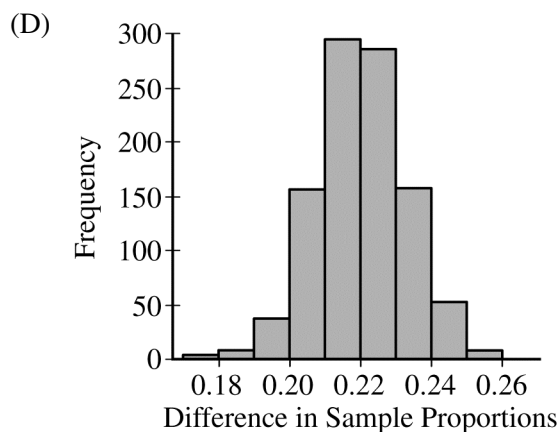
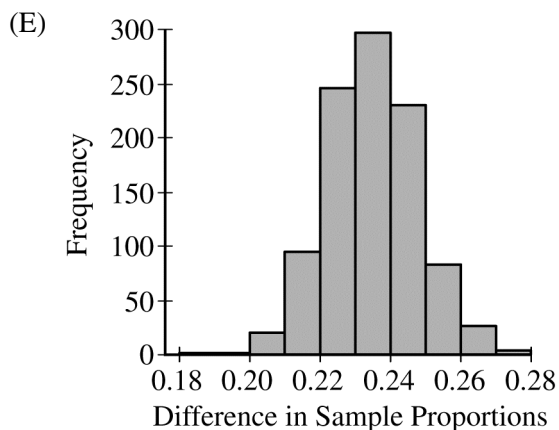
To investigate if the distribution has changed, the number of customers who visited the electronics department of the store was recorded for each of 50 randomly selected time periods. The results are shown in the table below.

Number of customers	0	1	2	3 or more
Number of time periods	4	13	14	19

A chi-square goodness-of-fit test was conducted to determine whether the data provide convincing evidence that the distribution has changed. The test statistic was 10.13 with a p -value of 0.0175. Which of the following statements is true?

- (A) At the significance level $\alpha = 0.05$, the data provide convincing evidence that the current distribution is different from the historical distribution.
- (B) At the significance level $\alpha = 0.10$, the data do not provide convincing evidence that the current distribution is different from the historical distribution.
- (C) The mean number of customers in a randomly selected time period is 12.5.
- (D) No valid conclusion can be made because the observed frequency for one cell is less than 5.
- (E) The chi-square statistic has $50 - 1 = 49$ degrees of freedom.

36. Suppose that 25 percent of women and 22 percent of men would answer yes to a particular question. In a simulation, a random sample of 100 women and a random sample of 100 men were selected, and the difference in sample proportions of those who answered yes, $\hat{p}_{\text{women}} - \hat{p}_{\text{men}}$, was calculated. The process was repeated 1,000 times. Which of the following is most likely to be a representation of the simulated sampling distribution of the difference between the two sample proportions?



37. According to government data, 22 percent of children in the United States under the age of 6 years live in households with incomes that are classified at a particular income level. A simple random sample of 300 children in the United States under the age of 6 years was selected for a study of learning in early childhood. If the government data are correct, which of the following best approximates the probability that at least 27 percent of the children in the sample live in households that are classified at the particular income level? (Note: z represents a standard normal random variable.)

(A) $P\left(z > \frac{0.27 - 0.22}{\sqrt{\frac{(0.50)(0.50)}{300}}}\right)$

(B) $P\left(z > \frac{0.27 - 0.22}{\sqrt{\frac{(0.22)(0.78)}{300}}}\right)$

(C) $P\left(z > \frac{0.27 - 0.22}{\sqrt{\frac{(0.27)(0.73)}{300}}}\right)$

(D) $P\left(z > \frac{0.22 - 0.27}{\sqrt{\frac{(0.22)(0.78)}{300}}}\right)$

(E) $P\left(z > \frac{0.22 - 0.27}{\sqrt{(0.27)(0.73)}}\right)$

38. A machine is designed to dispense at least 12 ounces of a beverage into a bottle. To test whether the machine is working properly, a random sample of 50 bottles was selected and the mean number of ounces for the 50 bottles was computed. A test of the hypotheses $H_0 : \mu = 12$ versus $H_a : \mu < 12$ was conducted, where μ represents the population mean number of ounces of the beverage dispensed per bottle by the machine. The p -value for the test was 0.08. Which of the following is the most appropriate conclusion to draw at the significance level of $\alpha = 0.05$?
- (A) Because the p -value is greater than the significance level, there is convincing evidence that the population mean number of ounces dispensed into a bottle is 12 ounces.
 - (B) Because the p -value is greater than the significance level, there is convincing evidence that the population mean number of ounces dispensed into a bottle is less than 12 ounces.
 - (C) Because the p -value is greater than the significance level, there is not convincing evidence that the population mean number of ounces dispensed into a bottle is less than 12 ounces.
 - (D) Because the p -value is less than the significance level, there is convincing evidence that the population mean number of ounces dispensed into a bottle is 12 ounces.
 - (E) Eight percent of the bottles will be filled with less than 12 ounces.

39. In a certain school, students can choose whether to eat in the school's cafeteria. A reporter working for the school's newspaper polled students on their reactions to changes in the menu at the cafeteria. For each student leaving the cafeteria in one 30-minute time period, the reporter used a coin to determine whether to stop the student and ask how he or she felt about the new menu. In the reporter's article it was stated that a random sample of the students showed that 89 percent of the school's student population was happy with the new menu. Which of the following statements is true?
- (A) Because each student leaving the cafeteria was randomly selected and could choose to answer or not, this is a random sample of the student population, and the 89% is an accurate measurement of the school population's view of the new menu.
 - (B) Because students self-selected whether to eat in the cafeteria, the sampling method might be biased and the sample might not be representative of all students in the school.
 - (C) The survey would have been more effective if the reporter had collected the data in one 15-minute time period rather than in one 30-minute time period.
 - (D) The survey would have been more effective if students who cared about the food could have called the reporter to tell how they felt about the new menu, so that only students with opinions on the subject would have been surveyed.
 - (E) Because no treatment was imposed on the students eating in the cafeteria, one cannot make any conclusions about the new menu.

40. Ms. Tucker travels through two intersections with traffic lights as she drives to the market. The traffic lights operate independently. The probability that both lights will be red when she reaches them is 0.22. The probability that the first light will be red and the second light will not be red is 0.33. What is the probability that the second light will be red when she reaches it?
- (A) 0.40
 - (B) 0.45
 - (C) 0.50
 - (D) 0.55
 - (E) 0.60

END OF SECTION I

**IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY
CHECK YOUR WORK ON THIS SECTION.**

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

MAKE SURE YOU HAVE DONE THE FOLLOWING.

- **PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET**
- **WRITTEN AND GRIDDED YOUR AP NUMBER CORRECTLY ON YOUR ANSWER SHEET**
- **TAKEN THE AP EXAM LABEL FROM THE FRONT OF THIS BOOKLET AND PLACED IT ON YOUR ANSWER SHEET**

Table entry for z is the probability lying below z .

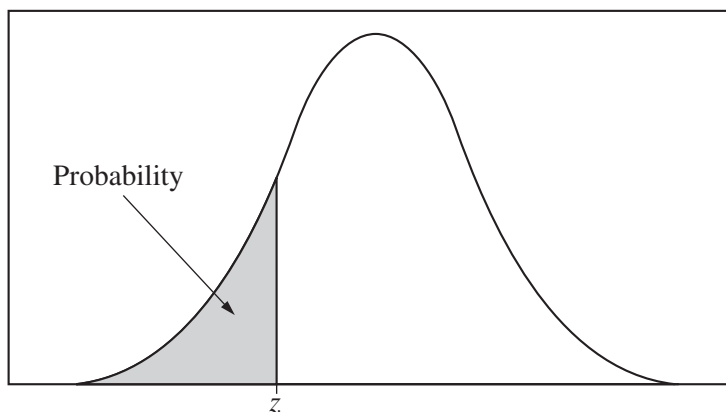


Table A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table entry for z is the probability lying below z .

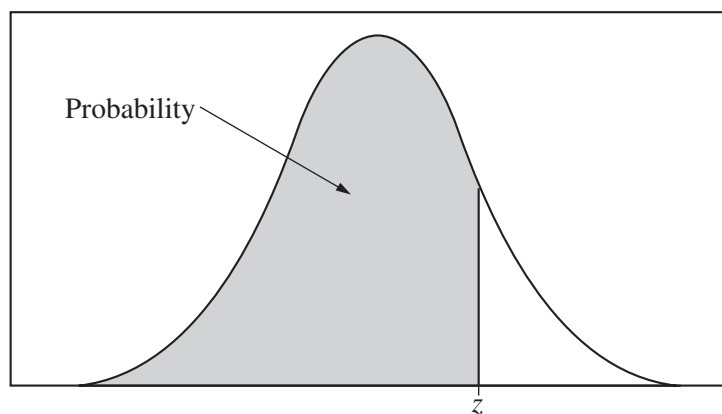


Table A (Continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^* .

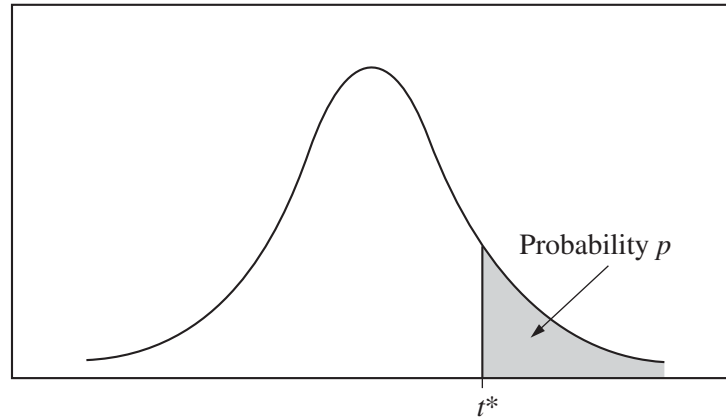


Table B t distribution critical values

df	Tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
Confidence level C												

Table entry for p is the point (χ^2) with probability p lying above it.

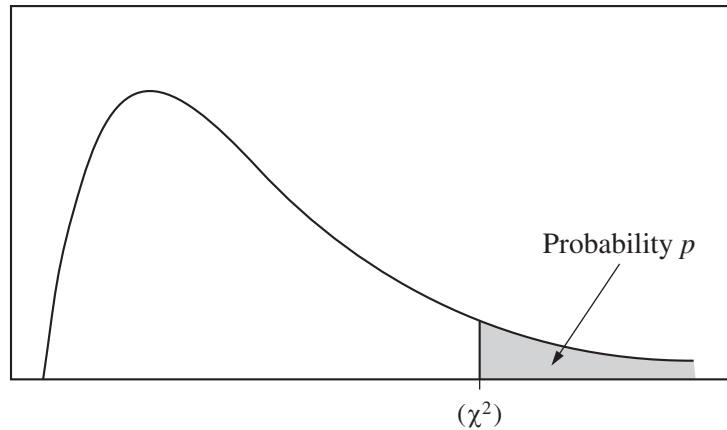


Table C χ^2 critical values

df	Tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

AP[®] Statistics Exam

SECTION II: Free Response

2015

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time

1 hour, 30 minutes

Number of Questions

6

Percent of Total Score

50%

Writing Instrument

Either pencil or pen with black or dark blue ink

Electronic Device

Graphing calculator expected

Part A

Number of Questions

5

Suggested Time

1 hour, 5 minutes

Percent of Section II Score

75%

Part B

Number of Questions

1

Suggested Time

25 minutes

Percent of Section II Score

25%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name

First letter of your first name

2. Date of birth

Month Day Year

3. Six-digit school code

4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.

No, I do not grant the College Board these rights. ☐

Instructions

The questions for both Part A and Part B are printed in this booklet. You may use any blank space in the booklet to organize your answers and for scratch work, but you must write your answers in the spaces provided for each answer. Pages containing statistical tables and useful formulas are printed in this booklet.

You may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions. Show all your work. Indicate clearly the methods you use because you will be scored on the correctness of your methods as well as the accuracy and completeness of your results and explanations. Correct answers without supporting work may not receive credit. Write your solution to each part of each question in the space provided for that part. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. The proctor will announce the suggested time for Part A and Part B, but you may proceed freely from one question to the next. You may review your responses if you finish before the end of the exam is announced.

Form I

Form Code 4GBP2-S2

90

Formulas begin on page 3.
Questions begin on page 6.
Tables begin on page 18.

GO ON TO THE NEXT PAGE.

Formulas

(I) Descriptive Statistics

$$\bar{x} = \frac{\sum x_i}{n}$$

$$s_x = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2}$$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}}$$

$$\hat{y} = b_0 + b_1 x$$

$$b_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$$b_0 = \bar{y} - b_1 \bar{x}$$

$$r = \frac{1}{n-1} \sum \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right)$$

$$b_1 = r \frac{s_y}{s_x}$$

$$s_{b_1} = \frac{\sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n-2}}}{\sqrt{\sum (x_i - \bar{x})^2}}$$

(II) Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$E(X) = \mu_x = \sum x_i p_i$$

$$\text{Var}(X) = \sigma_x^2 = \sum (x_i - \mu_x)^2 p_i$$

If X has a binomial distribution with parameters n and p , then:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

$$\mu_x = np$$

$$\sigma_x = \sqrt{np(1 - p)}$$

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1 - p)}{n}}$$

If $\bar{x}$ is the mean of a random sample of size n from an infinite population with mean μ and standard deviation σ , then:

$$\mu_{\bar{x}} = \mu$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

(III) Inferential Statistics

Standardized test statistic: $\frac{\text{statistic} - \text{parameter}}{\text{standard deviation of statistic}}$

Confidence interval: $\text{statistic} \pm (\text{critical value}) \cdot (\text{standard deviation of statistic})$

Single-Sample

Statistic	Standard Deviation of Statistic
Sample Mean	$\frac{\sigma}{\sqrt{n}}$
Sample Proportion	$\sqrt{\frac{p(1-p)}{n}}$

Two-Sample

Statistic	Standard Deviation of Statistic
Difference of sample means	$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ Special case when $\sigma_1 = \sigma_2$ $\sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$
Difference of sample proportions	$\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$ Special case when $p_1 = p_2$ $\sqrt{p(1-p)} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

$$\text{Chi-square test statistic} = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

STATISTICS

SECTION II

Part A

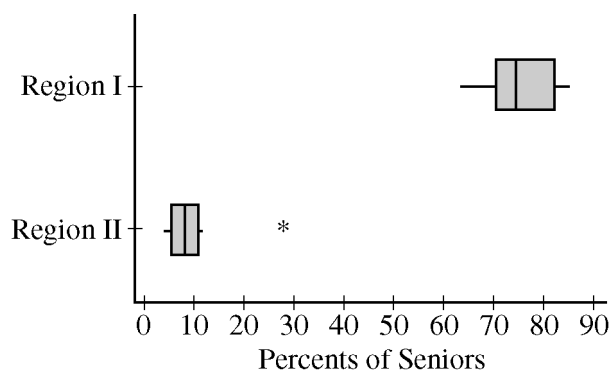
Questions 1-5

Spend about 65 minutes on this part of the exam.

Percent of Section II score—75

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

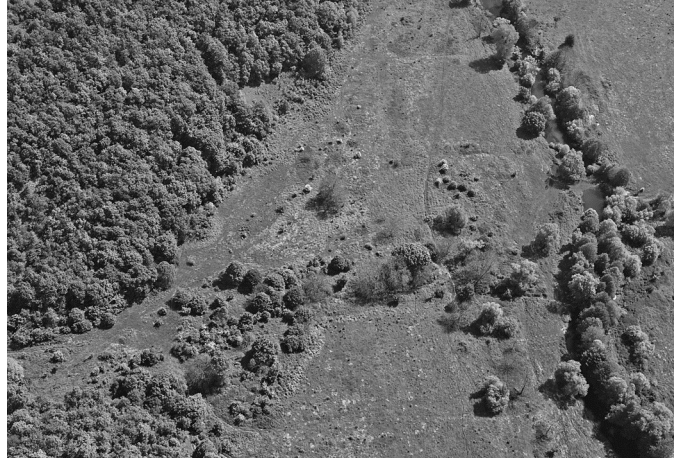
1. A university researcher is interested in comparing the percents of high school seniors who took a college entrance exam for two different regions of the country, Region I and Region II. The researcher recorded the percent of seniors taking the exam for each high school within the two regions. The boxplots of the distributions of the percents of seniors who took the college entrance exam are shown below.



- (a) Compare the distributions of percents of seniors who took the college entrance exam for the two regions.
- (b) In writing a report, the researcher produced a single histogram of the combined data for Region I and Region II. Describe the shape of the histogram for the combined data.

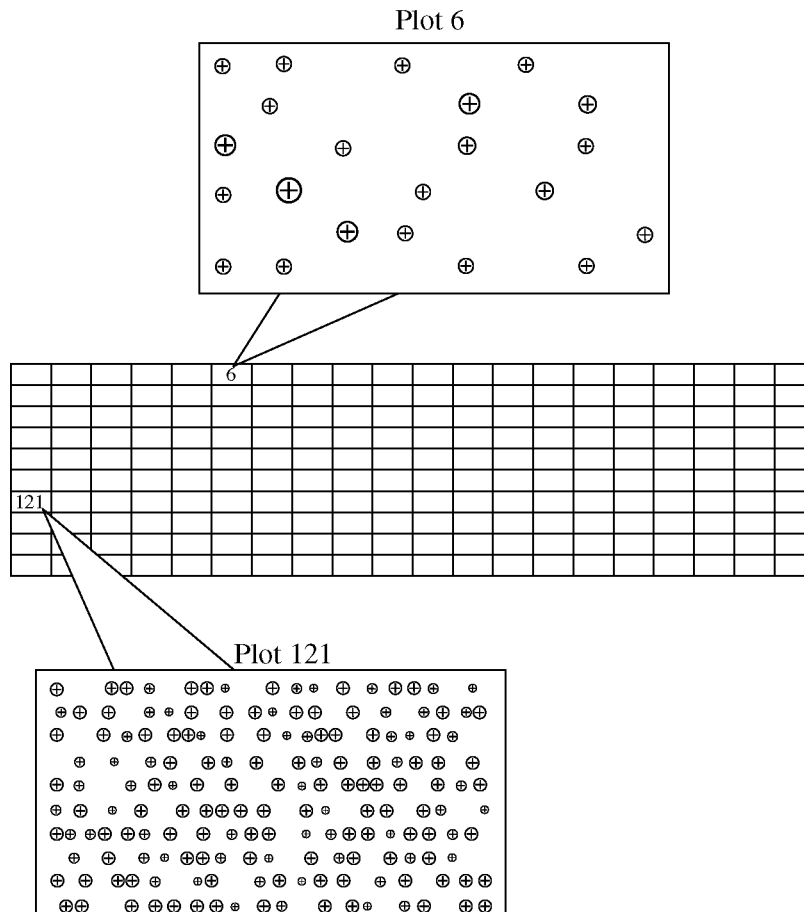
2. The weights of red delicious apples are approximately normally distributed with a mean of 9 ounces and a standard deviation of 0.75 ounce. An online gift store sells gift boxes containing 5 red delicious apples. At the time of packaging, 5 red delicious apples are randomly selected and packaged in a box.
- (a) Describe the distribution of the total weight of the 5 randomly selected apples.
- (b) What is the probability that the total weight of the 5 randomly selected apples will be less than 42 ounces?
- (c) The combined weight of the packing material and box in which the apples will be shipped is always 10 ounces. Let W represent the weight of a complete packaged gift box, which consists of the packing material, box, and 5 randomly selected apples. What are the mean and the standard deviation of W ?

3. Recently, a company acquired the rights to use a forest—like the one shown in the photograph below—to harvest trees to produce lumber.



The company wants to conduct a study to estimate the mean trunk diameter of the trees from the forest by taking a random sample of approximately 5 percent of the trees from the forest. For the study, the company divides the forest into 200 equally sized plots of approximately one acre each, as shown in the figure below.

Because of previous logging practices and growth patterns, plots with older trees, such as Plot 6, tend to have fewer trees but with larger trunk diameters, and plots with younger trees, such as Plot 121, tend to have more trees but with smaller trunk diameters. This is illustrated in the two figures of Plots 6 and 121 by the varying number and sizes of the symbol $\oplus$.



- (a) Describe a procedure for using cluster sampling to obtain a random sample of approximately 5 percent of the trees from the forest, using the plots as clusters.
- (b) Describe a procedure for using stratified sampling to obtain a random sample of approximately 5 percent of the trees from the forest, using the plots as strata.
- (c) For the study, give one advantage of using cluster sampling as described in part (a) over stratified sampling as described in part (b).
- (d) For the study, give one advantage of using stratified sampling as described in part (b) over cluster sampling as described in part (a).

4. The president of a large bank with many branch offices wanted to know if the quality of customer service at a new branch office was acceptable. One aspect of service that was examined was the length of time that customers had to wait in line before being helped by a member of the bank's staff. The bank decided on acceptable probabilities for the waiting-time categories, and these are given in the table below.

Waiting-time Category	Probability
No more than 1 minute	0.30
More than 1 minute but no more than 3 minutes	0.25
More than 3 minutes but no more than 5 minutes	0.20
More than 5 minutes but no more than 10 minutes	0.15
More than 10 minutes	0.10

To investigate whether the quality of customer service was acceptable, waiting times were recorded for a random sample of 100 customers at the new branch office. The table below shows the numbers of customers observed in the five waiting-time categories.

Waiting-time Category	Number of Customers
No more than 1 minute	25
More than 1 minute but no more than 3 minutes	21
More than 3 minutes but no more than 5 minutes	21
More than 5 minutes but no more than 10 minutes	20
More than 10 minutes	13
Total	100

Use the sample data for the 100 customers to conduct a statistical test to determine if the waiting times at the new branch office are inconsistent with the acceptable probabilities for the waiting-time categories.

If you need more room for your work for question 4, use the space below.

5. A company manufactures and markets a traditional type of disposable coffee cup that is used in many fast food restaurants. The company has created a new cup that it believes insulates better than the traditional cup. To investigate whether the new cup insulates better, the company plans to conduct a study. In the study, a random sample of cups for each of the two types will be selected. In each sample, each cup will be filled with the same amount of coffee that has been heated to 150 degrees Fahrenheit ($^{\circ}\text{F}$). The amount of time (in minutes) it takes for the coffee to cool to 100°F will be measured for each cup.

The hypotheses that the company will test are shown below, where μ_N is the true mean time it takes coffee to cool from 150°F to 100°F in the new cup and μ_T is the true mean time it takes coffee to cool from 150°F to 100°F in the traditional cup.

$$H_0: \mu_N = \mu_T$$

$$H_a: \mu_N > \mu_T$$

- (a) Describe a Type II error in the context of the study.
- (b) The company is concerned about the probability of a Type II error. Which test procedure, one that uses a significance level of $\alpha = 0.10$ or one that uses a significance level of $\alpha = 0.01$, would result in a smaller probability of a Type II error? Explain.
- (c) The marketing department in the company has suggested that a 2-minute increase in the time it takes the coffee to cool from 150°F to 100°F would be a noticeable improvement to customers. Suppose the company statistician estimates that the power of the appropriate significance test is 0.88 when the true mean cooling time for the new cups is 2 minutes greater than the true mean cooling time for the traditional cups. Interpret the value of 0.88 in the context of the study.

STATISTICS

SECTION II

Part B

Question 6

Spend about 25 minutes on this part of the exam.

Percent of Section II score—25

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. A consumer group compared the longevity of two types of razor blades: an inexpensive generic blade and a more expensive name-brand blade. Two independent samples were randomly selected—one consisted of ten generic blades and the other consisted of ten name-brand blades. The number of shaves obtained before a blade became too dull to provide a close shave was recorded for each blade in the two independent samples.

Number of Close Shaves for the Generic (G) Type

99	111	97	101	118	108	96	129	80	136
----	-----	----	-----	-----	-----	----	-----	----	-----

Number of Close Shaves for the Name-brand (B) Type

112	105	109	122	130	113	106	116	103	110
-----	-----	-----	-----	-----	-----	-----	-----	-----	-----

- (a) Create a back-to-back stemplot to display the distributions of the data.

(b) In a few sentences, compare the distributions in part (a).

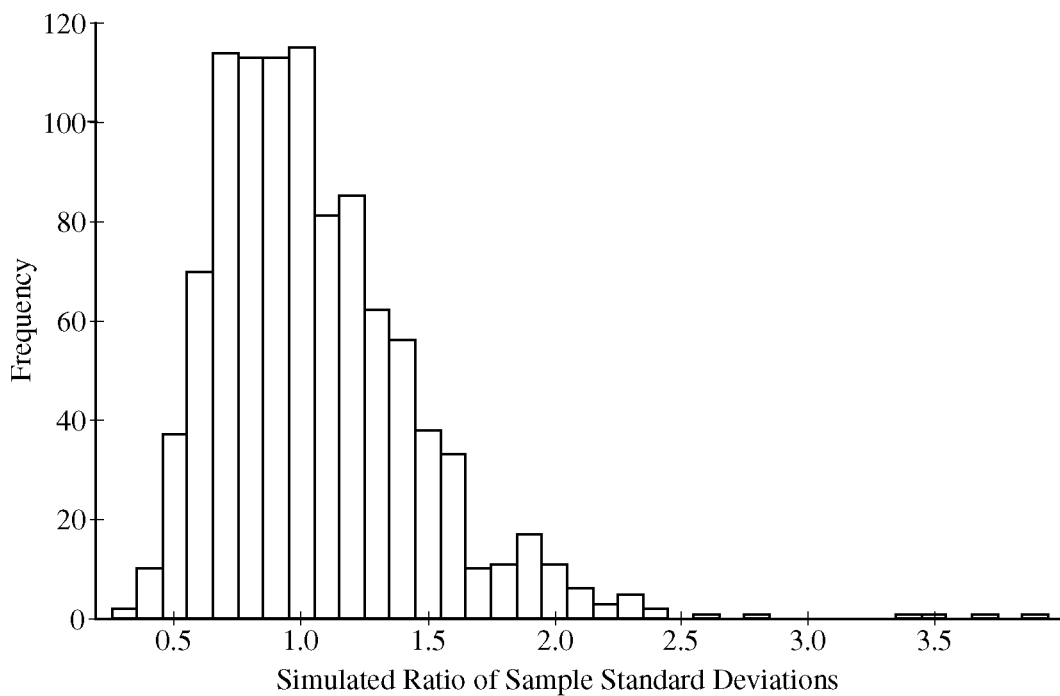
(c) To test whether the name-brand blades tend to last longer than the generic blades on average, a two-sample t -test was performed. The conditions for inference were checked and verified. The test statistic was $t = 0.87$, with a p -value of 0.20. What should be concluded about whether the name-brand blades last significantly longer than the generic blades? Explain.

Also of interest to the consumer group is whether the generic (G) blades have more variability in the number of close shaves than the name-brand (B) blades. The consumer group intends to use the ratio of the sample standard deviations, $\left(\frac{s_G}{s_B}\right)$, as a test statistic to test the hypothesis $H_0: \sigma_G = \sigma_B$ versus $H_a: \sigma_G > \sigma_B$.

The group conducted a simulation study to investigate the sampling distribution of the test statistic when the population standard deviations were actually the same. For each repetition of the simulation, two independent samples of size 10 were selected from the same normally distributed population, and the ratio of their sample standard deviations was computed.

(d) Explain why the median of the distribution of a sample of 1,000 of these simulated ratios will be about 1.

- (e) A histogram for the 1,000 simulated values of the ratio $\left(\frac{s_G}{s_B}\right)$ is shown below.



Summary statistics for the original data (number of close shaves) are provided in the table below.

	Sample Size	Sample Mean	Sample Standard Deviation
Generic (G)	10	107.5	16.70
Name Brand (B)	10	112.6	8.28

Is there convincing evidence that the generic blades have more variability in the number of close shaves than the name-brand blades? Explain.

STOP

END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT AND BACK COVERS OF THE SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX(ES) ON THE COVER(S).**
- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.**

Table entry for z is the probability lying below z .

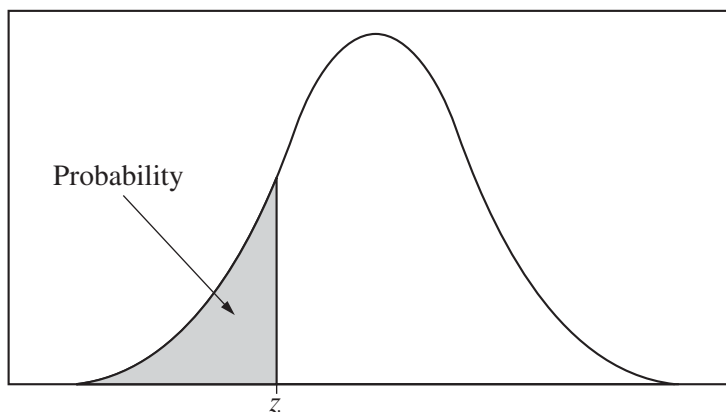


Table A Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table entry for z is the probability lying below z .

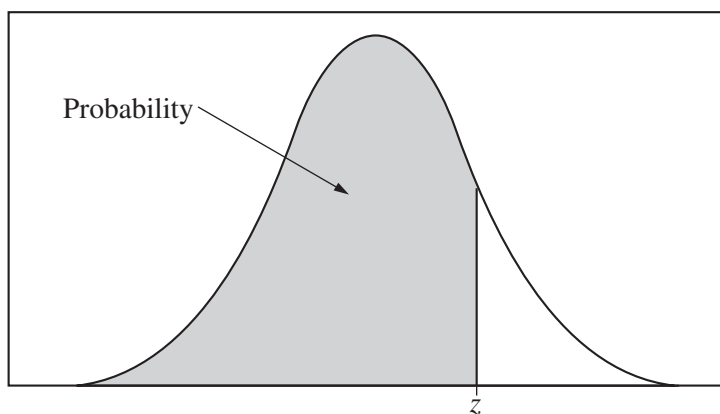


Table A (Continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^* .

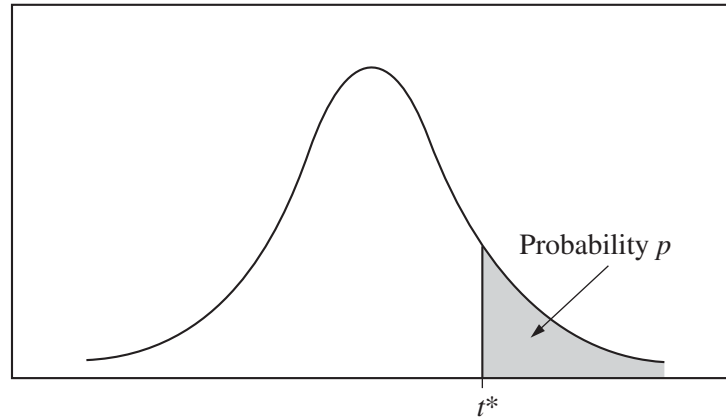


Table B t distribution critical values

df	Tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
Confidence level C												

Table entry for p is the point (χ^2) with probability p lying above it.

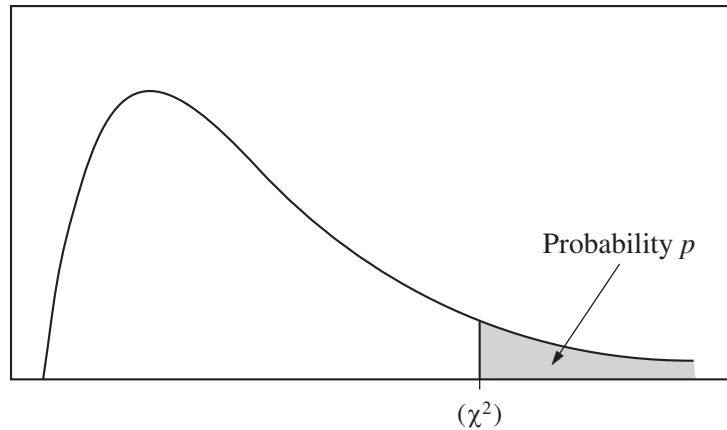


Table C χ^2 critical values

df	Tail probability p											
	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.32	1.64	2.07	2.71	3.84	5.02	5.41	6.63	7.88	9.14	10.83	12.12
2	2.77	3.22	3.79	4.61	5.99	7.38	7.82	9.21	10.60	11.98	13.82	15.20
3	4.11	4.64	5.32	6.25	7.81	9.35	9.84	11.34	12.84	14.32	16.27	17.73
4	5.39	5.99	6.74	7.78	9.49	11.14	11.67	13.28	14.86	16.42	18.47	20.00
5	6.63	7.29	8.12	9.24	11.07	12.83	13.39	15.09	16.75	18.39	20.51	22.11
6	7.84	8.56	9.45	10.64	12.59	14.45	15.03	16.81	18.55	20.25	22.46	24.10
7	9.04	9.80	10.75	12.02	14.07	16.01	16.62	18.48	20.28	22.04	24.32	26.02
8	10.22	11.03	12.03	13.36	15.51	17.53	18.17	20.09	21.95	23.77	26.12	27.87
9	11.39	12.24	13.29	14.68	16.92	19.02	19.68	21.67	23.59	25.46	27.88	29.67
10	12.55	13.44	14.53	15.99	18.31	20.48	21.16	23.21	25.19	27.11	29.59	31.42
11	13.70	14.63	15.77	17.28	19.68	21.92	22.62	24.72	26.76	28.73	31.26	33.14
12	14.85	15.81	16.99	18.55	21.03	23.34	24.05	26.22	28.30	30.32	32.91	34.82
13	15.98	16.98	18.20	19.81	22.36	24.74	25.47	27.69	29.82	31.88	34.53	36.48
14	17.12	18.15	19.41	21.06	23.68	26.12	26.87	29.14	31.32	33.43	36.12	38.11
15	18.25	19.31	20.60	22.31	25.00	27.49	28.26	30.58	32.80	34.95	37.70	39.72
16	19.37	20.47	21.79	23.54	26.30	28.85	29.63	32.00	34.27	36.46	39.25	41.31
17	20.49	21.61	22.98	24.77	27.59	30.19	31.00	33.41	35.72	37.95	40.79	42.88
18	21.60	22.76	24.16	25.99	28.87	31.53	32.35	34.81	37.16	39.42	42.31	44.43
19	22.72	23.90	25.33	27.20	30.14	32.85	33.69	36.19	38.58	40.88	43.82	45.97
20	23.83	25.04	26.50	28.41	31.41	34.17	35.02	37.57	40.00	42.34	45.31	47.50
21	24.93	26.17	27.66	29.62	32.67	35.48	36.34	38.93	41.40	43.78	46.80	49.01
22	26.04	27.30	28.82	30.81	33.92	36.78	37.66	40.29	42.80	45.20	48.27	50.51
23	27.14	28.43	29.98	32.01	35.17	38.08	38.97	41.64	44.18	46.62	49.73	52.00
24	28.24	29.55	31.13	33.20	36.42	39.36	40.27	42.98	45.56	48.03	51.18	53.48
25	29.34	30.68	32.28	34.38	37.65	40.65	41.57	44.31	46.93	49.44	52.62	54.95
26	30.43	31.79	33.43	35.56	38.89	41.92	42.86	45.64	48.29	50.83	54.05	56.41
27	31.53	32.91	34.57	36.74	40.11	43.19	44.14	46.96	49.64	52.22	55.48	57.86
28	32.62	34.03	35.71	37.92	41.34	44.46	45.42	48.28	50.99	53.59	56.89	59.30
29	33.71	35.14	36.85	39.09	42.56	45.72	46.69	49.59	52.34	54.97	58.30	60.73
30	34.80	36.25	37.99	40.26	43.77	46.98	47.96	50.89	53.67	56.33	59.70	62.16
40	45.62	47.27	49.24	51.81	55.76	59.34	60.44	63.69	66.77	69.70	73.40	76.09
50	56.33	58.16	60.35	63.17	67.50	71.42	72.61	76.15	79.49	82.66	86.66	89.56
60	66.98	68.97	71.34	74.40	79.08	83.30	84.58	88.38	91.95	95.34	99.61	102.7
80	88.13	90.41	93.11	96.58	101.9	106.6	108.1	112.3	116.3	120.1	124.8	128.3
100	109.1	111.7	114.7	118.5	124.3	129.6	131.1	135.8	140.2	144.3	149.4	153.2

AP Statistics

2015

APPENDIX - ANSWERS

Matching multiple-choice answers and free-response scoring follow.

Answer Key for AP Statistics Practice Exam, Section I

Question 1: B	Question 21: A
Question 2: E	Question 22: D
Question 3: C	Question 23: B
Question 4: D	Question 24: C
Question 5: B	Question 25: B
Question 6: B	Question 26: B
Question 7: D	Question 27: B
Question 8: B	Question 28: C
Question 9: C	Question 29: A
Question 10: B	Question 30: D
Question 11: D	Question 31: D
Question 12: E	Question 32: B
Question 13: E	Question 33: A
Question 14: A	Question 34: D
Question 15: E	Question 35: A
Question 16: D	Question 36: B
Question 17: C	Question 37: B
Question 18: E	Question 38: C
Question 19: D	Question 39: B
Question 20: A	Question 40: A

Free-Response Scoring Guidelines

The following contains the scoring guidelines for the free-response questions in this exam.

AP[®] STATISTICS

2015 SCORING GUIDELINES

Question 1

Intent of Question

The primary goals of this question were to assess a student's ability to (1) compare two distributions using boxplots; and (2) predict the shape of a single histogram when data for the two distributions are combined.

Solution

Part (a):

The median of the percentages of seniors who took the college entrance exam is more than 60 percentage points higher in Region I than in Region II. In fact, there is no overlap between the two distributions. A gap exists between the two distributions stretching from about 28% to about 63%, with Region I having the higher values. There is more variability in the percentages for Region I as measured by the IQR, but due to one large outlier in Region II, the range for Region II is slightly larger than the range for Region I.

Part (b):

The histogram will have two clusters with one centered around 8% and another around 74%, and a gap between the values of about 12% and 63% except for a solitary value around 28%.

AP[®] STATISTICS

2015 SCORING GUIDELINES

Question 1 (continued)

Scoring

The question is scored in four sections, at least one of which must be in context. Section 1 consists of a comparison of center in part (a), section 2 consists of a comparison of spread in part (a), section 3 consists of a comment on the gap in part (b), and section 4 consists of a description of two clusters in part (b). Sections 1, 2, and 3 are scored as essentially correct (E), partially correct (P), or incorrect (I). Section 4 is scored as essentially correct (E) or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the student correctly *compares* center (or location) of the two distributions. If a comparison is made, no numerical values need to be given.

Partially correct (P) if the student correctly gives the medians of the two distributions, but does not compare them or compares incorrectly.

Incorrect (I) if the response does not meet the criteria for E or P.

Section 2 is scored as follows:

Essentially correct (E) if the student correctly *compares* interquartile range *OR* range of the two distributions. If a correct comparison is made, no numerical values need to be given.

Such comparisons include:

- IQR of Region I is larger than that of Region II, or
- Range of Region II is slightly larger than that of Region I, or
- Range (or spread) of Region I is larger than that of Region II without the outlier.

Partially correct (P) if the student correctly gives the ranges or interquartile ranges of the two distributions, but does not compare them or compares them incorrectly;

OR

if the comparison is made using only a general term such as *spread* or *variability* without defining it to claim that Region I has more spread than Region II;

OR

if the student makes a correct comparison based on stated but incorrect values for the range or the IQR.

Incorrect (I) if the response does not meet the criteria for E or P.

AP[®] STATISTICS

2015 SCORING GUIDELINES

Question 1 (continued)

Section 3 is scored as follows:

Essentially correct (E) if the student correctly refers to the gap.

Partially correct (P) if the student refers to the gap only in part (a);

OR

says that no values occur between about 12% and 63%;

OR

says in part (b) that there is “no overlap” between the two distributions.

Incorrect (I) if the student does not mention the complete separation between the two clusters in either part (a) or in part (b).

Section 4 is scored as follows:

Essentially correct (E) if the student notes in part (b) that the combined distribution consists of two clusters. The locations of the two clusters do not need to be stated.

Incorrect (I) if the response does not meet the criteria for E.

Each essentially correct (E) section counts as 1 point. Each partially correct (P) section counts as $\frac{1}{2}$ point.

4 Complete Response

3 Substantial Response

2 Developing Response

1 Minimal Response

If a response is between two scores (for example, $2\frac{1}{2}$ points), use a holistic approach to decide whether to score up or down, depending on the overall strength of the response and communication.

Note: In at least one of the four sections, the response must refer to the variable of percentage of students in the high school taking the exam in addition to referring to Region I and Region II. If no reference is made, the response cannot earn a score of 4.

AP[®] STATISTICS
2015 SCORING GUIDELINES

Question 2

Intent of Question

The primary goals of this question were to assess a student's ability to (1) describe a distribution formed by adding together normal random variables; (2) calculate a probability from this new distribution; and (3) find the mean and standard deviation of a random variable when a constant is added to the values in the distribution.

Solution

Part (a):

Let T denote the total weight of 5 randomly selected red delicious apples. Because the apples are randomly selected, and each apple's weight is approximately normally distributed, T is also approximately normal with mean equal to the sum of the means of the distribution of each of the 5 apple's weights, $E(T) = 9 + 9 + 9 + 9 + 9 = 5(9) = 45$ ounces, and a variance equal to the sum of the variances of the five apples, $Var(T) = (0.75)^2 + (0.75)^2 + (0.75)^2 + (0.75)^2 + (0.75)^2 = 5(0.75)^2 = 2.8125$ ounces squared. The standard deviation is $\sigma_T = \sqrt{2.8125} \approx 1.677$ ounces.

Part (b):

Using the parameters from part (a), the appropriate normal probability can be calculated as follows:

$$P(T < 42) = P\left(Z < \frac{42 - 45}{1.677}\right) = P(Z < -1.79) = 0.0368.$$

Part (c):

Let W denote the weight for the packaged gift box, so $W = 10 + T$. The expected value (mean) of W is $E(W) = E(10 + T) = E(10) + E(T) = 10 + 45 = 55$ ounces. The variance of W is $Var(W) = Var(10 + T) = Var(10) + Var(T) = 0 + 2.8125 = 2.8125$ ounces squared. Thus, W has a standard deviation of $\sigma_W = \sqrt{2.8125} \approx 1.677$ ounces, because adding a constant to a random variable does not change its variance or standard deviation.

AP[®] STATISTICS

2015 SCORING GUIDELINES

Question 2 (continued)

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response correctly provides all three components of the distribution of the total weight, with appropriate justification: shape (approximately normal), center (mean is 45 ounces), and spread (standard deviation is 1.677 ounces).

Partially correct (P) if the response correctly gives only two of the three components with appropriate justification.

Incorrect (I) if the response does not meet the criteria for E or P.

Part (b) is scored as follows:

Essentially correct (E) if the response shows the correct probability with a correct normal probability calculation method;

OR

if the response shows the correct probability with a well-labeled sketch.

Partially correct (P) if the response sets up a correct normal probability calculation but does not carry it through correctly;

OR

if the response uses the standard deviation given in the stem of the problem and carries out a correct normal probability calculation.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- If the normal probability calculation in part (b) includes an incorrect value for the mean or standard deviation as obtained in part (a), part (b) should be scored as E if the probability calculation is carried out correctly.
- A response that arrives at the correct answer in the context of a significance test lowers the score by one level (that is, from E to P, or P to I).
- A response that includes an incorrect mathematical statement lowers the score by one level (that is, from E to P, or P to I).
- Because the probability distribution and its parameter values were asked for in part (a), the response does not have to repeat that information in part (b) to earn an E.

AP[®] STATISTICS

2015 SCORING GUIDELINES

Question 2 (continued)

Part (c) is scored as follows:

Essentially correct (E) if the response provides the correct values for the two parameters (mean and standard deviation) with appropriate justification for each.

Partially correct (P) if the response provides the correct values, with appropriate justification, for only one of the two parameters.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- An incorrect mean and/or standard deviation carried through into part (c) and used in calculating the mean and standard deviation, with appropriate justification, is acceptable for an E.
- Appropriate justification may be provided through a correct calculation or written explanation.

4 Complete Response

All three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and one part incorrect

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct and no parts partially correct

OR

No parts essentially correct and two parts partially correct

AP[®] STATISTICS

2015 SCORING GUIDELINES

Question 3

Intent of Question

The primary goals of this question were to assess a student's ability to (1) describe a cluster sampling procedure; (2) describe a stratified random sampling procedure; and (3) provide an advantage of each type of sampling (cluster and stratified) compared to the other type.

Solution

Part (a):

To obtain a random sample using cluster sampling, number the plots from 1 to 200 (or use the numbers implied in the stem of the problem). Using a random number generator, such as a random number table or a random number generator from a calculator or computer, generate 10 unique random integers from 1 to 200. Select the plots corresponding to the 10 integers. Measure the diameters of *all* trees in the 10 selected plots.

Part (b):

In each of the 200 plots, number each of the trees from 1 to n , where n is the number of trees in that plot. Within each plot, obtain a simple random sample of 5 percent of the trees by using a random number generator to generate unique random integers from 1 to n . Select the trees corresponding to the integers and measure the tree diameters.

Part (c):

An advantage of using the cluster sample instead of the stratified sample is that the cluster sample is much easier to obtain. For the cluster sample, only 10 plots must be visited and the trees do not need to be individually numbered.

Part (d):

If the distribution of tree diameters is different in different parts of the forest, an advantage of using stratified random sampling instead of cluster sampling is that the stratified sampling is more likely to result in a sample that is representative of the population of all tree diameters. Cluster sampling is more likely to yield a sample in which trees with large diameters or trees with small diameters are over-represented simply by chance.

AP[®] STATISTICS

2015 SCORING GUIDELINES

Question 3 (continued)

Scoring

The question is scored in three sections. Section 1 consists of part (a), section 2 consists of part (b), and section 3 consists of parts (c) and (d). Sections 1, 2, and 3 are scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response describes a correct sampling procedure for a cluster sample that contains the following three components:

1. Samples about 5% of the trees (by sampling 5% of the plots).
2. Indicates that *all* trees in the selected clusters will be included in the sample.
3. Provides sufficient detail about how clusters will be selected.

Note: To satisfy component 3, it is not enough to just say that 10 plots are selected at random.

Partially correct (P) if the response satisfies only two of the three components *OR* satisfies only component 2 *OR* satisfies only component 3.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: The statement of the problem in part (a) specifies “using the plots as clusters.” If the response defines different clusters, section 1 cannot be scored as E. To earn a score of P, the response must also:

- indicate that *all* units (trees, plots, etc.) in the randomly selected clusters will be included in the sample *AND*
- provide sufficient detail about how the clusters will be selected.

Section 2 is scored as follows:

Essentially correct (E) if the response describes a correct sampling procedure for a stratified random sample that contains the following three components:

1. Samples about 5% of the trees.
2. Indicates that the trees will be randomly selected from each stratum.
3. Provides sufficient detail about how trees will be selected from each stratum.

Note: To satisfy component 3, it is not enough to just say the trees are selected at random. More detail on how the selection will occur should be included.

Partially correct (P) if the response does not satisfy the three components but does satisfy component 2.

Note: The statement of the problem in part (b) specifies “using the plots as strata”. If the response defines different strata, section 2 cannot be scored as E. To earn a score of P, the response must also:

- clearly indicate that a random sample will be selected from *each* stratum *AND*
- provide sufficient detail about how the units that make up the strata will be selected.

Incorrect (I) if the response does not meet the criteria for E or P.

AP[®] STATISTICS

2015 SCORING GUIDELINES

Question 3 (continued)

Section 3 is scored as follows:

Essentially correct (E) if the response provides the following two components:

1. A reasonable advantage of cluster sampling that is not also true of stratified sampling, with justification and context.
2. A reasonable advantage of stratified sampling that is not also true of cluster sampling, with justification and context.

Partially correct (P) if the response provides only one of the two components;

OR

if the response provides a reasonable advantage for both components, but has no context and/or no justification.

Incorrect (I) if the response does not meet the criteria for E or P.

4 Complete Response

All three sections essentially correct

3 Substantial Response

Two sections essentially correct and one section partially correct

2 Developing Response

Two sections essentially correct and one section incorrect

OR

One section essentially correct and one or two sections partially correct

OR

Three sections partially correct

1 Minimal Response

One section essentially correct and two sections incorrect

OR

Two sections partially correct and one section incorrect

AP[®] STATISTICS

2015 SCORING GUIDELINES

Question 4

Intent of Question

The primary goal of this question was to assess a student's ability to identify, set up, perform, and interpret the results of an appropriate hypothesis test to address a particular question. More specific goals were to assess a student's ability to (1) state appropriate hypotheses; (2) determine an appropriate statistical test and check appropriate assumptions/conditions; (3) calculate an appropriate test statistic and p -value for a chi-square goodness-of-fit test; and (4) draw an appropriate conclusion, with justification, in context.

Solution

Step 1: States a correct pair of hypotheses.

H_0 : At the new branch, the probability distribution of waiting time categories is the same as the probability distribution of waiting time categories deemed acceptable by the bank.

H_a : The probability for at least one waiting time category at the new branch is different from the probabilities deemed acceptable by the bank.

OR

H_0 : $p_1 = 0.30$, $p_2 = 0.25$, $p_3 = 0.20$, $p_4 = 0.15$, $p_5 = 0.10$, where p_i is the probability that a customer's waiting time at the new branch office falls in the waiting-time category i .

H_a : At least one of the probabilities is incorrect.

Step 2: Identifies a correct test procedure (by name or by formula) and checks appropriate conditions.

The appropriate procedure is a chi-square goodness-of-fit test.

The formula for a chi-square goodness-of-fit test is:

$$\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

The conditions for the test were satisfied because:

1. It is stated in the problem that the 100 customers were randomly selected.
2. The sample size is large enough for the chi-square test to be used because the expected cell counts are 30, 25, 20, 15, and 10, which are all greater than 5.

AP[®] STATISTICS
2015 SCORING GUIDELINES

Question 4 (continued)

Step 3: Correct mechanics, including the value of the test statistic, degrees of freedom, and p -value (or rejection region)

$$\chi^2 = \frac{(25 - 30)^2}{30} + \frac{(21 - 25)^2}{25} + \frac{(21 - 20)^2}{20} + \frac{(20 - 15)^2}{15} + \frac{(13 - 10)^2}{10}$$

The test statistic is $= 0.833 + 0.640 + 0.050 + 1.67 + 0.900$
 $= 4.09$

The p -value, based $5 - 1 = 4$ degrees of freedom, is 0.3940.

Step 4: States a correct conclusion in the context of the study, using the result of the statistical test.

Because the p -value of 0.3940 is greater than any reasonable significance level such as $\alpha = 0.05$ or $\alpha = 0.10$, we fail to reject H_0 . Therefore, the sample data do not provide sufficient evidence to conclude that the probability for at least one waiting time category at the new branch office is different from the probabilities deemed acceptable by the bank.

OR

If the null hypothesis is true and the probability distribution of waiting time categories at the new branch is the same as the probability distribution deemed acceptable by the bank, we would observe a test statistic of 4.09 or greater in about 39% of random samples from the hypothesized distribution. Therefore at any reasonable significance level such as $\alpha = 0.05$ or $\alpha = 0.10$, there is not sufficient evidence to reject the null hypothesis that at the new branch the probability distribution of waiting time categories is the same as the probability distribution of waiting time categories deemed acceptable by the bank.

AP[®] STATISTICS

2015 SCORING GUIDELINES

Question 4 (continued)

Scoring

Each of steps 1, 2, 3, and 4 were scored as essentially correct (E), partially correct (P), or incorrect (I).

Step 1 is scored as follows:

Essentially correct (E) if the student correctly states the pair of hypotheses in context.

Partially correct (P) if the student correctly states the pair of hypotheses but not in context;

OR

if the student correctly states the hypotheses in terms of the p_i but does not clearly define p_i ;

OR

if the student correctly states one hypothesis in context but not the other;

OR

if the hypotheses refer to acceptable wait time and branch wait time, but it is not clear that the hypotheses are about the distribution of wait times.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- To earn a score of E, the null hypothesis must specify *equal to* or *the same as* or *equivalent*. Phrases such as *similar to* or *consistent with* in the null hypothesis cannot earn a score of E.
- The following hypotheses are scored as incorrect (I):

$$H_0 : \text{observed} = \text{expected}$$
$$H_a : \text{observed} \neq \text{expected}$$

Step 2 is scored as follows:

Essentially correct (E) if the response identifies the correct test procedure (by name or by formula) AND checks both conditions correctly.

Partially correct (P) if the response correctly completes two of the three components (identification of procedure, check of random sampling, check of expected count).

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- Chi-square test or χ^2 is sufficient for the name of the test. But if the response specifies a chi-square test for homogeneity or for independence, the name is considered incorrect.
- Because the question states the sample is random, a checked SRS is sufficient for the random sampling condition.
- Extraneous incorrect assumptions (such as normality, $n > 30$, etc.) lowers the score one level (that is, from E to P, or from P to I).

AP[®] STATISTICS

2015 SCORING GUIDELINES

Question 4 (continued)

Step 3 is scored as follows:

Essentially correct (E) if the response correctly calculates both the test statistic and the p -value.

Note: Degrees of freedom need to be reported only if the student uses a table to obtain the p -value.

Partially correct (P) if the response correctly calculates the test statistic but not the p -value;

OR

if the response calculates the test statistic incorrectly but then reports the correct p -value for the computed test statistic;

OR

if the response fails to report the test statistic but reports the correct degrees of freedom and the correct p -value.

Incorrect (I) if the response does not meet the criteria for E or P.

Step 4 is scored as follows:

Essentially correct (E) if the response provides a correct conclusion in context, also providing justification based on linkage between the p -value and the conclusion.

Partially correct (P) if the response provides a correct conclusion (or decision) with linkage to the p -value, but not in context;

OR

if the response provides a correct conclusion in context, but without justification based on linkage to the p -value.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- If the correct conclusion for the reported p -value is fail to reject H_0 , then a conclusion that is equivalent to *accept* H_0 either as a stated decision or as a conclusion in context cannot be scored E. Such a response is scored P if the conclusion is in context with linkage. If such a response lacks either context or linkage, it is scored I.
- If the p -value is computed incorrectly but the conclusion is consistent with the incorrect p -value and has context with linkage, the response is scored E.
- If both a significance level α and a p -value are given together, the linkage between the p -value and the conclusion is implied. If no significance level is given, the solution must be explicit about the linkage by giving a correct interpretation of the p -value OR explaining how the conclusion follows from the p -value, such as, "Because the p -value is small, we reject the null hypothesis" or "Because the p -value is large, we do not reject the null hypothesis."
- If the stated decision (reject, fail to reject) is inconsistent with a correct conclusion in context with linkage, the response is scored P.
- If the correct conclusion for the reported p -value is fail to reject, but the student says to retain the null hypothesis with linkage and context, the response is scored P. However, if the student goes on to say something equivalent to "fail to reject" in context, the response is scored E.

AP[®] STATISTICS
2015 SCORING GUIDELINES

Question 4 (continued)

Each essentially correct (E) step counts as 1 point. Each partially correct (P) step counts as $\frac{1}{2}$ point.

- 4 Complete Response**
- 3 Substantial Response**
- 2 Developing Response**
- 1 Minimal Response**

If a response is between two scores (for example, $2\frac{1}{2}$ points), use a holistic approach to decide whether to score up or down, depending on the overall strength of the response and communication.

AP[®] STATISTICS

2015 SCORING GUIDELINES

Question 5

Intent of Question

The primary goals of this question were to assess a student's ability to (1) describe a Type II error in the context of a specific study; (2) understand the relationship between the probability of a Type II error and the significance level; and (3) interpret in context a specific value for the power of a test.

Solution

Part (a):

A Type II error is the failure to reject the null hypothesis when it is false. For this study, a Type II error would be not concluding that the new cups insulate better on average than the traditional cups, when the new cups really do insulate better on average.

Part (b):

The probability of a Type II error and the probability of a Type I error (which is the significance level) are inversely related. As one of these probabilities is decreased, the other must necessarily increase. Thus, choosing a significance level of 0.10 would result in a smaller probability of Type II error because 0.10 is greater than 0.01.

Part (c):

The value of 0.88 represents the probability of correctly rejecting the null hypothesis and concluding that the true mean cooling time of the new cup is greater than that of the traditional cup, given that the true mean cooling time of the new cup is 2 minutes greater than that of the traditional cup. In other words, given that the mean cooling time of the new cup is actually 2 minutes greater than that of the traditional cup, if we were to apply the test on repeated samples of the same size, for about 88% of the samples we would expect to correctly reject the null hypothesis in favor of the alternative.

AP[®] STATISTICS

2015 SCORING GUIDELINES

Question 5 (continued)

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response includes a correct description of a Type II error in context, and with explicit correct mention of “mean” or “on average”.

Partially correct (P) if the response includes a correct description of a Type II error in context but without mention of “mean” or “on average”;

OR

if the response includes a correct description of a Type II error mentioning “mean” or “on average”, but having no context;

OR

if the response includes a correct description of Type I error in context, and with explicit correct mention of “mean” or “on average”.

Incorrect (I) if the response does not meet the criteria for E or P.

Note: If a response includes either wording that is equivalent to “accept H_0 ”, or a statement consistent with a two-sided alternative hypothesis, or both, then the response cannot be scored as E.

Part (b) is scored as follows:

Essentially correct (E) if the response includes the correct significance level and provides a reasonable justification in terms of both Type I and Type II errors.

Partially correct (P) if the response includes the correct significance level but provides vague or weak justification in terms of both Type I and Type II errors;

OR

if the response includes the incorrect significance level but provides an explanation indicating understanding of the relationship between Type II error probability and the significance level.

Incorrect (I) if the response does not meet the criteria for E or P.

AP[®] STATISTICS

2015 SCORING GUIDELINES

Question 5 (continued)

Part (c) is scored as follows:

Essentially correct (E) if the response includes a correct probability statement about rejecting the null hypothesis, a correct conditional statement about the null hypothesis being false, and explicit mention of the 2-minute increase.

Partially correct (P) if the response includes a correct probability statement about rejecting the null hypothesis and a correct conditional statement about the null hypothesis being false, but is missing explicit mention of the 2-minute increase.

Incorrect (I) if the response does not meet the criteria for E or P.

4 Complete Response

All three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and one part incorrect

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct and no parts partially correct

OR

No parts essentially correct and two parts partially correct

AP[®] STATISTICS

2015 SCORING GUIDELINES

Question 6

Intent of Question

The primary goals of this question were to assess a student's ability to understand, apply, and draw conclusions from a simulation analysis beyond what was previously studied. More specific goals are to assess a student's ability to (1) produce a graphical display and use it to compare distributions between two groups; (2) draw an appropriate conclusion from a test statistic and a p -value; and (3) understand and interpret simulation results to draw a conclusion about a hypothesis testing scenario not previously studied.

Solution

Part (a):

The back-to-back stemplot is as follows.

Number of shaves

Leaf unit = 1.0

0|8 represents 80 shaves.

Generic (G)		Name Brand (B)
0	8	
9 7 6	9	
8 1	10	3 5 6 9
8 1	11	0 2 3 6
9	12	2
6	13	0

Part (b):

The name brand blades generally produce more shaves than the generic blades; the median number of shaves for the name brand blades is 111.5, which is higher than the median number of shaves for the generic blades of 104.5. The generic blades display more variability in number of shaves than the name brand blades. Both distributions of number of shaves are slightly skewed to the right, and no outliers are apparent in either direction.

Part (c):

The p -value of 0.20 is greater than the conventional significance levels of $\alpha = 0.10$ or $\alpha = 0.05$. Therefore the sample data do not provide enough evidence to reject the null hypothesis that there is no difference in the mean number of shaves produced by the generic and the name brand blades. So we cannot conclude that the name brand blades last significantly longer than the generic blades, on average.

AP[®] STATISTICS
2015 SCORING GUIDELINES

Question 6 (continued)

Part (d):

The simulation was conducted under the assumption that the population standard deviations are equal (so, $\sigma_G / \sigma_B = 1$) and the parent population from which the samples were drawn is normal. Therefore we expect that with a very large number of random samples, the sample standard deviation for the name brand group will be greater than the sample standard deviation for the generic group about half of the time and will be less about half of the time. As a consequence, the *ratio* of the two sample standard deviations should be less than 1 about half of the time and greater than 1 about half of the time. Because the median of a distribution is the value that is exceeded half of the time and not exceeded half of the time, we expect the median of these ratios to equal 1.

Part (e):

Yes, there is evidence that the generic blades (G) have more variability than the name brand blades (B). The ratio of sample standard deviations for the observed sample data is $\frac{16.70}{8.28} \approx 2.017$.

The simulation was conducted assuming that the population standard deviations are equal. The value of 2.017 appears fairly far in the right tail of the distribution of simulated ratios. In fact, it appears that less than 5% of the simulated ratios are as large, or larger than, the observed ratio of 2.017. Because the observed ratio is unlikely to occur by random sampling alone when the population standard deviations are equal, we have fairly strong evidence that the generic blades (G) really do have more variability in the number of shaves than the name brand blades (B).

AP[®] STATISTICS

2015 SCORING GUIDELINES

Question 6 (continued)

Scoring

This question is scored in four sections. Section 1 consists of parts (a) and (b); section 2 consists of part (c); section 3 consists of part (d); and section 4 consists of part (e). Sections 1, 2, 3, and 4 are scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response constructs the back-to-back stemplot correctly AND correctly *compares* the two distributions on all three characteristics (center, spread, shape).

Partially correct (P) if the response:

- Correctly constructs the graph AND
 - Correctly compares one or two of the three characteristics;OR
 - Discusses all three characteristics without comparison.

OR

- Incorrectly constructs the graph AND correctly compares all three characteristics.

Incorrect (I) if the response does not meet the criteria for E or P.

Section 2 is scored as follows:

Essentially correct (E) if the response provides a correct conclusion in context with justification based on linkage to the p -value.

Partially correct (P) if the response provides a correct conclusion in context, but no justification based on linkage to the p -value;

OR

if the response provides a correct conclusion with justification based on linkage to the p -value, but no context.

Incorrect (I) if the response does not meet the criteria for E or P.

Notes:

- If the correct conclusion for the reported p -value is to fail to reject the null hypothesis, then a conclusion that is equivalent to accepting the null hypothesis, either as a stated decision or as a conclusion in context, cannot be scored as E (for example, “Name brand blades last the same length of time as the generic blades.”) Such a response is scored P provided that the conclusion is in context with linkage. If such a response lacks either context or linkage, it is scored I.
- If the correct conclusion for the reported p -value is to fail to reject, but the response says to “retain the null hypothesis” with linkage and context, the response is scored P. However, if the response goes on to say something equivalent to “fail to reject” in context, the response is scored E.
- If both a significance level α and a p -value are given together, the linkage between the p -value and the conclusion is implied. If no significance level is given, the solution must be explicit about the linkage by giving a correct interpretation of the p -value or explaining how the conclusion follows from the p -value, such as “Because the p -value is large, we do not reject the null hypothesis.”

AP[®] STATISTICS

2015 SCORING GUIDELINES

Question 6 (continued)

Section 3 is scored as follows:

Essentially correct (E) if the response correctly states that the population standard deviations are equal if the null hypothesis is true, so the (simulated) sample standard deviations should produce ratios centered around 1.

Partially correct (P) if the response does not appeal to the equality of the population standard deviations under the null hypothesis but makes the connection to the simulated ratios being centered around 1;

OR

if the response appeals to the equality of the population standard deviations under the null hypothesis, but does not make the connection that the simulated sample standard deviations should produce ratios centered around 1.

Incorrect (I) if the response does not meet the criteria for E or P.

Section 4 is scored as follows:

Essentially correct (E) if the response draws an appropriate conclusion with a well-explained justification based on the results of the simulation.

Partially correct (P) if the response draws an appropriate conclusion based on the results of the simulation, but with a weak or poorly communicated justification.

Incorrect (I) if the response does not meet the criteria for E or P.

Each essentially correct (E) section counts as 1 point. Each partially correct (P) section counts as $\frac{1}{2}$ point.

- 4 Complete Response**
- 3 Substantial Response**
- 2 Developing Response**
- 1 Minimal Response**

If a response is between two scores (for example, $2\frac{1}{2}$ points), use a holistic approach to decide whether to score up or down, depending on the overall strength of the response and communication.