

# AP<sup>®</sup> Statistics Practice Exam

---

From the 2014 Administration

This Practice Exam is provided by the College Board for AP Exam preparation. Teachers are permitted to download the materials and make copies to use with their students in a classroom setting only. To maintain the security of this exam, teachers should collect all materials after their administration and keep them in a secure location. **Exams may not be posted on school or personal websites, nor electronically redistributed for any reason.** Further distribution of these materials outside of the secure College Board site disadvantages teachers who rely on uncirculated questions for classroom testing. Any additional distribution is in violation of the College Board's copyright policies and may result in the termination of Practice Exam access for your school as well as the removal of access to other online services such as the AP Teacher Community and Online Score Reports.

PLACE SEAL HERE

# AP<sup>®</sup> Statistics Exam

## SECTION I: Multiple Choice

2014

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

### At a Glance

**Total Time**

1 hour, 30 minutes

**Number of Questions**

40

**Percent of Total Score**

50%

**Writing Instrument**

Pencil required

**Electronic Device**

Graphing calculator  
expected

### Instructions

Section I of this exam contains 40 multiple-choice questions. Fill in only the circles for numbers 1 through 40 on your answer sheet.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding circle on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question      Sample Answer

Chicago is a      (A) ● (C) (D) (E)  
(A) state  
(B) city  
(C) country  
(D) continent  
(E) village

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

PLACE SEAL HERE

Form I  
Form Code 4JBP6-S

90

PLACE SEAL HERE

DO NOT seal answer sheet inside

## Formulas

### (I) Descriptive Statistics

$$\bar{x} = \frac{\sum x_i}{n}$$

$$s_x = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2}$$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}}$$

$$\hat{y} = b_0 + b_1 x$$

$$b_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$$b_0 = \bar{y} - b_1 \bar{x}$$

$$r = \frac{1}{n-1} \sum \left( \frac{x_i - \bar{x}}{s_x} \right) \left( \frac{y_i - \bar{y}}{s_y} \right)$$

$$b_1 = r \frac{s_y}{s_x}$$

$$s_{b_1} = \frac{\sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n-2}}}{\sqrt{\sum (x_i - \bar{x})^2}}$$

(II) Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$E(X) = \mu_x = \sum x_i p_i$$

$$\text{Var}(X) = \sigma_x^2 = \sum (x_i - \mu_x)^2 p_i$$

If  $X$  has a binomial distribution with parameters  $n$  and  $p$ , then:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

$$\mu_x = np$$

$$\sigma_x = \sqrt{np(1 - p)}$$

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1 - p)}{n}}$$

If  $\bar{x}$  is the mean of a random sample of size  $n$  from an infinite population with mean  $\mu$  and standard deviation  $\sigma$ , then:

$$\mu_{\bar{x}} = \mu$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

(III) Inferential Statistics

Standardized test statistic:  $\frac{\text{statistic} - \text{parameter}}{\text{standard deviation of statistic}}$

Confidence interval: statistic  $\pm$  (critical value) • (standard deviation of statistic)

Single-Sample

| Statistic         | Standard Deviation of Statistic |
|-------------------|---------------------------------|
| Sample Mean       | $\frac{\sigma}{\sqrt{n}}$       |
| Sample Proportion | $\sqrt{\frac{p(1-p)}{n}}$       |

Two-Sample

| Statistic                        | Standard Deviation of Statistic                                                                                                                          |
|----------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------|
| Difference of sample means       | $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$<br><br>Special case when $\sigma_1 = \sigma_2$<br>$\sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ |
| Difference of sample proportions | $\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$<br><br>Special case when $p_1 = p_2$<br>$\sqrt{p(1-p)} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$    |

$$\text{Chi-square test statistic} = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

# STATISTICS

## SECTION I

**Time—1 hour and 30 minutes**

**Number of questions—40**

**Percent of total score—50**

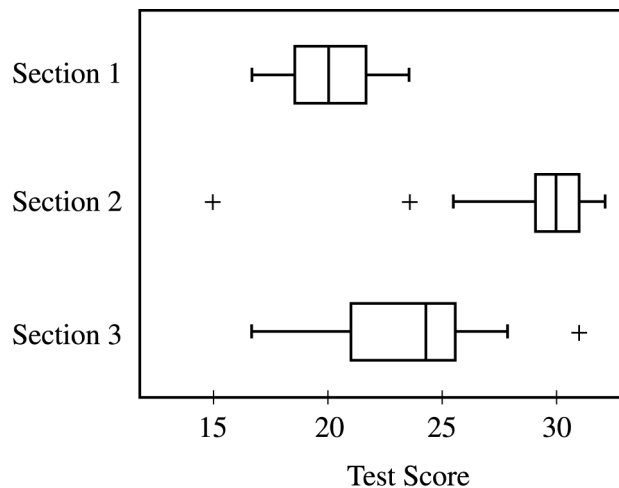
**Directions:** Solve each of the following problems, using the available space for scratch work. Decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

1. Heights, in inches, for the 200 graduating seniors from Washington High School are summarized in the frequency table below.

| Height                              | Frequency |
|-------------------------------------|-----------|
| Height < 60 inches                  | 22        |
| 60 inches $\leq$ height < 66 inches | 84        |
| 66 inches $\leq$ height < 72 inches | 62        |
| 72 inches $\leq$ height < 78 inches | 24        |
| Height $\geq$ 78 inches             | 8         |

Which of the following statements about the median height is true?

- (A) It is greater than or equal to 78 inches.
- (B) It is greater than or equal to 72 inches but less than 78 inches.
- (C) It is greater than or equal to 66 inches but less than 72 inches.
- (D) It is greater than or equal to 60 inches but less than 66 inches.
- (E) It is less than 60 inches.



2. Professor James gave the same test to his three sections of statistics students. On the 35-question test, the highest score was 32 and the lowest was 15. Based on the information displayed in the boxplots above, which of the following statements is true?
- (A) Section 1 has the smallest interquartile range.
  - (B) The lowest score in section 2 is higher than the highest score in either of the other sections.
  - (C) Section 2 has the smallest range of scores.
  - (D) The top 25% of scores in section 2 are lower than the highest score in section 3.
  - (E) At least 50% of the scores in section 3 are higher than all of the scores in section 1.

3. A well-designed experiment should have which of the following characteristics?

I. Subjects assigned randomly to treatments

II. A control group or at least two treatment groups

III. Replication

(A) I only

(B) I and II only

(C) I and III only

(D) II and III only

(E) I, II, and III

4. The distribution of colors of candies in a bag is as follows.

| Color      | Brown | Red | Yellow | Green | Orange |
|------------|-------|-----|--------|-------|--------|
| Proportion | 0.3   | 0.2 | 0.2    | 0.2   | 0.1    |

If two candies are randomly drawn from the bag with replacement, what is the probability that they are the same color?

- (A) 0.09
- (B) 0.22
- (C) 0.25
- (D) 0.75
- (E) 0.78

| Color<br>Selected | Age of Child |             |
|-------------------|--------------|-------------|
|                   | 2-year olds  | 3-year olds |
| Yellow            | 14           | 28          |
| Red               | 32           | 24          |
| Blue              | 19           | 16          |

5. Each of 133 children in a sample was asked to choose a pencil. Three different colors were available: yellow, red, and blue. The number of 2-year olds and 3-year olds who selected each color is shown in the table above. In a test of independence of age and color, which of the following is used as the expected cell count for 2-year olds who select a yellow pencil?

- (A) 14
- (B)  $\frac{1}{6}(133)$
- (C)  $\frac{1}{3}(14 + 32 + 19)$
- (D)  $\frac{1}{2}(14 + 28)$
- (E)  $\frac{(14 + 28)(14 + 32 + 19)}{133}$

6. Approximately 52 percent of all recent births were boys. In a simple random sample of 100 recent births, 49 were boys and 51 were girls. The most likely explanation for the difference between the observed results and the expected results in this case is
- (A) bias
  - (B) variability due to sampling
  - (C) nonsampling error
  - (D) a sampling frame that is incomplete
  - (E) confounding

7. Each person in a random sample of adults was asked how many DVDs he or she owned. Summary statistics are given below.

|          |         |       |         |        |       |         |
|----------|---------|-------|---------|--------|-------|---------|
| Variable | N       | Mean  | Median  | TrMean | StDev | SE Mean |
| DVDs     | 117     | 129.4 | 50.0    | 76.5   | 323.6 | 29.2    |
| Variable | Minimum |       | Maximum |        | Q1    | Q3      |
| DVDs     | 0.0     |       | 3000.0  |        | 30.0  | 95.0    |

Which of the following statements is true?

- (A) Seventy-five percent of the adults in the sample own more than 95 DVDs.
- (B) Fifty percent of the adults in the sample own between 0 and 129.4 DVDs.
- (C) The distribution of the number of DVDs owned appears to be approximately symmetric.
- (D) The interquartile range of the number of DVDs owned is 65.
- (E) The distribution of the number of DVDs owned contains outliers on both the low side and the high side.

8. For flights from a particular airport in January, there is a 30 percent chance of a flight being delayed because of icy weather. If a flight is delayed because of icy weather, there is a 10 percent chance the flight will also be delayed because of a mechanical problem. If a flight is not delayed because of icy weather, there is a 5 percent chance that it will be delayed because of a mechanical problem. If one flight is selected at random from the airport in January, what is the probability that the flight selected will have at least one of the two types of delays?
- (A) 0.065
  - (B) 0.335
  - (C) 0.350
  - (D) 0.450
  - (E) 0.665

- 
9. A survey was administered to parents of high school students in a certain state to see if the parents thought the students' academic needs were being met. To select the sample, the parents were divided into two groups—one group of parents who live in cities with populations of more than 100,000 and the other group of parents who live in cities with populations less than or equal to 100,000. A random sample of 100 parents from each group was taken. Which of the following statements about the sample of 200 parents is true?
- (A) It is a convenience sample because the sample of parents was easily obtained.
  - (B) It is a stratified random sample because parents were randomly selected from each group.
  - (C) It is a random cluster sample because parents were randomly selected from each group.
  - (D) It is a random cluster sample because groups of high schools were randomly selected.
  - (E) It is a systematic sample because the parents were systematically divided into two groups.

10. A popular computer card game keeps track of the number of games played and the number of games won on that computer. The cards are shuffled before each game, so the outcome of the game is independent from one game to the next and is based on the skill of the player. Let  $X$  represent the number of games that have been won out of 100 games. Under which of the following situations would  $X$  be a binomial random variable?
- (A) All games were played by the same player, whose skill improved over the course of the 100 games.
  - (B) A group of 5 players of different skill levels were each allowed to play 20 games in a row.
  - (C) A group of players of different skill levels were each allowed to play until they had lost 3 games and this resulted in 100 games played.
  - (D) Two players of equal skill level each played one game a day for 50 days and their skill level did not change from day to day.
  - (E) Two players of different skill levels competed by allowing one player to continue until a game was lost, then the other player to continue until a game was lost, and so on, until 100 games were played.
- 

11. A carnival game allows the player a choice of simultaneously rolling two, four, six, eight, or ten fair dice. Each die has six faces numbered 1 through 6, respectively. After the player rolls the dice, the numbers that appear on the faces that land up are recorded. The player wins if the greatest number recorded is 1 or 2. How many dice should the player choose to roll to maximize the chance of winning?
- (A) Two
  - (B) Four
  - (C) Six
  - (D) Eight
  - (E) Ten

12. A school is having a contest in which students guess the number of candies in a jar. The student whose guess is closest to the correct number of candies in the jar wins a prize. The number of candies guessed by male and female students is shown in the back-to-back stemplot below.

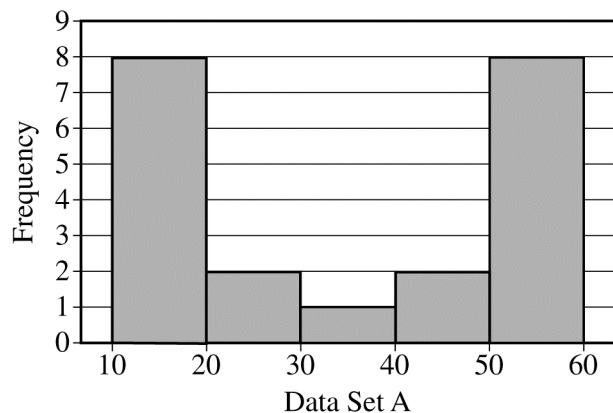
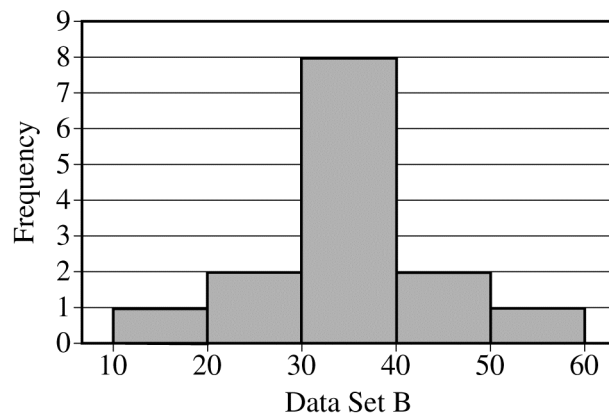
| Male          |    | Female      |
|---------------|----|-------------|
|               | 6  | 4           |
| 2             | 7  | 5 8         |
| 1             | 8  | 0 9         |
| 0             | 9  | 0 0 5       |
| 0 0           | 10 | 0 0 1 4     |
| 5 5 3 0       | 11 | 0 0 5 7 8 8 |
| 5 5 5 4 2 1 0 | 12 | 0 1 1 2     |
| 8 6 5 0 0 0   | 13 |             |
| 9 5 0         | 14 |             |

Key:  $6|4 = 64$

Which of the following statements is true about the distributions of guesses?

- (A) The distribution of guesses for male students is skewed to the left, and the distribution of guesses for female students is skewed to the right.
- (B) The distribution of guesses for male students is skewed to the right, and the distribution of guesses for female students is skewed to the left.
- (C) The distributions of guesses for male and female students are both skewed to the right.
- (D) The distributions of guesses for male and female students are both skewed to the left.
- (E) The distributions of guesses for male and female students are both symmetric.

13. The manager of a car company will select a random sample of its customers to create a 90 percent confidence interval to estimate the proportion of its customers who have children. Of the following, which is the smallest sample size that will result in a margin of error of no more than 6 percentage points?
- (A) 100
  - (B) 125
  - (C) 150
  - (D) 200
  - (E) 275



14. Which of the following statements must be true about the data sets A and B displayed in the histograms above?
- (A) The mean of data set A is equal to the mean of data set B.
  - (B) The median of data set A is equal to the median of data set B.
  - (C) The range of data set A is equal to the range of data set B.
  - (D) The standard deviation of data set A is less than the standard deviation of data set B.
  - (E) The standard deviation of data set A is greater than the standard deviation of data set B.

15. A box contains 10 tags, numbered 1 through 10, with a different number on each tag. A second box contains 8 tags, numbered 20 through 27, with a different number on each tag. One tag is drawn at random from each box. What is the expected value of the sum of the numbers on the two selected tags?

- (A) 13.5
- (B) 14.5
- (C) 15.0
- (D) 27.0
- (E) 29.0

---

16. A pollster is interested in comparing the proportions of women and men in a particular town who are in favor of a ban on fireworks within town borders. The pollster plans to test the hypothesis that the proportion of women in favor of the ban is different from the proportion of men in favor of the ban. There are 4,673 women and 4,502 men who live in the town. From a simple random sample of 40 women in the town, the pollster finds that 38 favor the ban. From an independent simple random sample of 50 men in the town, the pollster finds that 27 favor the ban. Which of the following statements is true about this situation?

- (A) Because the samples are from normal populations, a two-proportion  $z$ -test would be valid.
- (B) Because the size of each sample is greater than 30, a two-proportion  $z$ -test would be valid.
- (C) Because the number who favored the ban is greater than 10 in both groups, a two-proportion  $z$ -test would be valid.
- (D) Because of the relative sizes of the populations and samples, a two-proportion  $z$ -test would be valid.
- (E) A two-proportion  $z$ -test would not be valid for these data.

17. Employees at a large company can earn monthly bonuses. The distribution of monthly bonuses earned by all employees last year has mean 2.3 and standard deviation 1.3. Let  $z$  represent the standard normal distribution. If  $\bar{x}$  represents the mean number of monthly bonuses earned last year for a random sample of 40 employees, which of the following calculations will give the approximate probability that  $\bar{x}$  is less than 2 ?

(A)  $P\left(z < \frac{2 - 2.3}{\left(\frac{1.3}{\sqrt{40}}\right)}\right)$

(B)  $P\left(z < \frac{2 - 2.3}{1.3}\right)$

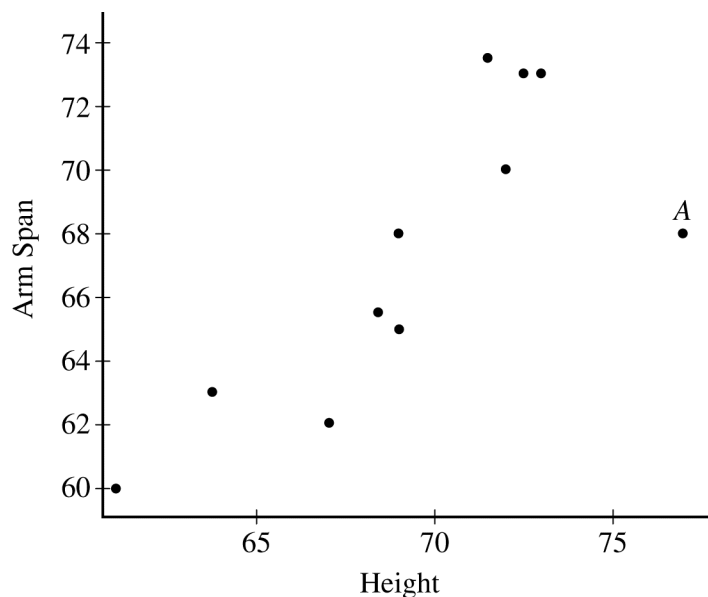
(C)  $P(z < 2)$

(D)  $P\left(z < \frac{2.3 - 2}{\left(\frac{1.3}{\sqrt{40}}\right)}\right)$

(E)  $P\left(z < \frac{2.3 - 2}{1.3}\right)$

18. The  $p$ -value for a one-sided  $t$ -test is 0.10. If the test had been two-sided, what would the  $p$ -value have been?
- (A) 0.05
  - (B) 0.20
  - (C) 0.90
  - (D) 0.95
  - (E) It depends on the direction of the alternative hypothesis.

19. A scatterplot of student height, in inches, versus corresponding arm span length, in inches, is shown below. One of the points in the graph is labeled *A*.



If the point labeled *A* is removed, which of the following statements would be true?

- (A) The slope of the least squares regression line is unchanged and the correlation coefficient increases.
- (B) The slope of the least squares regression line is unchanged and the correlation coefficient decreases.
- (C) The slope of the least squares regression line increases and the correlation coefficient increases.
- (D) The slope of the least squares regression line increases and the correlation coefficient decreases.
- (E) The slope of the least squares regression line decreases and the correlation coefficient increases.

20. Let  $X$  be a random variable whose values are the number of dots that appear on the uppermost face when a fair die is rolled. The possible values of  $X$  are 1, 2, 3, 4, 5, and 6. The mean of  $X$  is  $\frac{7}{2}$  and the variance of  $X$  is  $\frac{35}{12}$ . Let  $Y$  be the random variable whose value is the difference (first minus second) between the number of dots that appear on the uppermost face for the first and second rolls of a fair die that is rolled twice. What is the standard deviation of  $Y$ ?

(A)  $\sqrt{\frac{35}{12}}$

(B)  $\sqrt{\frac{35}{12}} + \sqrt{\frac{35}{12}}$

(C)  $\sqrt{\frac{35}{12} + \frac{35}{12}}$

(D)  $\sqrt{\frac{35}{12} - \frac{35}{12}}$

(E)  $\frac{35}{12} + \frac{35}{12}$

21. In a recent poll of 1,500 randomly selected eligible voters, only 525 (35 percent) said that they did not vote in the last election. However, a vote count showed that 80 percent of eligible voters actually did not vote in the last election. Which of the following types of bias is most likely to have occurred in the poll?
- (A) Nonresponse bias
  - (B) Sampling bias
  - (C) Selection bias
  - (D) Response bias
  - (E) Undercoverage bias

22. A distribution of scores is approximately normal with a mean of 78 and a standard deviation of 8.6. Which of the following equations can be used to find the score  $x$  above which 33 percent of the scores fall?

(A)  $0.44 = \frac{x - 78}{(8.6)^2}$

(B)  $0.67 = \frac{x - 78}{(8.6)^2}$

(C)  $0.33 = \frac{x - 78}{8.6}$

(D)  $0.44 = \frac{x - 78}{8.6}$

(E)  $0.67 = \frac{x - 78}{8.6}$

23. A random sample of 300 students is selected from a large group of students who use a computer-equipped classroom on a regular basis. Occasionally, students leave their USB drive in a computer. Of the 300 students questioned, 180 said that they write their name on their USB drive. Which of the following is a 98 percent confidence interval for the proportion of all students using the classroom who write their name on their USB drive?

(A)  $0.4 \pm 2.33\sqrt{\frac{(0.4)(0.6)}{300}}$

(B)  $0.4 \pm 1.96\sqrt{\frac{(0.4)(0.6)}{300}}$

(C)  $0.6 \pm 2.33\sqrt{\frac{(0.6)(0.4)}{300}}$

(D)  $0.6 \pm 1.96\sqrt{\frac{(0.6)(0.4)}{300}}$

(E)  $0.6 \pm 2.05\sqrt{\frac{(0.6)(0.4)}{300}}$

24. In a large set of data that are approximately normally distributed,

$r$  is the value in the data set that has a  $z$ -score of  $-1.00$ ,

$s$  is the value of the first quartile, and

$t$  is the value of the 20th percentile.

Which of the following is the correct order from least to greatest for the values of  $r$ ,  $s$ , and  $t$ ?

(A)  $r, s, t$

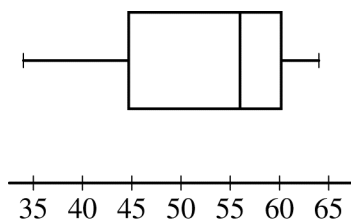
(B)  $r, t, s$

(C)  $s, t, r$

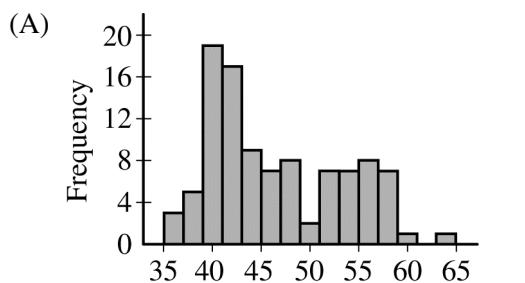
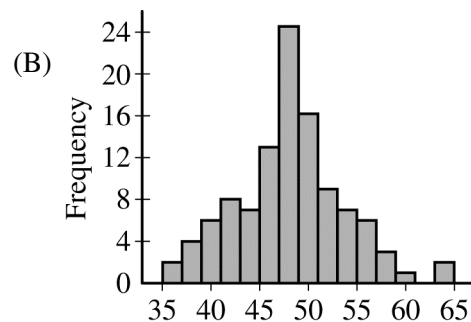
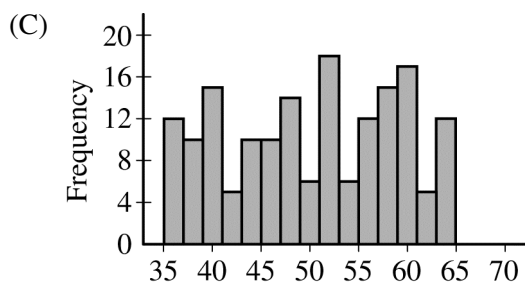
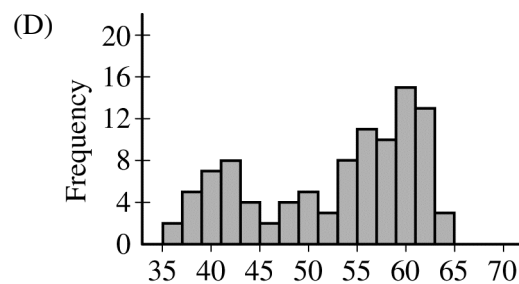
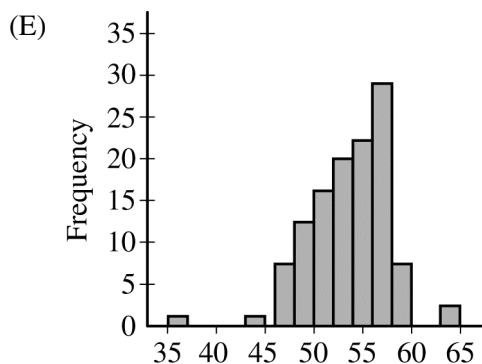
(D)  $t, r, s$

(E)  $t, s, r$

25. A researcher constructed a 95 percent confidence interval for the mean number of alfalfa weevils on an alfalfa plant within a field. Based on 80 randomly selected alfalfa plants, the researcher found an average of 2.5 alfalfa weevils per plant and computed the 95 percent confidence interval to be 1.50 to 3.50. Which of the following statements is a correct interpretation of the 95 percent confidence level?
- (A) Approximately 95 percent of alfalfa fields sampled will have an average of 1.50 to 3.50 alfalfa weevils per plant, but nothing can be said about the sample mean number of alfalfa weevils for this or any other field.
  - (B) Approximately 95 percent of alfalfa fields sampled will have an average of 1.50 to 3.50 alfalfa weevils per plant. The sample mean for this field was 2.50 alfalfa weevils per plant, but the sample means for other fields may be different.
  - (C) If we repeatedly sampled this field, taking samples of 80 plants and constructing 95% confidence intervals, then, approximately 95 percent of these intervals would include 2.5, the mean for the sample described above.
  - (D) If we repeatedly sampled this field, taking samples of 80 plants and constructing 95% confidence intervals, then, approximately 95 percent of these intervals would include the population mean number of alfalfa weevils on an alfalfa plant in this field.
  - (E) If we repeatedly sampled this field, taking samples of 80 plants and constructing 95% confidence intervals, then, approximately 95 percent of these intervals would include the sample mean for that sample.



26. Which of the following histograms could have been constructed from the same set of data summarized by the boxplot above?



27. An amateur men's swimming association is trying to decide whether times in the 100-meter breaststroke will be reduced if the men shave their heads. From the population of swimmers, six were selected at random and agreed to swim two races — one before shaving their heads and one after shaving their heads. The results for each race, with times in seconds, are given in the table below.

| Swimmer                       | I    | II   | III  | IV   | V    | VI   |
|-------------------------------|------|------|------|------|------|------|
| Time before shaving ( $x_1$ ) | 66.2 | 67.5 | 64.3 | 68.0 | 67.2 | 63.9 |
| Time after shaving ( $x_2$ )  | 65.9 | 67.0 | 66.3 | 67.1 | 67.5 | 64.2 |

Let  $\mu_1$  represent the population mean of swimming times before shaving and  $\mu_2$  represent the population mean of swimming times after shaving. Let  $\mu_d$  represent the population mean of the differences,  $X_1 - X_2$ . These differences follow a normal distribution. Which of the following would be the most appropriate test and alternative hypothesis to use in testing this theory?

- (A) A two-sample  $t$ -test with  $H_a: \mu_1 > \mu_2$
- (B) A two-sample  $t$ -test with  $H_a: \mu_1 \neq \mu_2$
- (C) A two-sample  $t$ -test with  $H_a: \mu_1 < \mu_2$
- (D) A paired  $t$ -test with  $H_a: \mu_d > 0$
- (E) A paired  $t$ -test with  $H_a: \mu_d \neq 0$

28. A newspaper poll found that 52 percent of the respondents in a large random sample of likely voters in a district intend to vote for candidate Smith rather than the opponent. A 95 percent confidence interval for the population proportion was computed to be  $0.52 \pm 0.04$ . Based on the confidence interval, which of the following should the newspaper report to its readers?
- (A) Smith will win because a majority of voters are in favor of Smith.
  - (B) There is a 95% chance that Smith will win.
  - (C) The poll predicts Smith will win, but there is a 5% chance that the prediction is incorrect due to sampling error.
  - (D) With 95% confidence, there is convincing evidence that Smith will win.
  - (E) No prediction about who will win can be made with 95% confidence.

29. Mating eagle pairs typically have two baby eagles (called eaglets). When there are two eaglets, the parents always feed the older eaglet until it has had its fill, and then they feed the younger eaglet. This results in an unequal chance of survival for the two eaglets. Suppose that the older eaglet has a 50 percent chance of survival. If the older eaglet survives, the younger eaglet has a 10 percent chance of survival. If the older eaglet does not survive, the younger eaglet has a 30 percent chance of survival. Let  $X$  be the number of eaglets that survive. Which of the following tables shows the probability distribution of  $X$ ?

(A)

|        |               |               |               |
|--------|---------------|---------------|---------------|
| $x$    | 0             | 1             | 2             |
| $p(x)$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ |

(B)

|        |               |               |               |
|--------|---------------|---------------|---------------|
| $x$    | 0             | 1             | 2             |
| $p(x)$ | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{1}{4}$ |

(C)

|        |      |      |      |
|--------|------|------|------|
| $x$    | 0    | 1    | 2    |
| $p(x)$ | 0.35 | 0.60 | 0.05 |

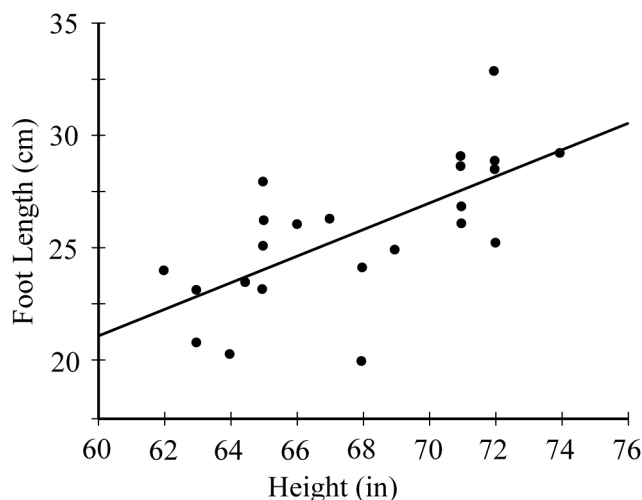
(D)

|        |      |      |      |
|--------|------|------|------|
| $x$    | 0    | 1    | 2    |
| $p(x)$ | 0.05 | 0.90 | 0.05 |

(E)

|        |      |      |      |
|--------|------|------|------|
| $x$    | 0    | 1    | 2    |
| $p(x)$ | 0.10 | 0.30 | 0.50 |

30. A statistics teacher wants to determine whether there is a linear relationship between high school students' heights, in inches (in), and the lengths of their feet, in centimeters (cm). The teacher obtains height and foot-length measurements for a random sample of 23 students at the high school and generates the following graph and computer output.



| Term      | Estimate | Std Error | t Ratio | Prob> t |
|-----------|----------|-----------|---------|---------|
| Intercept | -13.867  | 9.410     | -1.474  | 0.1554  |
| Height    | 0.583    | 0.138     | 4.208   | 0.0004  |

Provided that the assumptions for regression inference are satisfied, which of the following provides a 95 percent confidence interval estimate of the slope of the population regression line for predicting foot length from height?

(A)  $0.583 \pm (1.717)(0.138)$

(B)  $0.583 \pm (2.080)(0.138)$

(C)  $0.583 \pm (1.960)\left(\frac{0.138}{\sqrt{23}}\right)$

(D)  $0.583 \pm (2.074)\left(\frac{0.138}{\sqrt{23}}\right)$

(E)  $0.583 \pm (4.208)\left(\frac{0.138}{\sqrt{23}}\right)$

31. Each value in a sample has been transformed by multiplying by 3 and then adding 10. If the original sample had a variance of 4, what is the variance of the transformed sample?
- (A) 4
  - (B) 12
  - (C) 16
  - (D) 22
  - (E) 36

32. A controversial issue in the sport of professional soccer is the use of instant replay for making difficult goal line decisions. Each person in a representative sample of 102 players, fans, coaches, and officials was asked his or her opinion about the use of instant replay for goal-line decisions. The data are summarized in the two-way frequency table below.

|          |           | Opinion   |            |
|----------|-----------|-----------|------------|
|          |           | Favor Use | Oppose Use |
| Category | Players   | 22        | 2          |
|          | Fans      | 18        | 6          |
|          | Coaches   | 15        | 26         |
|          | Officials | 3         | 10         |

In testing to see whether opinion with respect to the use of instant replay is independent of the category of the person interviewed, a chi-square test statistic of 27.99 and a  $p$ -value less than 0.001 were calculated. Which of the following statements is correct?

- (A) The number of degrees of freedom for the test is  $8 - 1 = 7$ .
- (B) The chi-square test should not have been used because two of the counts in the table are less than 5.
- (C) The null hypothesis states that there is an association between category and opinion about the use of instant replay, and the small  $p$ -value suggests that the null hypothesis should be rejected.
- (D) The small  $p$ -value suggests that there is evidence of an association between category and opinion about the use of instant replay.
- (E) The chi-square test shows that fans favor the use of instant replay.

33. A large clinical trial was designed to determine whether a certain vitamin improves the general health of adults. The investigators first identified 85 variables that measure various aspects of the general health of adults. Because each adult in the clinical trial was to serve as his or her own control, the 85 variables were measured for each adult, both before taking the vitamin and after taking the vitamin for three months. The investigators then performed 85 matched-pair  $t$ -tests, one for each variable. They found statistically significant results at the 0.05 level in 2 of the variables, both in the direction of improved general health. Which of the following should the investigators conclude?
- (A) There is evidence that the vitamin improves the health of adults because 2 of the 85 tests were statistically significant.
  - (B) There is evidence that the vitamin improves the health of adults because in clinical trials investigators typically underestimate the proportion of the population that is helped by a vitamin.
  - (C) There is insufficient evidence that the vitamin improves the health of adults because at the 0.05 significance level, one could easily get statistically significant results in 2 out of 85 tests just due to chance variability.
  - (D) There is insufficient evidence that the vitamin improves the health of adults because the sample size of 85 is not large enough to draw a conclusion.
  - (E) No conclusion can be drawn because an even number of variables is needed for a matched-pairs design.

34. A sample of 942 homeowners are classified, in the two-way frequency table below, by the number of credit cards they have and the number of years they have owned their current homes.

| Number of Credit Cards | Number of Years Owning Current Home |                         |                       |       |
|------------------------|-------------------------------------|-------------------------|-----------------------|-------|
|                        | Less than One Year                  | One Year to Three Years | More than Three Years | Total |
| One                    | 265                                 | 53                      | 16                    | 334   |
| Two                    | 84                                  | 19                      | 4                     | 107   |
| Three                  | 201                                 | 68                      | 20                    | 289   |
| Four or more           | 134                                 | 58                      | 20                    | 212   |
| Total                  | 684                                 | 198                     | 60                    | 942   |

Of the homeowners in the sample who have four or more credit cards, what proportion have owned their current homes for at least one year?

- (A)  $\frac{78}{212}$
- (B)  $\frac{78}{258}$
- (C)  $\frac{78}{942}$
- (D)  $\frac{212}{942}$
- (E)  $\frac{258}{942}$

35. Administrators at a state university computed the mean GPA (grade point average) for juniors and seniors majoring in either physics or chemistry. The results are displayed in the table below. When juniors and seniors are grouped together, could physics majors have a higher mean GPA than chemistry majors?

|         | Physics | Chemistry |
|---------|---------|-----------|
| Juniors | 2.8     | 3.0       |
| Seniors | 3.2     | 3.6       |
| Overall | ?       | ?         |

- (A) No. The physics majors' mean GPA for juniors and seniors must be 3.0, while the chemistry majors' mean GPA for juniors and seniors must be 3.3.
- (B) No. There is not enough information to determine the mean GPA for each major, but it must be higher for chemistry majors than for physics majors.
- (C) Yes. It could happen. Whether it does happen depends on the number of juniors and seniors in each major.
- (D) Yes. It could happen. Whether it does happen depends on the variability of the GPAs within each of the four groups of students.
- (E) Yes. It could happen. Whether it does happen depends on the shapes of the distributions of the GPAs for each of the four groups of students.

36. A department store manager wants to know if a greater proportion of customers on the store's mailing list would redeem a coupon for \$5 off the price of an item than would redeem a coupon for 10 percent off the price of an item. The manager mails a \$5 off coupon to a random sample of 500 customers and mails a 10 percent off coupon to an independent random sample of 500 customers. The number of coupons of each type that were redeemed was recorded. Assuming that the conditions for inference are met, what test procedure should be used to answer the manager's question?
- (A) A one-sample  $t$ -test for a mean
  - (B) A one-sample  $z$ -test for a proportion
  - (C) A  $t$ -test for the slope of a regression line
  - (D) A matched-pairs  $t$ -test for a mean difference
  - (E) A two-sample  $z$ -test for a difference between two proportions
- 

37. A one-sided hypothesis test is to be performed with a significance level of 0.05. Suppose that the null hypothesis is false. If a significance level of 0.01 were to be used instead of a significance level of 0.05, which of the following would be true?
- (A) Neither the probability of a Type II error nor the power of the test would change.
  - (B) Both the probability of a Type II error and the power of the test would decrease.
  - (C) Both the probability of a Type II error and the power of the test would increase.
  - (D) The probability of a Type II error would decrease and the power of the test would increase.
  - (E) The probability of a Type II error would increase and the power of the test would decrease.

38. Which of the following distinguishes an observational study from a randomized experiment?

- (A) In an observational study volunteers are always used, whereas in a randomized experiment a random sample is always taken from the population.
- (B) In an observational study a random sample is always taken from the population, whereas in a randomized experiment volunteers are always used.
- (C) In an observational study treatments are not randomly assigned, whereas in a randomized experiment treatments are randomly assigned.
- (D) In an observational study a control group is never used, whereas in a randomized experiment a control group is always used.
- (E) An observational study can be double-blind, whereas a randomized experiment can only be single-blind because the experimenter determines who is randomly assigned to each treatment.

39. Frank, who lives in Texas, and his sister Lilly, who lives in Japan, correspond regularly. From what he can tell from the postmarks on both his and his sister's letters, it appears that it takes longer for Lilly's mail from Japan to reach him in Texas than it does for his letters from Texas to reach her in Japan. When Frank called his post office to ask if there was a reason for this, the postmaster told him that the delivery time of letters in both directions should be the same. Frank and his sister decided to collect data to see if letters from Japan to Texas take longer to be delivered than letters from Texas to Japan. They recorded the delivery time in days. After convincing themselves that the assumptions were reasonable, they performed a two-sample  $t$ -test and obtained the following computer output.

| Two sample T for To Texas vs To Japan                               |    |      |       |         |
|---------------------------------------------------------------------|----|------|-------|---------|
|                                                                     | N  | Mean | StDev | SE Mean |
| To Texas                                                            | 12 | 8.74 | 2.92  | 0.84    |
| To Japan                                                            | 9  | 6.75 | 2.56  | 0.85    |
| 95% CI for mu To Texas - mu To Japan: (-0.53, 4.51)                 |    |      |       |         |
| T-Test mu To Texas = mu To Japan (vs >): T = 1.66 P = 0.058 DF = 18 |    |      |       |         |

Using a significance level of 0.05, which of the following statements best describes the conclusion that can be drawn from these data?

- (A) There is convincing evidence that there is no difference in the mean delivery times.
- (B) There is convincing evidence that there is a difference in the mean delivery times.
- (C) There is convincing evidence that the mean delivery time from Japan to Texas is greater than the mean delivery time from Texas to Japan.
- (D) There is not convincing evidence that the mean delivery time from Japan to Texas is greater than the mean delivery time from Texas to Japan.
- (E) The  $t$ -test cannot be used for sample sizes that are this small.

40. In a national study on transportation patterns, 1,000 randomly selected adults will be asked the question: How many trips per week do you make to the grocery store? The sample mean will be computed. Let  $\mu$  denote the population mean response to the question if everyone in the population is to be asked the question. Is the sample mean  $\bar{x}$  unbiased for estimating  $\mu$ ?
- (A) Yes, because for random samples the mean (expected value) of the sample mean  $\bar{x}$  is equal to the population mean  $\mu$ .
- (B) Yes, because with a sample size of 1,000 the standard deviation of the sample mean  $\bar{x}$  is small.
- (C) Yes, because the wording of the question is not biased.
- (D) No, because the sample mean  $\bar{x}$  does not always equal the population mean  $\mu$ .
- (E) No, because number of trips to the grocery store is not normally distributed so the mean (expected value) of the sample mean  $\bar{x}$  does not equal the population mean  $\mu$ .

**END OF SECTION I**

**IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY  
CHECK YOUR WORK ON THIS SECTION.**

**DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.**

---

**MAKE SURE YOU HAVE DONE THE FOLLOWING.**

- **PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET**
- **WRITTEN AND GRIDDED YOUR AP NUMBER CORRECTLY ON YOUR ANSWER SHEET**
- **TAKEN THE AP EXAM LABEL FROM THE FRONT OF THIS BOOKLET AND PLACED IT ON YOUR ANSWER SHEET**

Table entry for  $z$  is the probability lying below  $z$ .

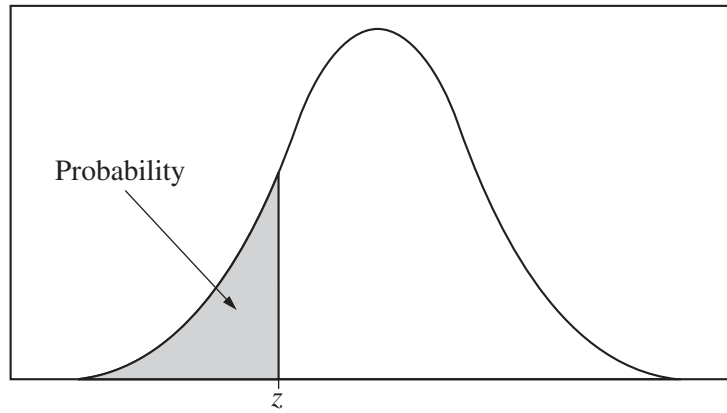


**Table A** Standard normal probabilities

| $z$  | .00   | .01   | .02   | .03   | .04   | .05   | .06   | .07   | .08   | .09   |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| -3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| -3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| -3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| -3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| -3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| -2.9 | .0019 | .0018 | .0018 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| -2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| -2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| -2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| -2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| -2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| -2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| -2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| -2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| -2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| -1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| -1.8 | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| -1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| -1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| -1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| -1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0721 | .0708 | .0694 | .0681 |
| -1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| -1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| -1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| -1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| -0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| -0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| -0.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| -0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| -0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| -0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| -0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| -0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| -0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| -0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |

Table entry for  $z$  is the probability lying below  $z$ .

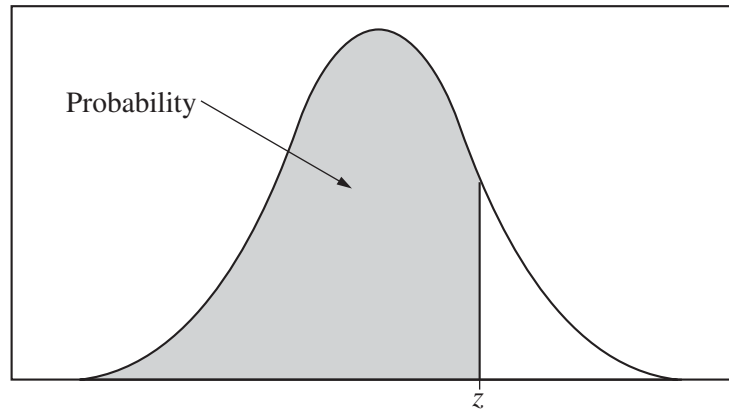


**Table A** (Continued)

| $z$ | .00   | .01   | .02   | .03   | .04   | .05   | .06   | .07   | .08   | .09   |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |

Table entry for  $p$  and  $C$  is the point  $t^*$  with probability  $p$  lying above it and probability  $C$  lying between  $-t^*$  and  $t^*$ .

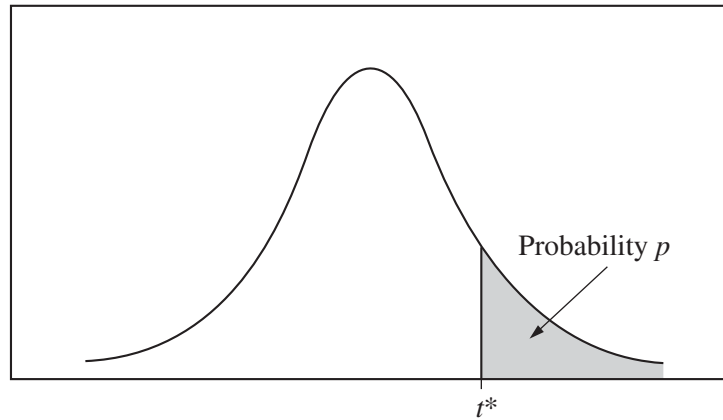


**Table B**  $t$  distribution critical values

| df                   | Tail probability $p$ |       |       |       |       |       |       |       |       |       |       |       |
|----------------------|----------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|                      | .25                  | .20   | .15   | .10   | .05   | .025  | .02   | .01   | .005  | .0025 | .001  | .0005 |
| 1                    | 1.000                | 1.376 | 1.963 | 3.078 | 6.314 | 12.71 | 15.89 | 31.82 | 63.66 | 127.3 | 318.3 | 636.6 |
| 2                    | .816                 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 | 4.849 | 6.965 | 9.925 | 14.09 | 22.33 | 31.60 |
| 3                    | .765                 | .978  | 1.250 | 1.638 | 2.353 | 3.182 | 3.482 | 4.541 | 5.841 | 7.453 | 10.21 | 12.92 |
| 4                    | .741                 | .941  | 1.190 | 1.533 | 2.132 | 2.776 | 2.999 | 3.747 | 4.604 | 5.598 | 7.173 | 8.610 |
| 5                    | .727                 | .920  | 1.156 | 1.476 | 2.015 | 2.571 | 2.757 | 3.365 | 4.032 | 4.773 | 5.893 | 6.869 |
| 6                    | .718                 | .906  | 1.134 | 1.440 | 1.943 | 2.447 | 2.612 | 3.143 | 3.707 | 4.317 | 5.208 | 5.959 |
| 7                    | .711                 | .896  | 1.119 | 1.415 | 1.895 | 2.365 | 2.517 | 2.998 | 3.499 | 4.029 | 4.785 | 5.408 |
| 8                    | .706                 | .889  | 1.108 | 1.397 | 1.860 | 2.306 | 2.449 | 2.896 | 3.355 | 3.833 | 4.501 | 5.041 |
| 9                    | .703                 | .883  | 1.100 | 1.383 | 1.833 | 2.262 | 2.398 | 2.821 | 3.250 | 3.690 | 4.297 | 4.781 |
| 10                   | .700                 | .879  | 1.093 | 1.372 | 1.812 | 2.228 | 2.359 | 2.764 | 3.169 | 3.581 | 4.144 | 4.587 |
| 11                   | .697                 | .876  | 1.088 | 1.363 | 1.796 | 2.201 | 2.328 | 2.718 | 3.106 | 3.497 | 4.025 | 4.437 |
| 12                   | .695                 | .873  | 1.083 | 1.356 | 1.782 | 2.179 | 2.303 | 2.681 | 3.055 | 3.428 | 3.930 | 4.318 |
| 13                   | .694                 | .870  | 1.079 | 1.350 | 1.771 | 2.160 | 2.282 | 2.650 | 3.012 | 3.372 | 3.852 | 4.221 |
| 14                   | .692                 | .868  | 1.076 | 1.345 | 1.761 | 2.145 | 2.264 | 2.624 | 2.977 | 3.326 | 3.787 | 4.140 |
| 15                   | .691                 | .866  | 1.074 | 1.341 | 1.753 | 2.131 | 2.249 | 2.602 | 2.947 | 3.286 | 3.733 | 4.073 |
| 16                   | .690                 | .865  | 1.071 | 1.337 | 1.746 | 2.120 | 2.235 | 2.583 | 2.921 | 3.252 | 3.686 | 4.015 |
| 17                   | .689                 | .863  | 1.069 | 1.333 | 1.740 | 2.110 | 2.224 | 2.567 | 2.898 | 3.222 | 3.646 | 3.965 |
| 18                   | .688                 | .862  | 1.067 | 1.330 | 1.734 | 2.101 | 2.214 | 2.552 | 2.878 | 3.197 | 3.611 | 3.922 |
| 19                   | .688                 | .861  | 1.066 | 1.328 | 1.729 | 2.093 | 2.205 | 2.539 | 2.861 | 3.174 | 3.579 | 3.883 |
| 20                   | .687                 | .860  | 1.064 | 1.325 | 1.725 | 2.086 | 2.197 | 2.528 | 2.845 | 3.153 | 3.552 | 3.850 |
| 21                   | .686                 | .859  | 1.063 | 1.323 | 1.721 | 2.080 | 2.189 | 2.518 | 2.831 | 3.135 | 3.527 | 3.819 |
| 22                   | .686                 | .858  | 1.061 | 1.321 | 1.717 | 2.074 | 2.183 | 2.508 | 2.819 | 3.119 | 3.505 | 3.792 |
| 23                   | .685                 | .858  | 1.060 | 1.319 | 1.714 | 2.069 | 2.177 | 2.500 | 2.807 | 3.104 | 3.485 | 3.768 |
| 24                   | .685                 | .857  | 1.059 | 1.318 | 1.711 | 2.064 | 2.172 | 2.492 | 2.797 | 3.091 | 3.467 | 3.745 |
| 25                   | .684                 | .856  | 1.058 | 1.316 | 1.708 | 2.060 | 2.167 | 2.485 | 2.787 | 3.078 | 3.450 | 3.725 |
| 26                   | .684                 | .856  | 1.058 | 1.315 | 1.706 | 2.056 | 2.162 | 2.479 | 2.779 | 3.067 | 3.435 | 3.707 |
| 27                   | .684                 | .855  | 1.057 | 1.314 | 1.703 | 2.052 | 2.158 | 2.473 | 2.771 | 3.057 | 3.421 | 3.690 |
| 28                   | .683                 | .855  | 1.056 | 1.313 | 1.701 | 2.048 | 2.154 | 2.467 | 2.763 | 3.047 | 3.408 | 3.674 |
| 29                   | .683                 | .854  | 1.055 | 1.311 | 1.699 | 2.045 | 2.150 | 2.462 | 2.756 | 3.038 | 3.396 | 3.659 |
| 30                   | .683                 | .854  | 1.055 | 1.310 | 1.697 | 2.042 | 2.147 | 2.457 | 2.750 | 3.030 | 3.385 | 3.646 |
| 40                   | .681                 | .851  | 1.050 | 1.303 | 1.684 | 2.021 | 2.123 | 2.423 | 2.704 | 2.971 | 3.307 | 3.551 |
| 50                   | .679                 | .849  | 1.047 | 1.299 | 1.676 | 2.009 | 2.109 | 2.403 | 2.678 | 2.937 | 3.261 | 3.496 |
| 60                   | .679                 | .848  | 1.045 | 1.296 | 1.671 | 2.000 | 2.099 | 2.390 | 2.660 | 2.915 | 3.232 | 3.460 |
| 80                   | .678                 | .846  | 1.043 | 1.292 | 1.664 | 1.990 | 2.088 | 2.374 | 2.639 | 2.887 | 3.195 | 3.416 |
| 100                  | .677                 | .845  | 1.042 | 1.290 | 1.660 | 1.984 | 2.081 | 2.364 | 2.626 | 2.871 | 3.174 | 3.390 |
| 1000                 | .675                 | .842  | 1.037 | 1.282 | 1.646 | 1.962 | 2.056 | 2.330 | 2.581 | 2.813 | 3.098 | 3.300 |
| $\infty$             | .674                 | .841  | 1.036 | 1.282 | 1.645 | 1.960 | 2.054 | 2.326 | 2.576 | 2.807 | 3.091 | 3.291 |
|                      | 50%                  | 60%   | 70%   | 80%   | 90%   | 95%   | 96%   | 98%   | 99%   | 99.5% | 99.8% | 99.9% |
| Confidence level $C$ |                      |       |       |       |       |       |       |       |       |       |       |       |

Table entry for  $p$  is the point ( $\chi^2$ ) with probability  $p$  lying above it.

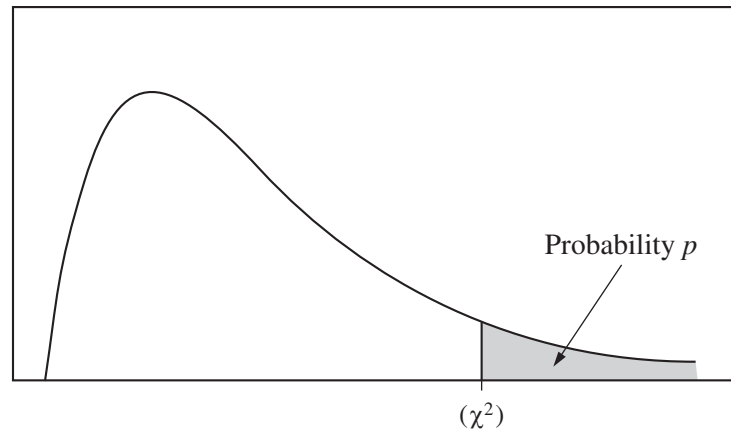


**Table C**  $\chi^2$  critical values

| df  | Tail probability $p$ |       |       |       |       |       |       |       |       |       |       |       |
|-----|----------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|     | .25                  | .20   | .15   | .10   | .05   | .025  | .02   | .01   | .005  | .0025 | .001  | .0005 |
| 1   | 1.32                 | 1.64  | 2.07  | 2.71  | 3.84  | 5.02  | 5.41  | 6.63  | 7.88  | 9.14  | 10.83 | 12.12 |
| 2   | 2.77                 | 3.22  | 3.79  | 4.61  | 5.99  | 7.38  | 7.82  | 9.21  | 10.60 | 11.98 | 13.82 | 15.20 |
| 3   | 4.11                 | 4.64  | 5.32  | 6.25  | 7.81  | 9.35  | 9.84  | 11.34 | 12.84 | 14.32 | 16.27 | 17.73 |
| 4   | 5.39                 | 5.99  | 6.74  | 7.78  | 9.49  | 11.14 | 11.67 | 13.28 | 14.86 | 16.42 | 18.47 | 20.00 |
| 5   | 6.63                 | 7.29  | 8.12  | 9.24  | 11.07 | 12.83 | 13.39 | 15.09 | 16.75 | 18.39 | 20.51 | 22.11 |
| 6   | 7.84                 | 8.56  | 9.45  | 10.64 | 12.59 | 14.45 | 15.03 | 16.81 | 18.55 | 20.25 | 22.46 | 24.10 |
| 7   | 9.04                 | 9.80  | 10.75 | 12.02 | 14.07 | 16.01 | 16.62 | 18.48 | 20.28 | 22.04 | 24.32 | 26.02 |
| 8   | 10.22                | 11.03 | 12.03 | 13.36 | 15.51 | 17.53 | 18.17 | 20.09 | 21.95 | 23.77 | 26.12 | 27.87 |
| 9   | 11.39                | 12.24 | 13.29 | 14.68 | 16.92 | 19.02 | 19.68 | 21.67 | 23.59 | 25.46 | 27.88 | 29.67 |
| 10  | 12.55                | 13.44 | 14.53 | 15.99 | 18.31 | 20.48 | 21.16 | 23.21 | 25.19 | 27.11 | 29.59 | 31.42 |
| 11  | 13.70                | 14.63 | 15.77 | 17.28 | 19.68 | 21.92 | 22.62 | 24.72 | 26.76 | 28.73 | 31.26 | 33.14 |
| 12  | 14.85                | 15.81 | 16.99 | 18.55 | 21.03 | 23.34 | 24.05 | 26.22 | 28.30 | 30.32 | 32.91 | 34.82 |
| 13  | 15.98                | 16.98 | 18.20 | 19.81 | 22.36 | 24.74 | 25.47 | 27.69 | 29.82 | 31.88 | 34.53 | 36.48 |
| 14  | 17.12                | 18.15 | 19.41 | 21.06 | 23.68 | 26.12 | 26.87 | 29.14 | 31.32 | 33.43 | 36.12 | 38.11 |
| 15  | 18.25                | 19.31 | 20.60 | 22.31 | 25.00 | 27.49 | 28.26 | 30.58 | 32.80 | 34.95 | 37.70 | 39.72 |
| 16  | 19.37                | 20.47 | 21.79 | 23.54 | 26.30 | 28.85 | 29.63 | 32.00 | 34.27 | 36.46 | 39.25 | 41.31 |
| 17  | 20.49                | 21.61 | 22.98 | 24.77 | 27.59 | 30.19 | 31.00 | 33.41 | 35.72 | 37.95 | 40.79 | 42.88 |
| 18  | 21.60                | 22.76 | 24.16 | 25.99 | 28.87 | 31.53 | 32.35 | 34.81 | 37.16 | 39.42 | 42.31 | 44.43 |
| 19  | 22.72                | 23.90 | 25.33 | 27.20 | 30.14 | 32.85 | 33.69 | 36.19 | 38.58 | 40.88 | 43.82 | 45.97 |
| 20  | 23.83                | 25.04 | 26.50 | 28.41 | 31.41 | 34.17 | 35.02 | 37.57 | 40.00 | 42.34 | 45.31 | 47.50 |
| 21  | 24.93                | 26.17 | 27.66 | 29.62 | 32.67 | 35.48 | 36.34 | 38.93 | 41.40 | 43.78 | 46.80 | 49.01 |
| 22  | 26.04                | 27.30 | 28.82 | 30.81 | 33.92 | 36.78 | 37.66 | 40.29 | 42.80 | 45.20 | 48.27 | 50.51 |
| 23  | 27.14                | 28.43 | 29.98 | 32.01 | 35.17 | 38.08 | 38.97 | 41.64 | 44.18 | 46.62 | 49.73 | 52.00 |
| 24  | 28.24                | 29.55 | 31.13 | 33.20 | 36.42 | 39.36 | 40.27 | 42.98 | 45.56 | 48.03 | 51.18 | 53.48 |
| 25  | 29.34                | 30.68 | 32.28 | 34.38 | 37.65 | 40.65 | 41.57 | 44.31 | 46.93 | 49.44 | 52.62 | 54.95 |
| 26  | 30.43                | 31.79 | 33.43 | 35.56 | 38.89 | 41.92 | 42.86 | 45.64 | 48.29 | 50.83 | 54.05 | 56.41 |
| 27  | 31.53                | 32.91 | 34.57 | 36.74 | 40.11 | 43.19 | 44.14 | 46.96 | 49.64 | 52.22 | 55.48 | 57.86 |
| 28  | 32.62                | 34.03 | 35.71 | 37.92 | 41.34 | 44.46 | 45.42 | 48.28 | 50.99 | 53.59 | 56.89 | 59.30 |
| 29  | 33.71                | 35.14 | 36.85 | 39.09 | 42.56 | 45.72 | 46.69 | 49.59 | 52.34 | 54.97 | 58.30 | 60.73 |
| 30  | 34.80                | 36.25 | 37.99 | 40.26 | 43.77 | 46.98 | 47.96 | 50.89 | 53.67 | 56.33 | 59.70 | 62.16 |
| 40  | 45.62                | 47.27 | 49.24 | 51.81 | 55.76 | 59.34 | 60.44 | 63.69 | 66.77 | 69.70 | 73.40 | 76.09 |
| 50  | 56.33                | 58.16 | 60.35 | 63.17 | 67.50 | 71.42 | 72.61 | 76.15 | 79.49 | 82.66 | 86.66 | 89.56 |
| 60  | 66.98                | 68.97 | 71.34 | 74.40 | 79.08 | 83.30 | 84.58 | 88.38 | 91.95 | 95.34 | 99.61 | 102.7 |
| 80  | 88.13                | 90.41 | 93.11 | 96.58 | 101.9 | 106.6 | 108.1 | 112.3 | 116.3 | 120.1 | 124.8 | 128.3 |
| 100 | 109.1                | 111.7 | 114.7 | 118.5 | 124.3 | 129.6 | 131.1 | 135.8 | 140.2 | 144.3 | 149.4 | 153.2 |

# AP<sup>®</sup> Statistics Exam

## SECTION II: Free Response

2014

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

### At a Glance

**Total Time**

1 hour, 30 minutes

**Number of Questions**

6

**Percent of Total Score**

50%

**Writing Instrument**

Either pencil or pen with black or dark blue ink

**Electronic Device**

Graphing calculator expected

### Part A

**Number of Questions**

5

**Suggested Time**

1 hour, 5 minutes

**Percent of Section II Score**

75%

### Part B

**Number of Questions**

1

**Suggested Time**

25 minutes

**Percent of Section II Score**

25%

### IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name

First letter of your first name

2. Date of birth

Month Day Year

3. Six-digit school code

4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.

No, I do not grant the College Board these rights. ☐

### Instructions

The questions for both Part A and Part B are printed in this booklet. You may use any blank space in the booklet to organize your answers and for scratch work, but you must write your answers in the spaces provided for each answer. Pages containing statistical tables and useful formulas are printed in this booklet.

You may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions. Show all your work. Indicate clearly the methods you use because you will be scored on the correctness of your methods as well as the accuracy and completeness of your results and explanations. Correct answers without supporting work may not receive credit. Write your solution to each part of each question in the space provided for that part. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. The proctor will announce the suggested time for Part A and Part B, but you may proceed freely from one question to the next. You may review your responses if you finish before the end of the exam is announced.

Form I

Form Code 4JBP6-S

90

Formulas begin on page 3.  
Questions begin on page 6.  
Tables begin on page 18.

## Formulas

### (I) Descriptive Statistics

$$\bar{x} = \frac{\sum x_i}{n}$$

$$s_x = \sqrt{\frac{1}{n-1} \sum (x_i - \bar{x})^2}$$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 - 1) + (n_2 - 1)}}$$

$$\hat{y} = b_0 + b_1 x$$

$$b_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$$b_0 = \bar{y} - b_1 \bar{x}$$

$$r = \frac{1}{n-1} \sum \left( \frac{x_i - \bar{x}}{s_x} \right) \left( \frac{y_i - \bar{y}}{s_y} \right)$$

$$b_1 = r \frac{s_y}{s_x}$$

$$s_{b_1} = \frac{\sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n-2}}}{\sqrt{\sum (x_i - \bar{x})^2}}$$

(II) Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$E(X) = \mu_x = \sum x_i p_i$$

$$\text{Var}(X) = \sigma_x^2 = \sum (x_i - \mu_x)^2 p_i$$

If  $X$  has a binomial distribution with parameters  $n$  and  $p$ , then:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

$$\mu_x = np$$

$$\sigma_x = \sqrt{np(1 - p)}$$

$$\mu_{\hat{p}} = p$$

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1 - p)}{n}}$$

If  $\bar{x}$  is the mean of a random sample of size  $n$  from an infinite population with mean  $\mu$  and standard deviation  $\sigma$ , then:

$$\mu_{\bar{x}} = \mu$$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

(III) Inferential Statistics

Standardized test statistic:  $\frac{\text{statistic} - \text{parameter}}{\text{standard deviation of statistic}}$

Confidence interval: statistic  $\pm$  (critical value) • (standard deviation of statistic)

Single-Sample

| Statistic         | Standard Deviation of Statistic |
|-------------------|---------------------------------|
| Sample Mean       | $\frac{\sigma}{\sqrt{n}}$       |
| Sample Proportion | $\sqrt{\frac{p(1-p)}{n}}$       |

Two-Sample

| Statistic                        | Standard Deviation of Statistic                                                                                                                          |
|----------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------|
| Difference of sample means       | $\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$<br><br>Special case when $\sigma_1 = \sigma_2$<br>$\sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ |
| Difference of sample proportions | $\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$<br><br>Special case when $p_1 = p_2$<br>$\sqrt{p(1-p)} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$    |

$$\text{Chi-square test statistic} = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

## STATISTICS

### SECTION II

#### Part A

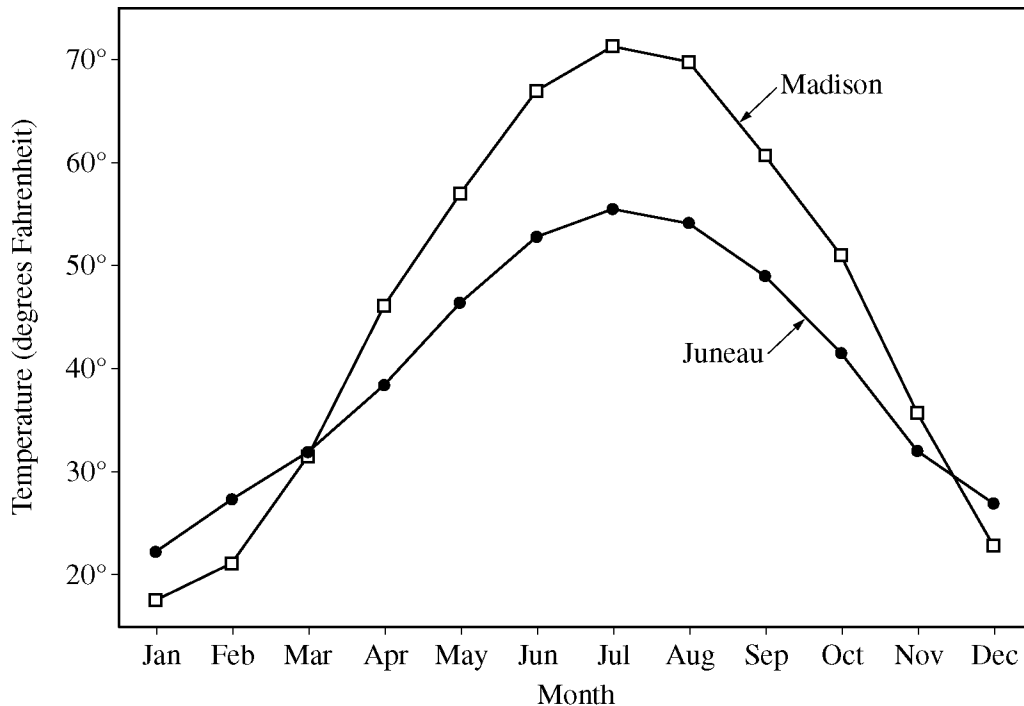
#### Questions 1-5

Spend about 65 minutes on this part of the exam.

Percent of Section II score—75

**Directions:** Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

1. The graph below shows the monthly average temperatures, in degrees Fahrenheit, for two cities—Madison, Wisconsin, and Juneau, Alaska—in the United States.



- (a) Based on the graph, compare the two cities with respect to the monthly average temperatures over the year. Address both similarities and differences in the overall patterns.

Similarities:

Differences:

- (b) For which of the two cities is the standard deviation of the 12 monthly average temperatures greater? Justify your answer without performing any calculations.

2. Researchers investigated the possible beneficial effect on heart health of drinking black tea and whether adding milk to the tea reduces any possible benefit. Twenty-four volunteers were randomly assigned to one of three groups. Every day for a month, participants in group 1 drank two cups of hot black tea without milk, participants in group 2 drank two cups of hot black tea with milk, and participants in group 3 drank two cups of hot water but no tea. At the end of the month, the researchers measured the change in each of the participants' heart health.

(a) Did the researchers conduct an experiment or an observational study? Explain.

(b) Why did the researchers include a group who drank hot water but no tea?

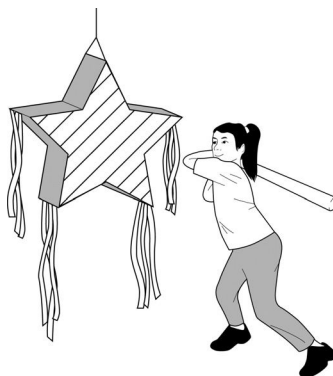
(c) Is it reasonable to generalize the results of the study beyond the 24 participants? Explain why or why not.

3. Patients experiencing symptoms of a heart attack are routinely transported to a hospital in an ambulance. In a study of a new treatment thought to reduce damage to the heart, patients experiencing symptoms of a heart attack were randomly assigned to one of two groups. During transportation to the hospital, patients in one group received standard care, and patients in the other group received the new treatment consisting of standard care and the application of a blood pressure cuff.

The response variable measured for each patient was a number between 0 and 1, referred to as the myocardial salvage index (MSI). A higher MSI value indicates a more positive outcome for the patient. Summary statistics for the MSI responses of the two groups are shown in the table below.

|               | Mean | Standard Deviation | Sample Size |
|---------------|------|--------------------|-------------|
| Standard care | 0.57 | 0.26               | 56          |
| New treatment | 0.69 | 0.27               | 56          |

Do the data provide convincing statistical evidence that the new treatment results in a higher mean MSI value than does the standard care among people similar to the patients in the study?



4. A piñata is a container filled with toys and candy and is broken open by hitting it with a stick. The drawing above shows Sophia trying to break a piñata. The probability that she will break the piñata on the first hit is 0.7. She will continue to hit the piñata until it breaks. If she does not break the piñata on a particular hit, the piñata is weakened and the probability that she will break it on the next hit is 0.1 greater than the probability on the previous hit. For example, if the piñata does not break on the first hit, the probability that it will break on the second hit is 0.8.
- (a) Calculate the probability that Sophia does not break the piñata on the first hit and does break the piñata on the second hit.

Let the random variable  $X$  represent the number of hits required for Sophia to break the piñata.

- (b) Complete the probability distribution of  $X$  in the table below.

| $x$                | 1   | 2 | 3 | 4 |
|--------------------|-----|---|---|---|
| Probability of $x$ | 0.7 |   |   |   |

(c) Calculate and interpret the expected value of  $X$ .

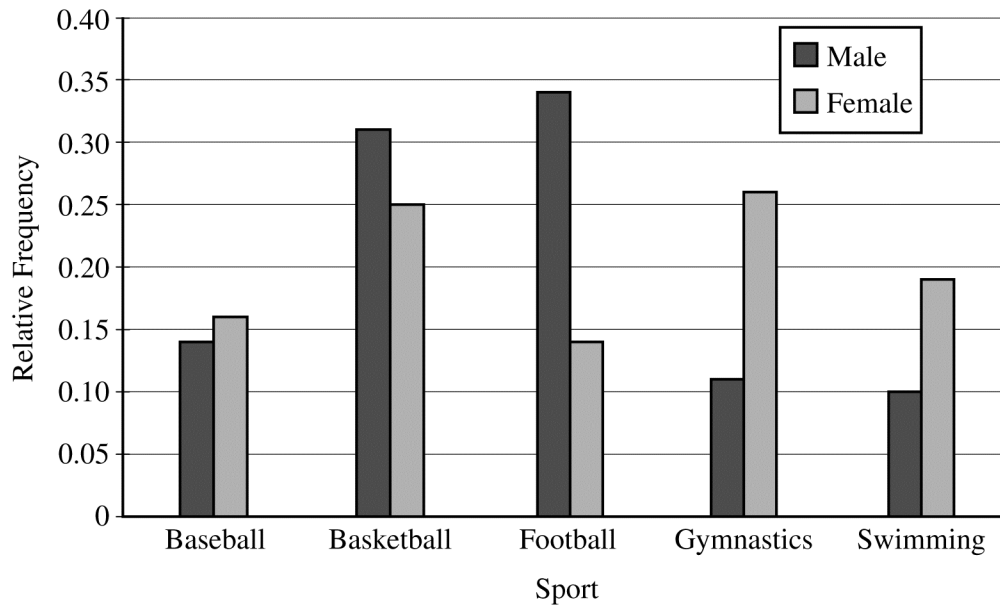
Calculation:

Interpretation:

5. The newspaper staff at a large high school randomly sampled students at the school to investigate the relationship between choice of favorite sport to watch and gender. Students in the sample were asked the following question.

Of the following five sports, which one is your favorite to watch:  
baseball, basketball, football, gymnastics, or swimming?

The number of responses by choice of favorite sport to watch and gender were tabulated. The graph below displays the relative frequencies of each sport by gender.



- (a) Write a few sentences summarizing what the graph reveals about the association between choice of favorite sport to watch and gender for the students in the sample.

- (b) What statistical test should be used to investigate whether there is an association between choice of favorite sport to watch and gender for the population of students at the high school? What is the appropriate number of degrees of freedom the test should be based on?

Test:

Degrees of freedom:

- (c) The conditions for inference were determined to be satisfied, and the appropriate statistical test was conducted. The value of the test statistic is 10.77. Is there convincing statistical evidence, at the  $\alpha = 0.05$  significance level, of an association between choice of favorite sport to watch and gender for all students at the high school?

## STATISTICS

### SECTION II

#### Part B

#### Question 6

**Spend about 25 minutes on this part of the exam.**

**Percent of Section II score—25**

**Directions:** Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

6. The United States one-cent coin is called a penny. Banks often use a machine to count and package pennies in rolls of 50 pennies. Occasionally a machine malfunctions and begins putting fewer than 50 pennies in each roll. The quality control inspector at a certain bank wants to investigate whether its machine is functioning properly but does not want to break open the rolls to count pennies. The inspector will measure the weight of a roll of pennies to test the following hypotheses.

$H_0$  : The machine is putting 50 pennies in each roll.

$H_a$  : The machine is putting fewer than 50 pennies in each roll.

- (a) For testing the hypotheses above, describe what a Type II error would be and describe a consequence of making a Type II error.

Description of error:

Consequence:

For pennies produced since 1983, the distribution of weight is approximately normal with a mean of 2.5 grams and a standard deviation of 0.04 gram. Assume that the pennies in a roll can be considered a random sample of pennies produced since 1983.

Let the random variable  $W$  represent the total weight of a roll of pennies. Ignore the weight of the packaging of the roll.

- (b) Describe the distribution of  $W$  if the selected roll contains 50 pennies.

The inspector will use the following rule for each selected roll.

If the total weight of the roll of pennies is 124 grams or less, I will conclude that the roll contains fewer than 50 pennies. Otherwise, I will conclude that the roll contains 50 pennies.

- (c) Suppose the selected roll contains 50 pennies. Using the inspector's rule, what is the probability that the inspector will conclude that the selected roll contains fewer than 50 pennies?
- (d) Suppose the selected roll actually contains 49 pennies. Using the inspector's rule, is the inspector likely to conclude that the selected roll contains fewer than 50 pennies? Justify your answer.
- (e) Based on your answers to (c) and (d), comment on whether the inspector's rule will be effective at distinguishing a roll that contains 50 pennies from a roll that contains fewer than 50 pennies.

**STOP**

**END OF EXAM**

---

**THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.**

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT AND BACK COVERS OF THE SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX(ES) ON THE COVER(S).**
- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.**

Table entry for  $z$  is the probability lying below  $z$ .

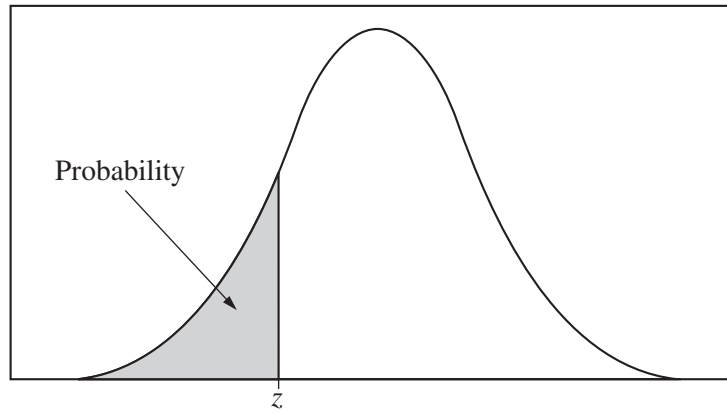


**Table A** Standard normal probabilities

| $z$  | .00   | .01   | .02   | .03   | .04   | .05   | .06   | .07   | .08   | .09   |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| -3.4 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0003 | .0002 |
| -3.3 | .0005 | .0005 | .0005 | .0004 | .0004 | .0004 | .0004 | .0004 | .0004 | .0003 |
| -3.2 | .0007 | .0007 | .0006 | .0006 | .0006 | .0006 | .0006 | .0005 | .0005 | .0005 |
| -3.1 | .0010 | .0009 | .0009 | .0009 | .0008 | .0008 | .0008 | .0008 | .0007 | .0007 |
| -3.0 | .0013 | .0013 | .0013 | .0012 | .0012 | .0011 | .0011 | .0011 | .0010 | .0010 |
| -2.9 | .0019 | .0018 | .0018 | .0017 | .0016 | .0016 | .0015 | .0015 | .0014 | .0014 |
| -2.8 | .0026 | .0025 | .0024 | .0023 | .0023 | .0022 | .0021 | .0021 | .0020 | .0019 |
| -2.7 | .0035 | .0034 | .0033 | .0032 | .0031 | .0030 | .0029 | .0028 | .0027 | .0026 |
| -2.6 | .0047 | .0045 | .0044 | .0043 | .0041 | .0040 | .0039 | .0038 | .0037 | .0036 |
| -2.5 | .0062 | .0060 | .0059 | .0057 | .0055 | .0054 | .0052 | .0051 | .0049 | .0048 |
| -2.4 | .0082 | .0080 | .0078 | .0075 | .0073 | .0071 | .0069 | .0068 | .0066 | .0064 |
| -2.3 | .0107 | .0104 | .0102 | .0099 | .0096 | .0094 | .0091 | .0089 | .0087 | .0084 |
| -2.2 | .0139 | .0136 | .0132 | .0129 | .0125 | .0122 | .0119 | .0116 | .0113 | .0110 |
| -2.1 | .0179 | .0174 | .0170 | .0166 | .0162 | .0158 | .0154 | .0150 | .0146 | .0143 |
| -2.0 | .0228 | .0222 | .0217 | .0212 | .0207 | .0202 | .0197 | .0192 | .0188 | .0183 |
| -1.9 | .0287 | .0281 | .0274 | .0268 | .0262 | .0256 | .0250 | .0244 | .0239 | .0233 |
| -1.8 | .0359 | .0351 | .0344 | .0336 | .0329 | .0322 | .0314 | .0307 | .0301 | .0294 |
| -1.7 | .0446 | .0436 | .0427 | .0418 | .0409 | .0401 | .0392 | .0384 | .0375 | .0367 |
| -1.6 | .0548 | .0537 | .0526 | .0516 | .0505 | .0495 | .0485 | .0475 | .0465 | .0455 |
| -1.5 | .0668 | .0655 | .0643 | .0630 | .0618 | .0606 | .0594 | .0582 | .0571 | .0559 |
| -1.4 | .0808 | .0793 | .0778 | .0764 | .0749 | .0735 | .0721 | .0708 | .0694 | .0681 |
| -1.3 | .0968 | .0951 | .0934 | .0918 | .0901 | .0885 | .0869 | .0853 | .0838 | .0823 |
| -1.2 | .1151 | .1131 | .1112 | .1093 | .1075 | .1056 | .1038 | .1020 | .1003 | .0985 |
| -1.1 | .1357 | .1335 | .1314 | .1292 | .1271 | .1251 | .1230 | .1210 | .1190 | .1170 |
| -1.0 | .1587 | .1562 | .1539 | .1515 | .1492 | .1469 | .1446 | .1423 | .1401 | .1379 |
| -0.9 | .1841 | .1814 | .1788 | .1762 | .1736 | .1711 | .1685 | .1660 | .1635 | .1611 |
| -0.8 | .2119 | .2090 | .2061 | .2033 | .2005 | .1977 | .1949 | .1922 | .1894 | .1867 |
| -0.7 | .2420 | .2389 | .2358 | .2327 | .2296 | .2266 | .2236 | .2206 | .2177 | .2148 |
| -0.6 | .2743 | .2709 | .2676 | .2643 | .2611 | .2578 | .2546 | .2514 | .2483 | .2451 |
| -0.5 | .3085 | .3050 | .3015 | .2981 | .2946 | .2912 | .2877 | .2843 | .2810 | .2776 |
| -0.4 | .3446 | .3409 | .3372 | .3336 | .3300 | .3264 | .3228 | .3192 | .3156 | .3121 |
| -0.3 | .3821 | .3783 | .3745 | .3707 | .3669 | .3632 | .3594 | .3557 | .3520 | .3483 |
| -0.2 | .4207 | .4168 | .4129 | .4090 | .4052 | .4013 | .3974 | .3936 | .3897 | .3859 |
| -0.1 | .4602 | .4562 | .4522 | .4483 | .4443 | .4404 | .4364 | .4325 | .4286 | .4247 |
| -0.0 | .5000 | .4960 | .4920 | .4880 | .4840 | .4801 | .4761 | .4721 | .4681 | .4641 |

Table entry for  $z$  is the probability lying below  $z$ .

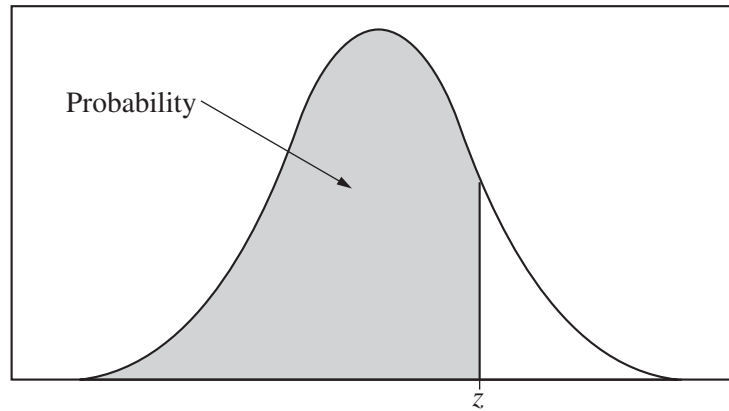


**Table A** (Continued)

| $z$ | .00   | .01   | .02   | .03   | .04   | .05   | .06   | .07   | .08   | .09   |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |

Table entry for  $p$  and  $C$  is the point  $t^*$  with probability  $p$  lying above it and probability  $C$  lying between  $-t^*$  and  $t^*$ .

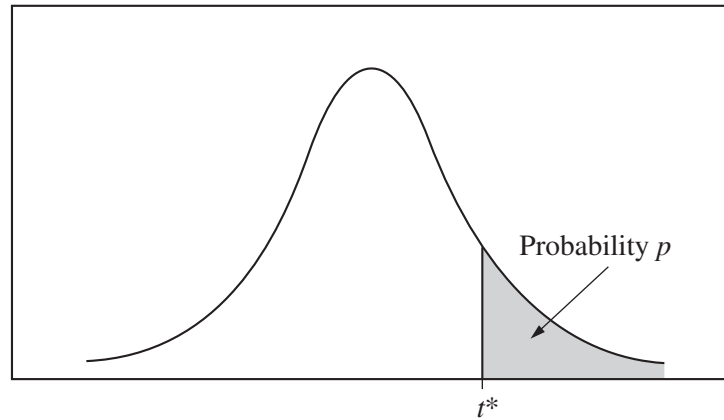


**Table B**  $t$  distribution critical values

| df                   | Tail probability $p$ |       |       |       |       |       |       |       |       |       |       |       |
|----------------------|----------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|                      | .25                  | .20   | .15   | .10   | .05   | .025  | .02   | .01   | .005  | .0025 | .001  | .0005 |
| 1                    | 1.000                | 1.376 | 1.963 | 3.078 | 6.314 | 12.71 | 15.89 | 31.82 | 63.66 | 127.3 | 318.3 | 636.6 |
| 2                    | .816                 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303 | 4.849 | 6.965 | 9.925 | 14.09 | 22.33 | 31.60 |
| 3                    | .765                 | .978  | 1.250 | 1.638 | 2.353 | 3.182 | 3.482 | 4.541 | 5.841 | 7.453 | 10.21 | 12.92 |
| 4                    | .741                 | .941  | 1.190 | 1.533 | 2.132 | 2.776 | 2.999 | 3.747 | 4.604 | 5.598 | 7.173 | 8.610 |
| 5                    | .727                 | .920  | 1.156 | 1.476 | 2.015 | 2.571 | 2.757 | 3.365 | 4.032 | 4.773 | 5.893 | 6.869 |
| 6                    | .718                 | .906  | 1.134 | 1.440 | 1.943 | 2.447 | 2.612 | 3.143 | 3.707 | 4.317 | 5.208 | 5.959 |
| 7                    | .711                 | .896  | 1.119 | 1.415 | 1.895 | 2.365 | 2.517 | 2.998 | 3.499 | 4.029 | 4.785 | 5.408 |
| 8                    | .706                 | .889  | 1.108 | 1.397 | 1.860 | 2.306 | 2.449 | 2.896 | 3.355 | 3.833 | 4.501 | 5.041 |
| 9                    | .703                 | .883  | 1.100 | 1.383 | 1.833 | 2.262 | 2.398 | 2.821 | 3.250 | 3.690 | 4.297 | 4.781 |
| 10                   | .700                 | .879  | 1.093 | 1.372 | 1.812 | 2.228 | 2.359 | 2.764 | 3.169 | 3.581 | 4.144 | 4.587 |
| 11                   | .697                 | .876  | 1.088 | 1.363 | 1.796 | 2.201 | 2.328 | 2.718 | 3.106 | 3.497 | 4.025 | 4.437 |
| 12                   | .695                 | .873  | 1.083 | 1.356 | 1.782 | 2.179 | 2.303 | 2.681 | 3.055 | 3.428 | 3.930 | 4.318 |
| 13                   | .694                 | .870  | 1.079 | 1.350 | 1.771 | 2.160 | 2.282 | 2.650 | 3.012 | 3.372 | 3.852 | 4.221 |
| 14                   | .692                 | .868  | 1.076 | 1.345 | 1.761 | 2.145 | 2.264 | 2.624 | 2.977 | 3.326 | 3.787 | 4.140 |
| 15                   | .691                 | .866  | 1.074 | 1.341 | 1.753 | 2.131 | 2.249 | 2.602 | 2.947 | 3.286 | 3.733 | 4.073 |
| 16                   | .690                 | .865  | 1.071 | 1.337 | 1.746 | 2.120 | 2.235 | 2.583 | 2.921 | 3.252 | 3.686 | 4.015 |
| 17                   | .689                 | .863  | 1.069 | 1.333 | 1.740 | 2.110 | 2.224 | 2.567 | 2.898 | 3.222 | 3.646 | 3.965 |
| 18                   | .688                 | .862  | 1.067 | 1.330 | 1.734 | 2.101 | 2.214 | 2.552 | 2.878 | 3.197 | 3.611 | 3.922 |
| 19                   | .688                 | .861  | 1.066 | 1.328 | 1.729 | 2.093 | 2.205 | 2.539 | 2.861 | 3.174 | 3.579 | 3.883 |
| 20                   | .687                 | .860  | 1.064 | 1.325 | 1.725 | 2.086 | 2.197 | 2.528 | 2.845 | 3.153 | 3.552 | 3.850 |
| 21                   | .686                 | .859  | 1.063 | 1.323 | 1.721 | 2.080 | 2.189 | 2.518 | 2.831 | 3.135 | 3.527 | 3.819 |
| 22                   | .686                 | .858  | 1.061 | 1.321 | 1.717 | 2.074 | 2.183 | 2.508 | 2.819 | 3.119 | 3.505 | 3.792 |
| 23                   | .685                 | .858  | 1.060 | 1.319 | 1.714 | 2.069 | 2.177 | 2.500 | 2.807 | 3.104 | 3.485 | 3.768 |
| 24                   | .685                 | .857  | 1.059 | 1.318 | 1.711 | 2.064 | 2.172 | 2.492 | 2.797 | 3.091 | 3.467 | 3.745 |
| 25                   | .684                 | .856  | 1.058 | 1.316 | 1.708 | 2.060 | 2.167 | 2.485 | 2.787 | 3.078 | 3.450 | 3.725 |
| 26                   | .684                 | .856  | 1.058 | 1.315 | 1.706 | 2.056 | 2.162 | 2.479 | 2.779 | 3.067 | 3.435 | 3.707 |
| 27                   | .684                 | .855  | 1.057 | 1.314 | 1.703 | 2.052 | 2.158 | 2.473 | 2.771 | 3.057 | 3.421 | 3.690 |
| 28                   | .683                 | .855  | 1.056 | 1.313 | 1.701 | 2.048 | 2.154 | 2.467 | 2.763 | 3.047 | 3.408 | 3.674 |
| 29                   | .683                 | .854  | 1.055 | 1.311 | 1.699 | 2.045 | 2.150 | 2.462 | 2.756 | 3.038 | 3.396 | 3.659 |
| 30                   | .683                 | .854  | 1.055 | 1.310 | 1.697 | 2.042 | 2.147 | 2.457 | 2.750 | 3.030 | 3.385 | 3.646 |
| 40                   | .681                 | .851  | 1.050 | 1.303 | 1.684 | 2.021 | 2.123 | 2.423 | 2.704 | 2.971 | 3.307 | 3.551 |
| 50                   | .679                 | .849  | 1.047 | 1.299 | 1.676 | 2.009 | 2.109 | 2.403 | 2.678 | 2.937 | 3.261 | 3.496 |
| 60                   | .679                 | .848  | 1.045 | 1.296 | 1.671 | 2.000 | 2.099 | 2.390 | 2.660 | 2.915 | 3.232 | 3.460 |
| 80                   | .678                 | .846  | 1.043 | 1.292 | 1.664 | 1.990 | 2.088 | 2.374 | 2.639 | 2.887 | 3.195 | 3.416 |
| 100                  | .677                 | .845  | 1.042 | 1.290 | 1.660 | 1.984 | 2.081 | 2.364 | 2.626 | 2.871 | 3.174 | 3.390 |
| 1000                 | .675                 | .842  | 1.037 | 1.282 | 1.646 | 1.962 | 2.056 | 2.330 | 2.581 | 2.813 | 3.098 | 3.300 |
| $\infty$             | .674                 | .841  | 1.036 | 1.282 | 1.645 | 1.960 | 2.054 | 2.326 | 2.576 | 2.807 | 3.091 | 3.291 |
|                      | 50%                  | 60%   | 70%   | 80%   | 90%   | 95%   | 96%   | 98%   | 99%   | 99.5% | 99.8% | 99.9% |
| Confidence level $C$ |                      |       |       |       |       |       |       |       |       |       |       |       |

Table entry for  $p$  is the point ( $\chi^2$ ) with probability  $p$  lying above it.

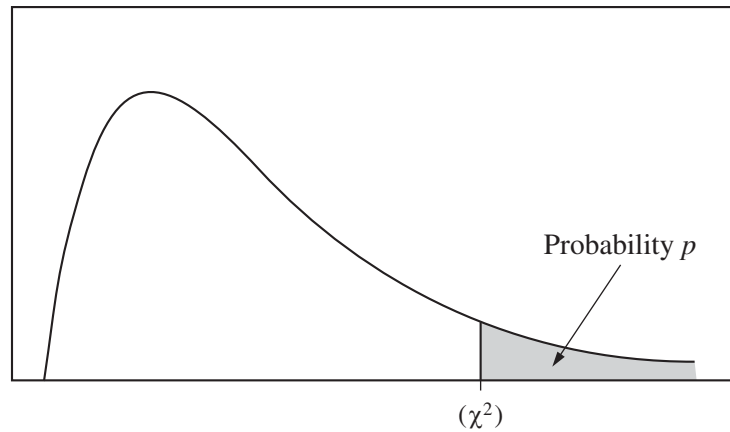


**Table C**  $\chi^2$  critical values

| df  | Tail probability $p$ |       |       |       |       |       |       |       |       |       |       |       |
|-----|----------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|     | .25                  | .20   | .15   | .10   | .05   | .025  | .02   | .01   | .005  | .0025 | .001  | .0005 |
| 1   | 1.32                 | 1.64  | 2.07  | 2.71  | 3.84  | 5.02  | 5.41  | 6.63  | 7.88  | 9.14  | 10.83 | 12.12 |
| 2   | 2.77                 | 3.22  | 3.79  | 4.61  | 5.99  | 7.38  | 7.82  | 9.21  | 10.60 | 11.98 | 13.82 | 15.20 |
| 3   | 4.11                 | 4.64  | 5.32  | 6.25  | 7.81  | 9.35  | 9.84  | 11.34 | 12.84 | 14.32 | 16.27 | 17.73 |
| 4   | 5.39                 | 5.99  | 6.74  | 7.78  | 9.49  | 11.14 | 11.67 | 13.28 | 14.86 | 16.42 | 18.47 | 20.00 |
| 5   | 6.63                 | 7.29  | 8.12  | 9.24  | 11.07 | 12.83 | 13.39 | 15.09 | 16.75 | 18.39 | 20.51 | 22.11 |
| 6   | 7.84                 | 8.56  | 9.45  | 10.64 | 12.59 | 14.45 | 15.03 | 16.81 | 18.55 | 20.25 | 22.46 | 24.10 |
| 7   | 9.04                 | 9.80  | 10.75 | 12.02 | 14.07 | 16.01 | 16.62 | 18.48 | 20.28 | 22.04 | 24.32 | 26.02 |
| 8   | 10.22                | 11.03 | 12.03 | 13.36 | 15.51 | 17.53 | 18.17 | 20.09 | 21.95 | 23.77 | 26.12 | 27.87 |
| 9   | 11.39                | 12.24 | 13.29 | 14.68 | 16.92 | 19.02 | 19.68 | 21.67 | 23.59 | 25.46 | 27.88 | 29.67 |
| 10  | 12.55                | 13.44 | 14.53 | 15.99 | 18.31 | 20.48 | 21.16 | 23.21 | 25.19 | 27.11 | 29.59 | 31.42 |
| 11  | 13.70                | 14.63 | 15.77 | 17.28 | 19.68 | 21.92 | 22.62 | 24.72 | 26.76 | 28.73 | 31.26 | 33.14 |
| 12  | 14.85                | 15.81 | 16.99 | 18.55 | 21.03 | 23.34 | 24.05 | 26.22 | 28.30 | 30.32 | 32.91 | 34.82 |
| 13  | 15.98                | 16.98 | 18.20 | 19.81 | 22.36 | 24.74 | 25.47 | 27.69 | 29.82 | 31.88 | 34.53 | 36.48 |
| 14  | 17.12                | 18.15 | 19.41 | 21.06 | 23.68 | 26.12 | 26.87 | 29.14 | 31.32 | 33.43 | 36.12 | 38.11 |
| 15  | 18.25                | 19.31 | 20.60 | 22.31 | 25.00 | 27.49 | 28.26 | 30.58 | 32.80 | 34.95 | 37.70 | 39.72 |
| 16  | 19.37                | 20.47 | 21.79 | 23.54 | 26.30 | 28.85 | 29.63 | 32.00 | 34.27 | 36.46 | 39.25 | 41.31 |
| 17  | 20.49                | 21.61 | 22.98 | 24.77 | 27.59 | 30.19 | 31.00 | 33.41 | 35.72 | 37.95 | 40.79 | 42.88 |
| 18  | 21.60                | 22.76 | 24.16 | 25.99 | 28.87 | 31.53 | 32.35 | 34.81 | 37.16 | 39.42 | 42.31 | 44.43 |
| 19  | 22.72                | 23.90 | 25.33 | 27.20 | 30.14 | 32.85 | 33.69 | 36.19 | 38.58 | 40.88 | 43.82 | 45.97 |
| 20  | 23.83                | 25.04 | 26.50 | 28.41 | 31.41 | 34.17 | 35.02 | 37.57 | 40.00 | 42.34 | 45.31 | 47.50 |
| 21  | 24.93                | 26.17 | 27.66 | 29.62 | 32.67 | 35.48 | 36.34 | 38.93 | 41.40 | 43.78 | 46.80 | 49.01 |
| 22  | 26.04                | 27.30 | 28.82 | 30.81 | 33.92 | 36.78 | 37.66 | 40.29 | 42.80 | 45.20 | 48.27 | 50.51 |
| 23  | 27.14                | 28.43 | 29.98 | 32.01 | 35.17 | 38.08 | 38.97 | 41.64 | 44.18 | 46.62 | 49.73 | 52.00 |
| 24  | 28.24                | 29.55 | 31.13 | 33.20 | 36.42 | 39.36 | 40.27 | 42.98 | 45.56 | 48.03 | 51.18 | 53.48 |
| 25  | 29.34                | 30.68 | 32.28 | 34.38 | 37.65 | 40.65 | 41.57 | 44.31 | 46.93 | 49.44 | 52.62 | 54.95 |
| 26  | 30.43                | 31.79 | 33.43 | 35.56 | 38.89 | 41.92 | 42.86 | 45.64 | 48.29 | 50.83 | 54.05 | 56.41 |
| 27  | 31.53                | 32.91 | 34.57 | 36.74 | 40.11 | 43.19 | 44.14 | 46.96 | 49.64 | 52.22 | 55.48 | 57.86 |
| 28  | 32.62                | 34.03 | 35.71 | 37.92 | 41.34 | 44.46 | 45.42 | 48.28 | 50.99 | 53.59 | 56.89 | 59.30 |
| 29  | 33.71                | 35.14 | 36.85 | 39.09 | 42.56 | 45.72 | 46.69 | 49.59 | 52.34 | 54.97 | 58.30 | 60.73 |
| 30  | 34.80                | 36.25 | 37.99 | 40.26 | 43.77 | 46.98 | 47.96 | 50.89 | 53.67 | 56.33 | 59.70 | 62.16 |
| 40  | 45.62                | 47.27 | 49.24 | 51.81 | 55.76 | 59.34 | 60.44 | 63.69 | 66.77 | 69.70 | 73.40 | 76.09 |
| 50  | 56.33                | 58.16 | 60.35 | 63.17 | 67.50 | 71.42 | 72.61 | 76.15 | 79.49 | 82.66 | 86.66 | 89.56 |
| 60  | 66.98                | 68.97 | 71.34 | 74.40 | 79.08 | 83.30 | 84.58 | 88.38 | 91.95 | 95.34 | 99.61 | 102.7 |
| 80  | 88.13                | 90.41 | 93.11 | 96.58 | 101.9 | 106.6 | 108.1 | 112.3 | 116.3 | 120.1 | 124.8 | 128.3 |
| 100 | 109.1                | 111.7 | 114.7 | 118.5 | 124.3 | 129.6 | 131.1 | 135.8 | 140.2 | 144.3 | 149.4 | 153.2 |

# **AP Statistics**

**2014**

## **APPENDIX - ANSWERS**

Matching multiple-choice answers and free-response scoring follow.

**Answer Key for AP Statistics  
Practice Exam, Section I**

|                |                |
|----------------|----------------|
| Question 1: D  | Question 21: D |
| Question 2: E  | Question 22: D |
| Question 3: E  | Question 23: C |
| Question 4: B  | Question 24: B |
| Question 5: E  | Question 25: D |
| Question 6: B  | Question 26: D |
| Question 7: D  | Question 27: D |
| Question 8: B  | Question 28: E |
| Question 9: B  | Question 29: C |
| Question 10: D | Question 30: B |
| Question 11: A | Question 31: E |
| Question 12: D | Question 32: D |
| Question 13: D | Question 33: C |
| Question 14: E | Question 34: A |
| Question 15: E | Question 35: C |
| Question 16: E | Question 36: E |
| Question 17: A | Question 37: E |
| Question 18: B | Question 38: C |
| Question 19: C | Question 39: D |
| Question 20: C | Question 40: A |

---

## Free-Response Scoring Guidelines

The following contains the scoring guidelines for the free-response questions in this exam.

# AP<sup>®</sup> STATISTICS

## 2014 SCORING GUIDELINES

### Question 1

#### **Intent of Question**

The primary goals of this question were to assess a student's ability to (1) interpret and summarize information from a graph; and (2) assess differences in variability between two groups from a graph.

#### **Solution**

##### **Part (a):**

Similarities: Both cities have the same pattern of average monthly temperatures throughout the year. More specifically, both cities are colder in the winter months at the beginning and end of the year and warmer in the summer months during the middle part of the year.

Differences: Madison is warmer during more months of the year than Juneau. Madison's average monthly temperatures vary more than Juneau's. For example, at the beginning and end of the year Madison's temperatures are lower than Juneau's but from April to November Madison temperatures are higher than Juneau's.

##### **Part (b):**

Madison has the greater standard deviation of average monthly temperatures because Madison's temperatures deviate more from its mean than Juneau's temperatures deviate from its corresponding mean. Standard deviation is (roughly) the average deviation from the mean, and Madison's average monthly temperatures tend to vary from the mean much more than Juneau's do.

#### **Scoring**

This question was scored in three sections. Section 1 consists of the description of the similarities in part (a); section 2 consists of the description of the differences in part (a); and section 3 consists of part (b). Each section was scored as essentially correct (E), partially correct (P), or incorrect (I).

**Section 1** is scored as follows:

Essentially correct (E) if the response states that both cities are cold in both the beginning and end of the year *AND* both cities are warm in months in the middle part of the year;

*OR*

if the response states that the temperatures for both cities increase from January to July and decrease from July to December.

Partially correct (P) if the response describes similarities between the two cities in at least two months (for example, both cities are hottest in July, both cities are coldest in January, the cities have similar temperatures in March);

*OR*

if the response states that both cities are cold in both the beginning and end of the year *OR* that both cities are warm in months in the middle part of the year, but not both;

*OR*

if the response states that temperatures in both cities increase and then decrease without identifying July as the peak.

# AP<sup>®</sup> STATISTICS

## 2014 SCORING GUIDELINES

### Question 1 (continued)

Incorrect (I) if the response does not meet the criteria for an E or a P.

*Note:* If the response mentions normality then the score is reduced one level (that is, from E to P or from P to I).

**Section 2** is scored as follows:

Essentially correct (E) if the response includes the following two components:

1. A correct comparison of the centers; for example, Madison has a tendency for higher average monthly temperatures overall, or Madison is warmer than Juneau during more months of the year (*April to November*).
2. A correct comparison of the variabilities; for example, Madison has greater variability than Juneau.

Partially correct (P) if the response includes only one of the two components listed above.

Incorrect (I) if the response does not meet the criteria for an E or P;

*OR*

only discusses differences month-by-month without providing a larger summary

**Section 3** is scored as follows:

Essentially correct (E) if the response selects Madison with the greater standard deviation *AND* provides a justification that states that Madison's average monthly temperatures vary more from its mean.

Partially correct (P) if the response selects Madison with the greater standard deviation *BUT* provides justification that only refers to a feature of the graph (for example, steeper, taller, bigger jump, range, more variation) without explaining how that relates to variability around the mean.

Incorrect (I) if the response does not meet the criteria for an E or P;

*OR*

selects Madison without justification;

*OR*

selects Madison and uses only calculations for justification;

*OR*

selects Juneau.

**AP<sup>®</sup> STATISTICS**  
**2014 SCORING GUIDELINES**

**Question 1 (continued)**

**4 Complete Response**

All three sections essentially correct

**3 Substantial Response**

Two sections essentially correct and one section partially correct

**2 Developing Response**

Two sections essentially correct and one section incorrect

*OR*

One section essentially correct and one or two sections partially correct

*OR*

Three sections partially correct

**1 Minimal Response**

One section essentially correct and two sections incorrect

*OR*

Two sections partially correct and one section incorrect

# AP<sup>®</sup> STATISTICS

## 2014 SCORING GUIDELINES

### Question 2

#### **Intent of Question**

The primary goals of this question were to assess a student's ability to: (1) distinguish between an observational study and an experiment; (2) explain the benefit of including a particular treatment group in an experiment; and (3) describe whether or not results from an experiment can reasonably be generalized to a larger population.

#### **Solution**

##### **Part (a):**

This is an experiment, because the participants were randomly assigned to treatment groups that drank different beverages.

##### **Part (b):**

The purpose of the “hot water” group is to investigate whether drinking tea is more effective than simply drinking hot water (essentially a placebo) for improving heart health. This group also allows for investigating whether drinking tea with milk is better than simply drinking hot water.

##### **Part (C):**

These 24 participants are volunteers, who may not be representative of a larger population with regard to the effect of tea/milk on heart health, so it is not necessarily reasonable to generalize the results of this study to a larger population.

#### **Scoring**

Part (a) was scored as either essentially correct (E) or incorrect (I). Parts (b) and (c) were scored as essentially correct (E), partially correct (P), or incorrect (I).

##### **Part (a)** is scored as follows:

Essentially correct (E) if the response correctly says that this is an experiment *AND* provides a reasonable explanation based on assignment of participants to treatment groups.

Incorrect (I) if the response incorrectly states that this is an observational study;

*OR*

if the response says that this is an experiment but provides no explanation;

*OR*

if the response says that this is an experiment but provides an incorrect explanation

# AP<sup>®</sup> STATISTICS

## 2014 SCORING GUIDELINES

### Question 2 (continued)

**Part (b)** is scored as follows:

Essentially correct (E) if the response correctly states that a benefit is determining whether drinking tea is better for heart health than simply drinking hot water

*Note:* To earn an E, the response must refer to one or more treatments in context.

Partially correct (P) if the response refers to the hot water group as a control group, but does not specify an appropriate benefit of including that group;

*OR*

if the response correctly describes a benefit of a control group but the explanation does not refer to the treatments in context.

Incorrect (I) if the response does not meet the criteria for an E or a P.

**Part (c)** is scored as follows:

Essentially correct (E) if the response answers “no” *AND* notes that the subjects were volunteers *AND* comments that volunteers may not be representative of a larger population;

*OR*

if the response essentially says “yes, if the participants are representative of a larger population.”

*Note:* If the essentially correct “no” response above is accompanied by the additional statement that the sample size is too small to allow generalizing, then the score is reduced from E to P.

Partially correct (P) if the response answers “no” *AND* simply notes that the subjects are volunteers without commenting that volunteers may not be representative of a larger population

*Note:* If the partially correct “no” response above is accompanied by the additional statement that the sample size is too small to allow generalizing, then the score is reduced from P to I.

Incorrect (I) if the response says “no, because 24 is such a small sample size”;

*OR*

if the response says “yes, because of random assignment”;

*OR*

if the response essentially says “yes, because the volunteers are representative of a larger population”;

*OR*

if the response does not meet the criteria for an E or P.

**AP<sup>®</sup> STATISTICS**  
**2014 SCORING GUIDELINES**

**Question 2 (continued)**

**4 Complete Response**

All three parts essentially correct

**3 Substantial Response**

Two parts essentially correct and one part partially correct

**2 Developing Response**

Two parts essentially correct and one part incorrect

*OR*

One part essentially correct and one or two parts partially correct

*OR*

Three parts partially correct

**1 Minimal Response**

One part essentially correct and two parts incorrect

*OR*

Two parts partially correct and one part incorrect

# AP<sup>®</sup> STATISTICS

## 2014 SCORING GUIDELINES

### Question 3

#### **Intent of Question**

The primary goal of this question was to assess students' ability to identify, set up, perform, and interpret the results of an appropriate hypothesis test to address a particular question. More specific goals were to assess students' ability to (1) state appropriate hypotheses; (2) identify the appropriate statistical test procedure and check appropriate conditions for inference; (3) calculate the appropriate test statistic and  $p$ -value; and (4) draw an appropriate conclusion, with justification, in the context of the study.

#### **Solution**

Step 1: States a correct pair of hypotheses.

$\mu_n$  represents the mean patient response (myocardial salvage index) to the new treatment.

$\mu_s$  represents the mean patient response (myocardial salvage index) to the standard care treatment.

$\mu_n - \mu_s$  represents the difference in mean response for the two treatments.

We want to test whether the data provide convincing evidence that the new treatment produces a higher mean myocardial salvage index than the standard care, so the hypotheses to be tested are:

$$H_0 : \mu_n = \mu_s \text{ vs. } H_a : \mu_n > \mu_s, \text{ or } H_0 : \mu_n - \mu_s = 0 \text{ vs. } H_a : \mu_n - \mu_s > 0.$$

Step 2: Identifies a correct test procedure (by name or by formula) and checks appropriate conditions.

Provided the conditions are satisfied, the appropriate procedure is a two-sample  $t$ -test.

Because this data is from an experiment, the first condition is random assignment to treatment groups. We are told that the 112 patients in the study were randomly assigned to one of the two treatment groups, so this condition is satisfied.

The second condition is that the populations of myocardial salvage index values follow normal distributions or the sample sizes are large. We are not given enough information to assess whether the myocardial salvage index values follow normal distributions, but both sample sizes  $n_n = 56$  and  $n_s = 56$  are large enough ( $n > 30$ ) for the two-sample  $t$ -test to be valid.

Step 3: Correct mechanics, including the value of the test statistic value and  $p$ -value (or rejection region).

$$\text{The test statistic is: } t = \frac{\bar{x}_n - \bar{x}_s}{\sqrt{\frac{s_n^2}{n_n} + \frac{s_s^2}{n_s}}} = \frac{0.69 - 0.57}{\sqrt{\frac{0.27^2}{56} + \frac{0.26^2}{56}}} \approx 2.40.$$

The  $p$ -value = 0.009, based on 109 degrees of freedom.

(The  $p$ -value = 0.0099, based on conservative df = smaller sample size – 1 = 55).

# AP<sup>®</sup> STATISTICS

## 2014 SCORING GUIDELINES

### Question 3 (continued)

Step 4: States a correct conclusion in the context of the study, using the result of the statistical test.

Because this p-value is small, smaller than any common significance level such as  $\alpha = 0.05$  or  $\alpha = 0.01$ , we reject  $H_0$  and conclude that the data provide convincing evidence that the new treatment produces a higher mean myocardial salvage index than the standard care.  
(The cause-and-effect nature of this conclusion is warranted because this is a randomized experiment.)

### **Scoring**

Each of steps 1, 2, 3, and 4 were scored as essentially correct (E), partially correct (P), or incorrect (I).

**Step 1** is scored as follows:

Essentially correct (E) if the response includes the following two components:

1. Correctly defines both parameters
2. Correct hypotheses.

Partially correct (P) if the response includes only one of the two components listed above

Incorrect (I) if the response does not meet the criteria for an E or a P.

*Note:* Defining the parameter symbols in context or using common parameter notation with subscripts clearly relevant to the context is sufficient for defining parameters.

**Step 2** is scored as follows:

Essentially correct (E) if the response includes the following three components:

1. Identifies the correct test procedure (by name or by formula)
2. Checks for random assignment
3. Checks the sample size is large enough for both groups

*Notes:*

1. The check of the randomness condition must refer to random assignment, not random sampling. Because random assignment is stated in the problem stem, any statement about random assignment is sufficient.
2. Approximate (large samples) two-sample z-test is an acceptable solution.  
( $z = 2.4$ ,  $p\text{-value} = 0.0083$ )

Partially correct (P) if the response correctly completes two of the three components listed above.

Incorrect (I) if the response correctly completes at most one of the three components listed above.

**Step 3** is scored as follows:

Essentially correct (E) if the response includes the following two components:

1. States correct value of the test statistic. Supporting work is not required, but if included it must be correct.
2. Gives a correct p-value.

# AP<sup>®</sup> STATISTICS

## 2014 SCORING GUIDELINES

### Question 3 (continued)

Partially correct (P) if the response includes only one of the two component listed above;

*OR*

if test statistic value is incorrect, but for this statistic, the  $p$ -value is correct and consistent with alternate hypothesis.

Incorrect (I) if the response does not meet the criteria for an E or a P.

**Step 4** is scored as follows:

Essentially correct (E) if the response provides the following three components:

1. correct conclusion
2. linkage between the  $p$ -value and conclusion
3. conclusion is stated in context

Partially correct (P) if the response provides a correct conclusion (or decision) but only one of the other two components listed above.

Incorrect (I) if the response does not provide a correct conclusion (or decision) or is missing both linkage and context

*Notes:*

1. If the conclusion in context does not say “mean” or equivalent, Step 4 cannot be scored as essentially correct (E).
2. If the conclusion in context uses “different” rather than “greater than”, Step 4 cannot be scored as essentially correct (E).
3. If the correct conclusion for the reported  $p$ -value is to fail to reject  $H_0$ , then a conclusion that is equivalent to “accept  $H_0$ ” either as a stated decision or as a conclusion in context cannot be scored as essentially correct (E). Such a response will be scored partially correct (P) provided that the conclusion is in context with linkage. Such a response will be scored as incorrect (I) if it lacks either context or linkage.
4. If the conclusion is consistent with an incorrect  $p$ -value from Step 3, and also in context with justification based on linkage to the  $p$ -value, then Step 4 is scored as essentially correct (E).
5. If a rejection region approach is used, the response must include the critical value and the observed value of the test statistic must be compared to the critical value.

# AP<sup>®</sup> STATISTICS

## 2014 SCORING GUIDELINES

### Question 3 (continued)

Each essentially correct (E) step counts as 1 point, and a partially correct (P) step counts as  $\frac{1}{2}$  point.

- 4      Complete Response**
- 3      Substantial Response**
- 2      Developing Response**
- 1      Minimal Response**

If a response is between two scores (for example,  $2\frac{1}{2}$  points), use a holistic approach to decide whether to score up or down, depending on the overall strength of the response and communication. A  $3\frac{1}{2}$  response should not be scored up to a 4 if it is missing something important.

# AP<sup>®</sup> STATISTICS

## 2014 SCORING GUIDELINES

### Question 4

#### Intent of Question

The primary goals of this question were to assess a student's ability to: (1) use conditional probability rules to calculate a probability; (2) determine a probability distribution; and (3) calculate and interpret an expected value.

#### Solution

##### Part (a):

The probability that the piñata does not break on the first hit *and* breaks on the second hit is equal to the probability that the piñata does not break on the first hit ( $1 - 0.7 = 0.3$ ) times the probability that the piñata does break on the second hit given that it did not break on the first hit (0.8)  
Thus,  $0.3 \times 0.8 = 0.24$ .

##### Part (b):

$$P(X = 2) = 0.3 \times 0.8 = 0.24$$

$$\begin{aligned} P(X = 3) &= P(\text{no break on first hit and no break on second hit and break on third hit}) \\ &= P(\text{no break on first hit}) \times P(\text{no break on second hit} \mid \text{no break on first hit}) \\ &\quad \times P(\text{break on third hit} \mid \text{no break on first hit and no break on second hit}) \\ &= 0.3 \times 0.2 \times 0.9 = 0.054 \\ P(X = 4) &= 1 - P(X = 1) - P(X = 2) - P(X = 3) \\ &= 1 - 0.7 - 0.24 - 0.054 = 0.006 \end{aligned}$$

| $x$                | 1   | 2    | 3     | 4     |
|--------------------|-----|------|-------|-------|
| Probability of $x$ | 0.7 | 0.24 | 0.054 | 0.006 |

##### Part (c):

$$E(X) = 1(0.7) + 2(0.24) + 3(0.054) + 4(0.006) = 1.366$$

Interpretation: If Sophia repeats this process for a very large number of piñatas, the average number of hits required to break the piñata will be very close to 1.366 hits per piñata.

# AP<sup>®</sup> STATISTICS

## 2014 SCORING GUIDELINES

### Question 4 (continued)

#### **Scoring**

Part (a), (b) and (c) were scored as essentially correct (E), partially correct (P), or incorrect (I).

**Part (a)** is scored as follows:

Essentially correct (E) if the response correctly performs the calculation *AND* shows a correct method of solution.

Partially correct (P) if the response indicates knowledge to use the multiplication rule but multiplies two wrong, but reasonable, values based on information from the stem of the problem (for example,  $0.7 \times 0.8$ , or  $0.7 \times 0.1$ , or  $0.3 \times 0.1$ , or  $0.3 \times 0.7$ ).

Incorrect (I) if the response does not meet the criteria for an E or a P, including if the response gives the correct answer with no work shown.

**Part (b)** is scored as follows:

Essentially correct (E) if the response correctly calculates all three probabilities *AND* shows sufficient work for at least two of the probabilities, which can include work coming from part (a) or part (c).

*Note:* If the answer to part (a) is not correct and is carried forward to part b), then the response to part (b) can earn an E if one additional probability is calculated correctly with supporting work and the four probabilities sum to 1.

Partially correct (P) if the response correctly calculates at least two of the three probabilities but shows sufficient work for only one of the probabilities, which can include work coming from part (a) or part (c);  
*OR*

if the response correctly calculates only one of the three probabilities *AND* shows sufficient work, which can include work coming from part (a) or part (c) *AND* reports probabilities that sum to 1;  
*OR*

if the responses calculates geometric probabilities *AND* shows work *AND* makes the probabilities sum to 1.

*Note:* The response in part b) can receive a score of P if the response correctly calculates all three probabilities with no supporting work shown.

Incorrect (I) if the response does not meet the criteria for an E or a P.

**Part (c)** is scored as follows:

Essentially correct (E) if the response calculates the expected value correctly *AND* provides a correct interpretation that refers to “long run” or “average value” and context.

# AP<sup>®</sup> STATISTICS

## 2014 SCORING GUIDELINES

### Question 4 (continued)

Notes:

1. An expected value calculation that is consistent with probabilities reported in (b) gets credit, even if the result is not a reasonable value.
2. If the expected value calculation is rounded to an integer, the response does not receive credit for the calculation.

Partially correct (P) if the response includes only one of the two aspects (calculation, interpretation) correctly.

Incorrect (I) if the response includes neither of the two aspects (calculation, interpretation) correctly

#### **4 Complete Response**

All three parts essentially correct

#### **3 Substantial Response**

Two parts essentially correct and one part partially correct

#### **2 Developing Response**

Two parts essentially correct and one part incorrect

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

#### **1 Minimal Response**

One part essentially correct and two parts incorrect

OR

Two parts partially correct and one part incorrect

# AP<sup>®</sup> STATISTICS

## 2014 SCORING GUIDELINES

### Question 5

#### **Intent of Question**

The primary goals of this question were to assess a student's ability to: (1) interpret information displayed in comparative bar graphs; (2) identify an appropriate statistical test to address a research question; and (3) draw an appropriate conclusion from a test statistic.

#### **Solution**

##### **Part (a):**

The graph reveals that males and females tend to differ with regard to which is their favorite sport to watch. Males tend to select football and basketball more often than females, whereas females tend to select gymnastics and swimming more often than males. The relative frequencies are similar for baseball between males and females.

##### **Part (b):**

The appropriate statistical test is a chi-squared test for independence. The number of degrees of freedom is  $(5 - 1) \times (2 - 1) = 4$ .

##### **Part (c):**

The  $p$ -value, based on a chi-squared test statistic of 10.77 and determined using technology, is approximately 0.029. This  $p$ -value is smaller than the significance level of 0.05. Therefore, the sample data provides convincing statistical evidence that there is an association between gender and sport preference in the population of all students at this school.

# AP<sup>®</sup> STATISTICS

## 2014 SCORING GUIDELINES

### Question 5 (continued)

#### **Scoring**

Parts (a), (b), and (c) were scored as essentially correct (E), partially correct (P), or incorrect (I):

**Part (a)** is scored as follows:

Essentially correct (E) if the response correctly identifies at least one sport for which males exhibit a higher relative frequency than females, and at least one sport for which females exhibit a higher relative frequency than males;

*OR*

correctly identifies at least one sport for which the relative frequencies for males and females are different, and at least one sport for which the relative frequencies for males and females are similar;

*OR*

correctly identifies at least two differences between the distributions of sport preferences for males and females.

Partially correct (P) if the response correctly identifies at least one sport for which males have a higher relative frequency or at least one sport for which females have a higher relative frequency, but not both;

*OR*

correctly identifies one difference between the distributions of sport preferences for males and females.

Incorrect (I) if the response does not meet the criteria for an E or a P

*Notes:*

1. An ambiguous statements such as “Males like football more while females like gymnastics more” is interpreted as contrasting gender differences across different sports (that is, males like football more [than females do] while females like gymnastics more [than males do]).
2. A response containing incorrect statistical statements can be scored no higher than P.

**Part (b)** is scored as follows:

Essentially correct (E) if the response correctly provides the following three components:

1. Name of test
2. Identification of the two-variable nature of the test
3. Degrees of freedom

Partially correct (P) if the response correctly provides two of the three components listed above.

Incorrect (I) if the response correctly provides at most one of the three components listed above.

*Notes:*

1. It is not necessary for the response to include supporting work for the calculation of the degrees of freedom. However if supporting work is shown, it must be correct.

# AP<sup>®</sup> STATISTICS

## 2014 SCORING GUIDELINES

### Question 5 (continued)

2. In the following table, check marks (✓) indicate components satisfied by each of the given responses.

|                            | Name of Test | Two-Variables |
|----------------------------|--------------|---------------|
| Test of association        | ✓            | ✓             |
| Test of independence       | ✓            | ✓             |
| Chi-square test            | ✓            |               |
| $df = (5-1)(2-1) = 4$      |              | ✓             |
| Homogeneity of proportions |              | ✓             |
| Goodness-of-fit            |              |               |

**Part (c)** is scored as follows:

Essentially correct (E) if the response gives the correct conclusion and includes the following three components:

1. Reports the correct  $p$ -value or indicates a range including the correct  $p$ -value, identified from a chi-squared table
2. Provides linkage/justification between the  $p$ -value and the conclusion
3. Gives the conclusion in context

Partially correct (P) if the response gives the correct conclusion and includes two of the three components listed above.

Incorrect (I) if the response provides an incorrect conclusion or includes at most one of the three component listed above.

*Notes:*

1. A critical-region approach is acceptable for full credit in part (c), if done correctly. Such an approach receives an E if it gives the correct conclusion and includes the following three components:
  1. Correctly identifies the threshold value of chi-squared for rejection, which is 9.49
  2. Provides linkage/justification between the rejection threshold and the test statistic (that is,  $10.77 > 9.49$ )
  3. Gives the conclusion in context
2. A conclusion in part (c) that is equivalent to asserting a causal relationship between the variables is scored as incorrect for the conclusion in context component. If both a significance level and a  $p$ -value are included together, then the linkage between the  $p$ -value and the conclusion is implied. If no significance level is given, the solution must be explicit about the linkage by giving a correct interpretation of the  $p$ -value or by explaining how the conclusion follows by stating something like “Because the  $p$ -value is large, there is insufficient evidence of an association between choice of favorite sport to watch and gender.”
3. If the correct conclusion for the reported  $p$ -value is to fail to reject  $H_0$ , then a conclusion that is equivalent to “accept  $H_0$ ” either as a stated decision or as a conclusion in context (for example, “there is no association between choice of favorite sport to watch and gender”) may not be scored essentially correct (E). Such a response will be scored partially correct (P) provided that the conclusion is in context with linkage. Such a response will be scored as incorrect (I) if it lacks either context or linkage.

**AP<sup>®</sup> STATISTICS**  
**2014 SCORING GUIDELINES**

**Question 5 (continued)**

4. The following table shows, for some degrees of freedom that students may identify in part (b), the  $p$ -value that corresponds to the observed chi-squared statistic of 10.77:

| df | $p$ -value | Reject $H_0$ ? |          |
|----|------------|----------------|----------|
| 3  | 0.013      | yes            |          |
| 4  | 0.029      | yes            | Correct! |
| 5  | 0.056      | no             |          |
| 8  | 0.215      | no             |          |
| 9  | 0.292      | no             |          |
| 10 | 0.376      | no             |          |

5. A response referring to a  $p$ -value from Table C or “from the table” but in which no range of  $p$ -values is provided can be scored as partially correct (P) if linkage and a conclusion in context are present.
6. If the correct conclusion for the reported  $p$ -value is to fail to reject  $H_0$ , and the response includes wording equivalent to “retain the null hypothesis” with linkage and context, this should be scored partially correct (P). If the student goes on to make a statement that is equivalent to “fail to reject” in context (for example, “We should not conclude that there is an association between choice of favorite sport to watch and gender), then the response should be scored essentially correct (E).

**4 Complete Response**

All three parts essentially correct

**3 Substantial Response**

Two parts essentially correct and one part partially correct

**2 Developing Response**

Two parts essentially correct and one part incorrect

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

**1 Minimal Response**

One part essentially correct and two parts incorrect

OR

Two parts partially correct and one part incorrect

# AP<sup>®</sup> STATISTICS

## 2014 SCORING GUIDELINES

### Question 6

#### **Intent of Question**

The primary goals of this question were to assess a student's ability to: (1) describe Type II error and a consequence in context; (2) determine the sampling distribution for a total and use it to determine a test procedure; and (3) perform and interpret a normal probability calculation related to a hypothesis test.

#### **Solution**

##### **Part (a):**

Description: Type II error would be failing to reject the null hypothesis that the machine is putting 50 pennies in each roll when in reality the machine is putting fewer than 50 pennies in each roll.

Consequence: A consequence would be that the machine would not be repaired even though it is malfunctioning, so the machine would continue to put fewer than 50 pennies in each roll.

##### **Part (b):**

The sampling distribution of the total weight  $W$  is approximately normal with mean  $50 \times 2.5 = 125$  grams and standard deviation  $\sqrt{50} \times 0.04 \approx 0.2828$  grams.

##### **Part (c):**

The z-score for a total weight of 124 grams is:  $z = \frac{(124 - 125)}{0.2828} \approx -3.54$ .

The probability to the left of  $-3.54$  under a standard normal curve is 0.0002.

Therefore, using the rule given, the probability is 0.0002 that the inspector will conclude that the selected roll contains fewer than 50 pennies, when in fact the roll really does contain 50 pennies.

##### **Part (d):**

If the roll actually contains 49 pennies, then the sampling distribution of the total weight is approximately normal with mean  $49 \times 2.5 = 122.5$  grams and standard deviation  $\sqrt{49} \times 0.04 = 0.2800$  grams. The inspector will conclude that the roll contains fewer than 50 pennies if the total weight is less than 124 grams. The z-score associated with this value, when the roll actually contains 49 pennies, is:  $z = \frac{(124 - 122.5)}{0.2800} \approx 5.36$ . Such an extremely large z-score reveals that it would be extremely likely for 49 randomly selected pennies to have a total weight less than 124 grams. Therefore, when the roll actually contains 49 pennies, it is extremely likely (probability very close to 1) that the inspector will correctly conclude that the roll contains fewer than 50 pennies.

# AP<sup>®</sup> STATISTICS

## 2014 SCORING GUIDELINES

### Question 6 (continued)

#### Part (e):

The inspector's rule is very effective for distinguishing a roll that contains 50 pennies from one that contains fewer than 50 pennies. Part (c) reveals that if the roll actually contains 50 pennies, the probability is only 0.0002 that the inspector will mistakenly conclude that the roll contains fewer than 50 pennies. Part (d) reveals that if the roll actually contains fewer than 50 pennies, then it is extremely likely that the inspector will correctly conclude that the roll contains fewer than 50 pennies. Therefore, whether the roll contains 50 pennies or fewer, the inspector's rule is extremely likely to lead to the correct decision.

#### **Scoring**

This question was scored in four sections. Section 1 consists of part (a); section 2 consists of parts (b) and (c) combined; section 3 consists of part (d); and section 4 consists of part (e). Each section was scored as essentially correct (E), partially correct (P), or incorrect (I).

**Section 1** is scored as follows:

Essentially correct (E) if the response includes the following two components:

1. Correct description of Type II error
2. Correct consequence of the error in context

Partially correct (P) if the response includes only one of the two components listed above;

OR

if the response includes both components, but for a Type I error.

Incorrect (I) if the response does not meet the criteria for an E or a P.

**Section 2** is scored as follows:

Essentially correct (E) if the response includes the following three components:

1. Correct description of the shape of the sampling distribution as approximately normal
2. Correct calculations, with work shown, of the mean and standard deviation of the sampling distribution
3. Correct calculation of the relevant probability

Partially correct (P) if the response includes two of the three components listed above.

Incorrect (I) if the response includes at most one of the three components listed above.

**Section 3** is scored as follows:

Essentially correct (E) if the response includes the following two components:

1. Correct calculations of the mean and standard deviation of the sampling distribution
2. Correct conclusion that answers the question, with linkage to a z-score calculation, or a probability, or similar reasonable justification

# AP<sup>®</sup> STATISTICS

## 2014 SCORING GUIDELINES

### Question 6 (continued)

Partially correct (P) if the response includes only one of the two components listed above.

Incorrect (I) if the response does not meet the criteria for an E or P.

*Note:* The standard deviation is scored as correct if it incorporates the sample size 49 in the same form as was used in part (b).

**Section 4** is scored as follows:

Essentially correct (E) if the response provides a correct conclusion about the effectiveness of the inspector's rule with clear, reasonable justification linked to both of parts (c) and (d)

Partially correct (P) if the response provides a correct conclusion about the effectiveness of the inspector's rule with weak, but reasonable justification, such as unclear or missing linkage, and with reference to both of parts (c) and (d)

Incorrect (I) if the response does not meet the criteria for an E or P

Each essentially correct (E) section counts as 1 point. Each partially correct (P) section counts as  $\frac{1}{2}$  point.

- 4      Complete Response**
- 3      Substantial Response**
- 2      Developing Response**
- 1      Minimal Response**

If a response is between two scores (for example,  $2\frac{1}{2}$  points), use a holistic approach to determine whether to score up or down, depending on the strength of the response and communication, particularly in parts (d) and (e).