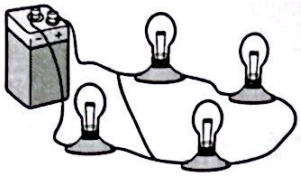
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION I

Time—45 minutes

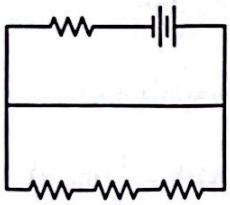
35 Questions

Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and then fill in the corresponding circle on the answer sheet.



1. The circuit shown consists of four identical lightbulbs and a battery. Which of the following diagrams best represents the circuit?

(A)



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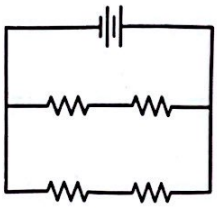

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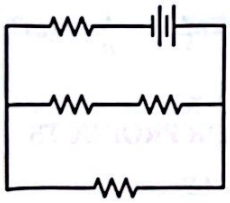

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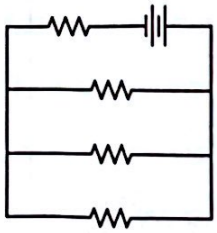

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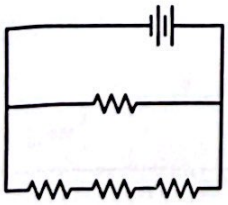

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2. A solid, nonconducting sphere of radius R has charge Q distributed uniformly throughout its volume. Which of the following is equal to the magnitude of the electric field at distance $\frac{R}{2}$ from the center of the sphere?

(A) Zero

(B) $\frac{1}{2}\frac{kQ}{R^2}$

(C) $\frac{kQ}{R^2}$

(D) $2\frac{kQ}{R^2}$

(E) $4\frac{kQ}{R^2}$

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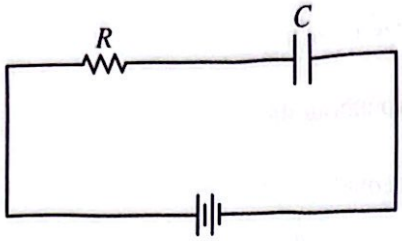
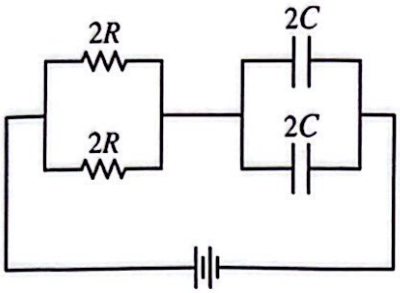


3. A resistor of resistance R and a capacitor of capacitance C are connected to an ideal battery, as shown. This circuit has time constant τ. Which of the following circuits also has time constant τ ?

(A)



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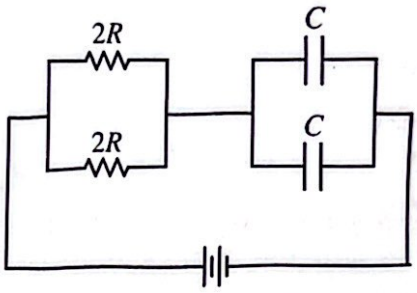

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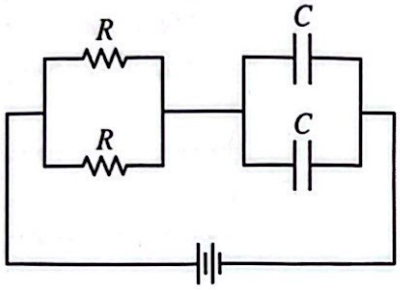

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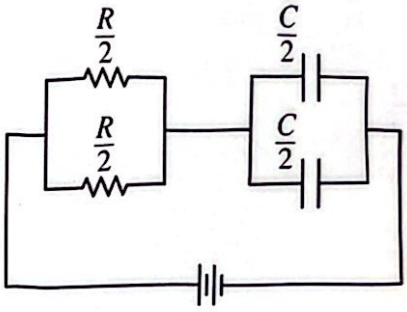

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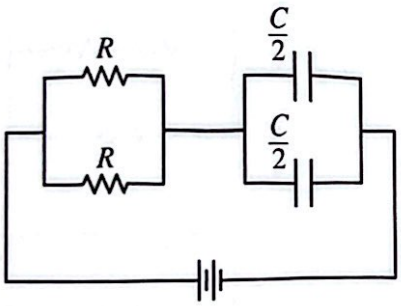

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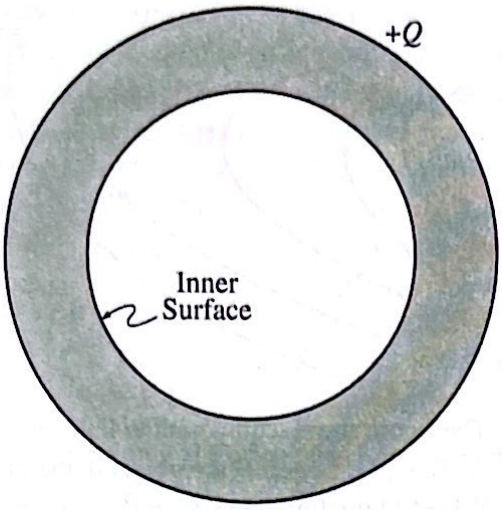

4. A conducting spherical shell has a net positive charge $+Q$, as shown. Which of the following indicates the net charge on the inner surface of the shell and the net charge inside of the shell (shaded region)?

	Inner Surface	Inside Shell
A	Zero	Zero
B	$-Q$	$-Q$
C	Zero	$-Q$
D	$-Q$	Zero
E	$+Q$	Zero

(A) A
(B) B
(C) C
(D) D
(E) E

5. An air-filled parallel plate capacitor stores 8.85×10^{-10} J of energy when the potential difference across the plates is 20.0 V and the plates are separated by a distance of 1.00×10^{-3} m. What is the area of each plate of the capacitor?

(A) $1.25 \times 10^{-4}\ m^2$
(B) $2.50 \times 10^{-4}\ m^2$
(C) $5.00 \times 10^{-4}\ m^2$
(D) $1.00 \times 10^{-3}\ m^2$
(E) $1.50 \times 10^{-3}\ m^2$

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d (m)	a (m/s^2)
0.1	90
0.2	90
0.3	90
0.4	90

6. Identical nonconducting spheres 1 and 2, each with equal amounts of positive net charge, are initially held at rest near each other so that the force between the spheres is nonnegligible. Sphere 2 is then released while Sphere 1 remains at rest. The table shows the acceleration a of Sphere 2 along a straight path as a function of the separation distance d between the centers of the spheres. Which of the following claims can be supported by data in the table?

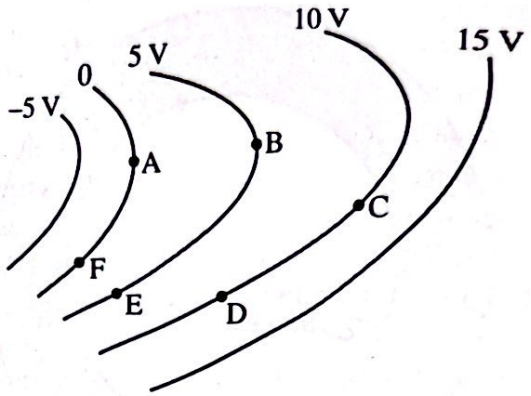
(A) The spheres are isolated from other objects and fields.

(B) The spheres are in a region of external uniform electric field only.

(C) The spheres are in a region of external nonuniform electric field only.

(D) The spheres are in a region of external uniform magnetic field only.

(E) The spheres are in a region of external uniform electric and magnetic fields only.



7. The figure shows equipotential lines in a plane. A, B, C, D, E, and F are points in the plane. Which of the following claims is correct?

(A) The electric field at Point A is directed toward Point B.

(B) The electric field at Point C is directed toward Point D.

(C) The magnitude of the electric field is greater at Point B than at Point E.

(D) The magnitude of the average electric field is greater between points F and E than between points A and B.

(E) The magnitude of the average electric field is greater between points B and C than between points E and D.

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8. Nonconducting Sphere 1 has a positive net charge and is held at rest. Conducting Sphere 2, which is near Sphere 1, has no net charge and is given an initial speed moving away from Sphere 1. All forces other than the electric force are negligible. Which of the following claims is correct for when Sphere 2 moves away from Sphere 1 ?

(A) The acceleration of Sphere 2 is toward Sphere 1.

(B) The momentum of Sphere 2 is increasing.

(C) The net force exerted on Sphere 2 is zero.

(D) The kinetic energy of Sphere 2 is constant.

(E) The electric potential energy of the Sphere 1-Sphere 2 system is decreasing.

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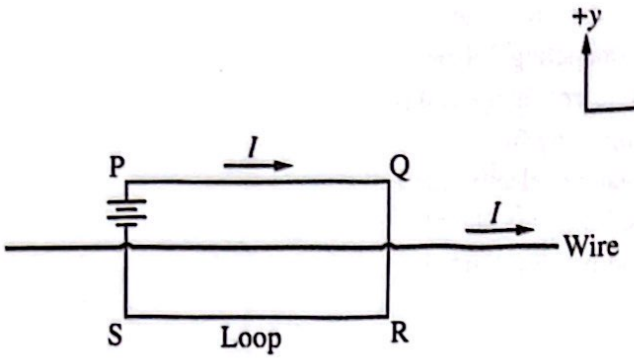

Top View

9. A conducting loop is beneath a long, straight, insulated wire. The loop and the wire are in the plane of the page. At the instant shown, the current in the wire and the current in Segment PQ of the loop are I in the $+x$-direction. Which of the following indicates the direction, if any, of the net magnetic force $\vec{F}_M$ exerted on Segment RS of the loop and provides a valid justification?

(A) $+y$, because the currents in Segment RS and the wire create magnetic fields that produce an attractive force.

(B) $+y$, because the currents in segments RS and PQ create magnetic fields that produce an attractive force.

(C) No direction, because the currents in Segment RS and the wire are constant.

(D) $-y$, because the currents in Segment RS and the wire create magnetic fields that produce a repulsive force.

(E) $-y$, because in the region between Segment RS and the wire, the magnetic field from Segment RS is directed in the opposite direction from the magnetic field from the wire.

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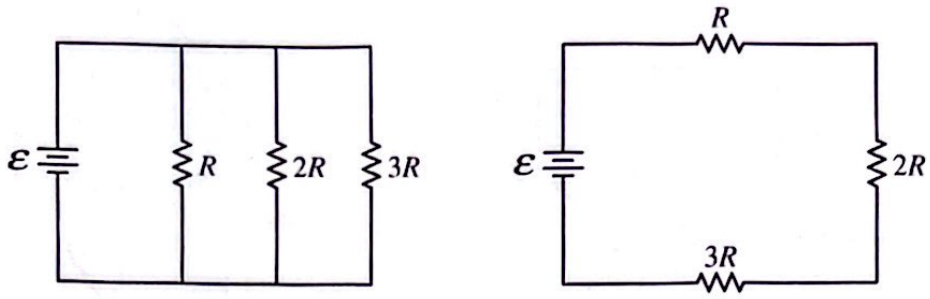

Circuit 1 Circuit 2

10. Circuits 1 and 2 shown each consist of an ideal battery of emf $\mathcal{E}$ and resistors with resistances of R, $2R$, and $3R$. The rates at which thermal energy is dissipated by the resistor with resistance R in circuits 1 and 2 are P_1 and P_2, respectively. Which of the following correctly compares P_2 to P_1 and provides a valid justification?

(A) $P_2 = P_1$, because the resistance of the resistor is the same in each circuit.

(B) $P_2 > P_1$, because the equivalent resistance of Circuit 1 is smaller than the equivalent resistance of Circuit 2.

(C) $P_2 > P_1$, because the current in the resistor for Circuit 1 is smaller than the current in the resistor for Circuit 2.

(D) $P_2 < P_1$, because the equivalent resistance of Circuit 1 is larger than the equivalent resistance of Circuit 2.

(E) $P_2 < P_1$, because the current in the resistor for Circuit 1 is larger than the current in the resistor for Circuit 2.

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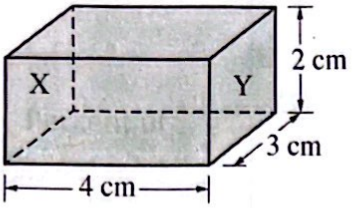
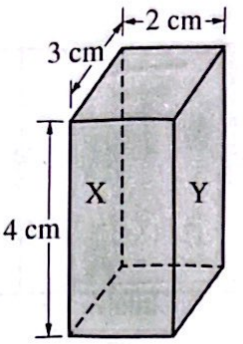


Piece 1



Piece 2

11. The resistance between the left side X and right side Y of two separate pieces, 1 and 2, of the same resistive material is measured. The dimensions of the pieces are shown. The resistances of pieces 1 and 2 are R_1 and R_2, respectively. Which of the following correctly relates R_2 and R_1 ?

(A) $R_2 = \frac{1}{4} R_1$

(B) $R_2 = \frac{1}{2} R_1$

(C) $R_2 = R_1$

(D) $R_2 = 2R_1$

(E) $R_2 = 4R_1$

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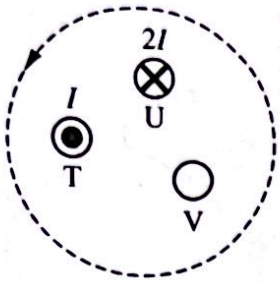
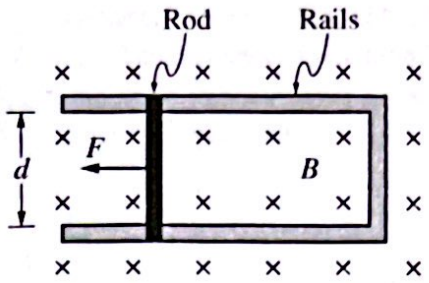


12. Wires T, U, and V each carry current and are oriented perpendicular to the plane of the page. Wire T carries current I out of the page. Wire U carries current $2I$ into the page. The value and direction of the current in Wire V are unknown. The line integral of the magnetic field $\vec{B}$ taken counterclockwise around the dashed path shown, $\oint \vec{B} \cdot d\vec{\ell}$, is equal to $2\mu_0 I$. Which of the following indicates the value and direction of the current in Wire V ?

(A) I out of the page
(B) $2I$ out of the page
(C) $2I$ into the page
(D) $3I$ out of the page
(E) $3I$ into the page



Top View

13. A metal rod of mass m and resistance R is in contact with the rails of a U-shaped conductor of width d and negligible resistance. The rails rest on a horizontal surface. The coefficient of kinetic friction between the rod and rails is fi. The rod and rails are in a uniform magnetic field of magnitude B that is directed into the surface, as shown. An external force of magnitude F is exerted on the rod so that the rod has constant speed v to the left. Which of the following expressions is equal to the power delivered to the rod by the force?

(A) $\frac{Bdv}{R}$
(B) $\frac{Bdv}{R} + \mu mgv$
(C) $\frac{B^2d^2v^2}{R}$
(D) $\frac{B^2d^2v^2}{R} + \mu mgv$
(E) $\frac{B^2d^2v^2}{R} - \mu mgv$

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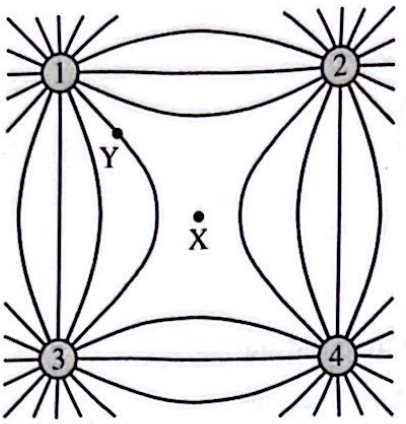

Point	Direction of $\vec{E}$
X	No direction
Y	Toward Sphere 4

14. The figure shown represents the electric field resulting from the square arrangement of charged spheres 1, 2, 3, and 4. The directions, if any, of the electric field $\vec{E}$ at points X and Y are given in the table. Which of the following identifies the signs of the charges on the spheres, consistent with the collected data?

	Positively Charged	Negatively Charged
A	1, 2, 3, and 4	None
B	1, 2, and 3	4
C	1 and 4	2 and 3
D	2 and 3	1 and 4
E	None	1, 2, 3, and 4

(A) A
(B) B
(C) C
(D) D
(E) E

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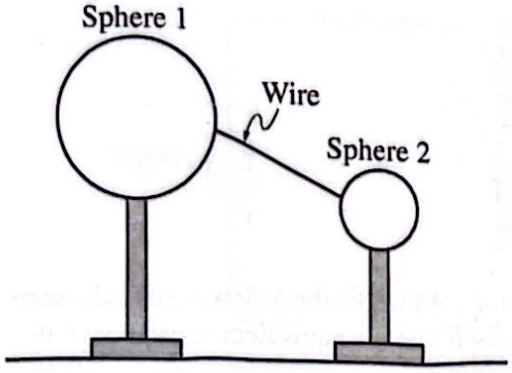

Note: Figure not drawn to scale.

15. Conducting spheres 1 and 2 are each on an insulating stand. The distance between the spheres is large compared to their diameters. Sphere 1 has a known amount of charge on it, and Sphere 2 is initially neutral. A wire is then connected between the spheres for a long time, as shown. Which of the following indicates the fewest measurable quantities necessary to calculate the electric potential at the surface of each sphere?

(A) The diameter of Sphere 1
(B) The diameter of Sphere 2
(C) The diameters of both spheres
(D) The diameters of both spheres and the length of the wire
(E) The diameters of both spheres, the length of the wire, and the heights of both stands

16. A long, cylindrical-shaped solenoid has length ℓ and N loops. The diameter of one loop is d. Which of the following expressions is equal to the inductance of the solenoid?

(A) $\frac{1}{4}\frac{N\mu_0\pi d^2}{\ell}$
(B) $\frac{1}{2}\frac{N\mu_0\pi d^2}{\ell}$
(C) $\frac{1}{4}\frac{N^2\mu_0\pi d^2}{\ell}$
(D) $\frac{1}{2}\frac{N^2\mu_0\pi d^2}{\ell}$
(E) $\frac{N^2\mu_0\pi d^2}{\ell}$

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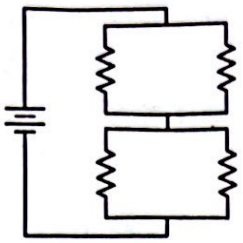
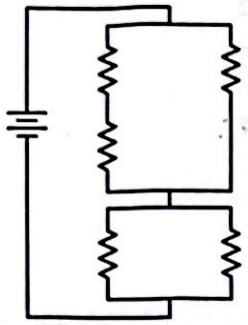
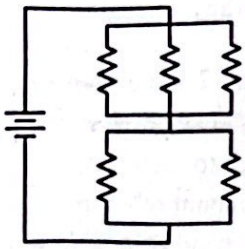


17. The circuit shown consists of a battery and four identical resistors. Which of the following circuits correctly shows how a fifth identical resistor can be added to the circuit such that the equivalent resistance of the circuit is doubled?

(A)



(B)



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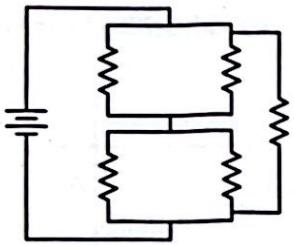

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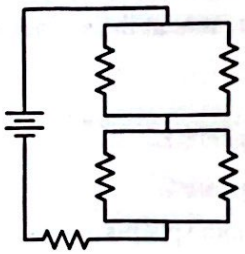

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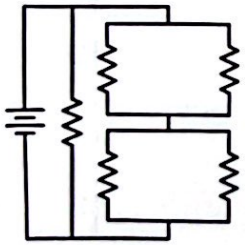

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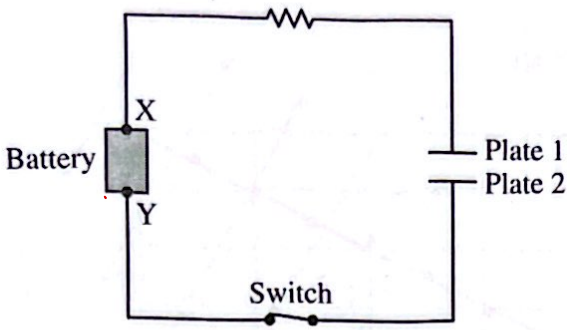

Figure 1

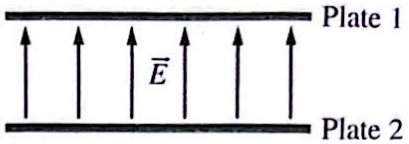

Figure 2

18. The circuit represented in Figure 1 consists of a battery, a resistor, a switch that has been closed for a long time, and a capacitor. Electric potentials V_X and V_Y are measured at positions X and Y, respectively, at the terminals of the battery. Figure 2 shows the direction of the electric field $\vec{E}$ between plates 1 and 2 of the capacitor. Which of the following correctly indicates whether Plate 1 or Plate 2 contains positive charge and indicates the correct sign, if any, of the potential difference $\Delta V_{XY} = V_X - V_Y$?

	Plate Containing Positive Charge	Sign of ΔV_{XY}
A	1	+
B	1	−
C	2	None
D	2	+
E	2	−

(A) A
(B) B
(C) C
(D) D
(E) E

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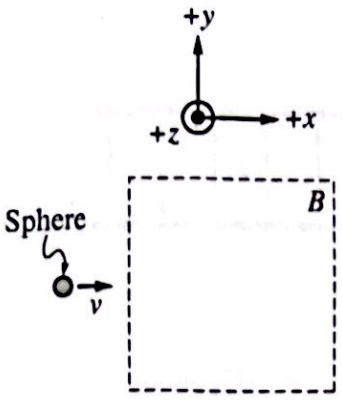

Figure 1

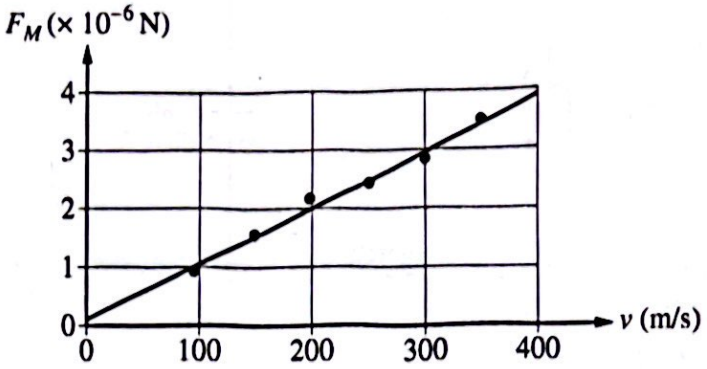

Figure 2

19. A sphere of charge $+2.0 \times 10^{-8}$ C is launched with different speeds v in the $+x$-direction toward a region of external uniform magnetic field of magnitude B represented by the dashed square in Figure 1. The magnetic field is perpendicular to the velocity of the sphere but not shown. The resulting magnetic force exerted on the sphere has magnitude F_M and is directed in the $+y$-direction immediately after the sphere enters the region. The trend line in Figure 2 represents the best-fit line to the graph of F_M as a function of v immediately after the sphere enters the region. Which of the following best approximates B and indicates the direction of the magnetic field?

	B (T)	Direction
A	0.5	$+z$
B	0.5	$-z$
C	1.0	$+z$
D	1.0	$-z$
E	1.0	$+y$

(A) A
(B) B
(C) C
(D) D
(E) E

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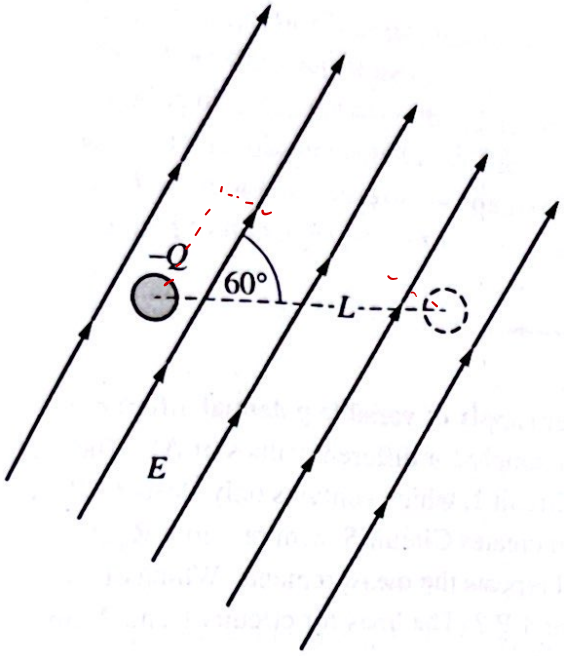

20. An external force moves a sphere with negative charge $-Q$ at a constant speed through a uniform electric field of magnitude E that is directed 60° above the horizontal, as shown. Which of the following represents the work done by the external force to move the sphere a horizontal distance L ?

(A) $-\frac{\sqrt{3}}{2}QEL$

(B) $-QEL$

(C) $-\frac{1}{2}QEL$

(D) $\frac{1}{2}QEL$

(E) $\frac{\sqrt{3}}{2}QEL$

21. Spheres 1, 2, and 3 each have net positive charge $+Q$ and radius r. Sphere 1 is a thin conducting shell. Sphere 2 is a solid conductor. Sphere 3 is a solid nonconductor with uniform charge distribution throughout its volume. Each sphere is isolated from the other spheres. The magnitudes of the electric fields at a distance $\frac{r}{5}$ from the centers of the spheres are E_1, E_2, and E_3 for spheres 1, 2, and 3, respectively. Which of the following correctly compares E_1, E_2, and E_3 ?

(A) $E_1 = E_2 = E_3$

(B) $(E_1 = E_2) < E_3$

(C) $E_1 < (E_2 = E_3)$

(D) $E_1 < E_2 < E_3$

(E) $E_1 > (E_2 = E_3)$

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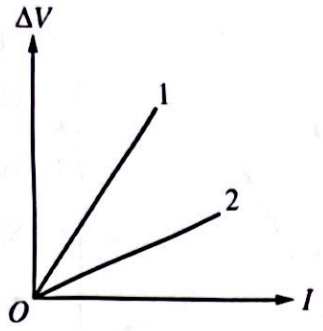
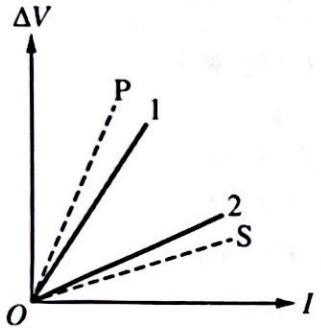


22. A student connects resistors R_1 and R_2 separately to a power supply of variable potential difference ΔV. For each circuit, the student measures the current I in the power supply for different values of ΔV. The student then creates the graph shown of ΔV as a function of I for Circuit 1, which contains only Resistor R_1, and Circuit 2, which contains only Resistor R_2. The student then creates Circuit S with resistors R_1 and R_2 in series and Circuit P with resistors R_1 and R_2 in parallel and repeats the measurements. Which of the following graphs best represents ΔV as a function of I for circuits S and P ? (The lines for circuits 1 and 2 are included for reference.)

(A)


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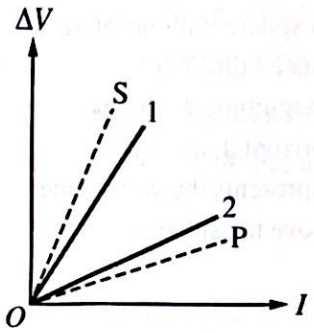

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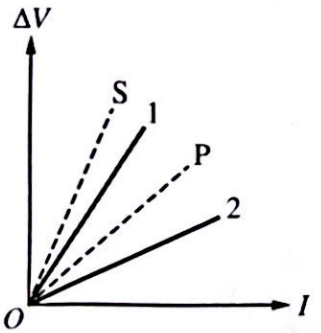

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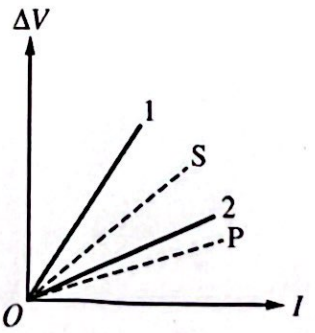
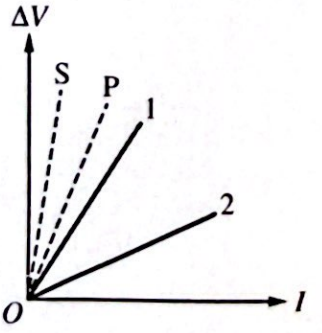


(E)


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23. A parallel plate capacitor is connected to a battery for a long time and stores 5 J of energy. The separation distance between the plates is d. The battery is then disconnected from the capacitor. Which of the following is equal to the work required to move the plates to separation distance $2d$?

(A) -10 J
(B) -5 J
(C) 0 J
(D) $+5$ J
(E) $+10$ J

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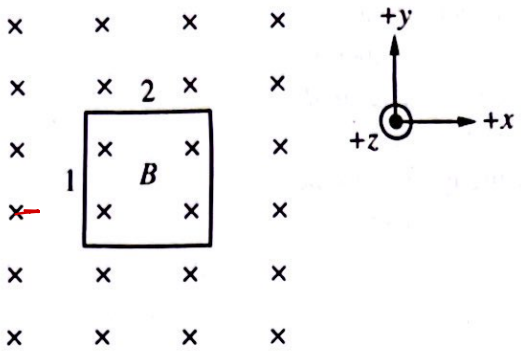

24. A conducting loop with labeled sides 1 and 2 is held at rest in a region of an external uniform magnetic field of magnitude B that is directed in the $-z$-direction, as shown. Starting at time $t = 0$, B is increased linearly as a function of t. The magnetic forces exerted on sides 1 and 2 are $\vec{F}_1$ and $\vec{F}_2$, respectively. Which of the following best describes $\vec{F}_1$ and $\vec{F}_2$ immediately after $t = 0$?

	$\vec{F}_1$	$\vec{F}_2$
A	In the $+x$-direction and increasing	In the $-y$-direction and increasing
B	In the $+y$-direction and increasing	In the $+x$-direction and increasing
C	In the $-y$-direction and increasing	In the $-x$-direction and increasing
D	In the $+x$-direction and decreasing	In the $-y$-direction and decreasing
E	In the $-y$-direction and decreasing	In the $-x$-direction and decreasing

(A) A
(B) B
(C) C
(D) D
(E) E

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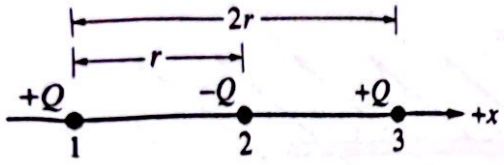

Figure 1

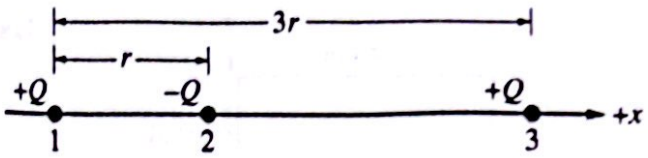

Figure 2

25. Small spheres 1, 2, and 3 of charges $+Q$, $-Q$, and $+Q$, respectively, are held at rest on the x-axis. When the centers of spheres 1 and 2 are distance r apart and the centers of spheres 1 and 3 are $2r$ apart, as shown in Figure 1, the magnitude of the net electric force exerted on Sphere 1 is F. Sphere 3 is then moved in the $+x$-direction and held at rest so that the centers of spheres 1 and 3 are $3r$ apart, as shown in Figure 2. Which of the following is equal to the magnitude of the resulting net electric force exerted on Sphere 1 in Figure 2 ?

(A) $\frac{2}{3}F$

(B) $\frac{8}{9}F$

(C) F

(D) $\frac{32}{27}F$

(E) $3F$

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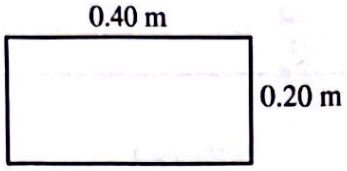

Figure 1. Top View

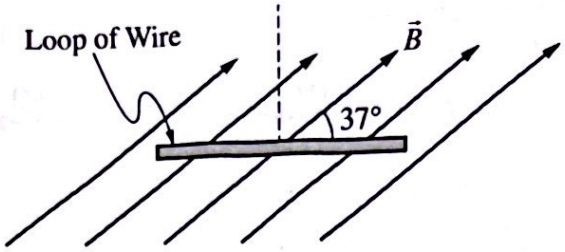

Figure 2. Edge View

26. A rectangular loop of wire has side lengths of 0.40 m and 0.20 m, as shown in Figure 1. The loop is placed in a uniform magnetic field $\vec{B}$ of magnitude 10.0 mT that is directed at an angle of 37° to the plane of and through the loop, as shown in Figure 2. What is the magnetic flux through the loop?

(A) $0.48\ \text{mT} \cdot \text{m}^2$

(B) $0.60\ \text{mT} \cdot \text{m}^2$

(C) $0.64\ \text{mT} \cdot \text{m}^2$

(D) $0.80\ \text{mT} \cdot \text{m}^2$

(E) $1.39\ \text{mT} \cdot \text{m}^2$

27. Parallel-plate capacitors 1 and 2 each have identical plate areas. Capacitor 1 is filled with air, has a plate separation of 4.0×10^{-3} m, and has a capacitance of 2.0 μF. Capacitor 2 is filled with a material of dielectric constant $\kappa = 3$, and the plates are separated by a distance of 2.0×10^{-3} m. Which of the following is equal to the capacitance of Capacitor 2 ?

(A) 0.20 μF
(B) 0.30 μF
(C) 2.0 μF
(D) 6.0 μF
(E) 12 μF

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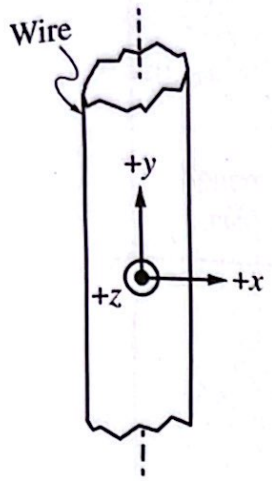

Figure 1

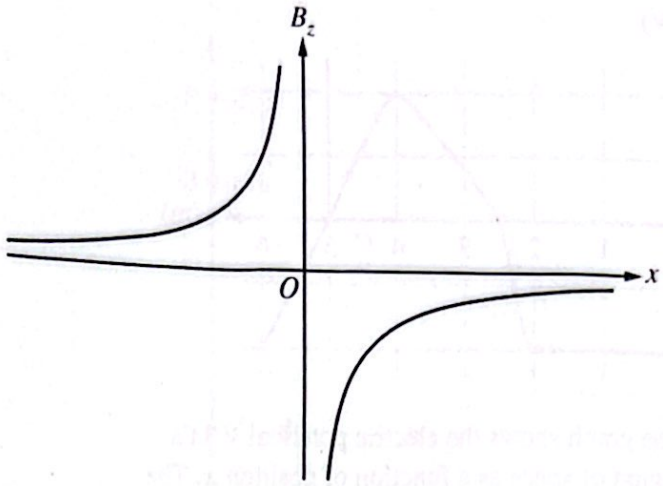

Figure 2

28. A long, straight wire is parallel to the y-axis, as shown in Figure 1. The wire carries current I. A graph of the z-component B_z of the magnetic field as a function of position x outside of the wire is shown in Figure 2. Which of the following correctly indicates the direction of I and provides a valid justification?

(A) $+y$-direction, because B_z approaches zero as x approaches $\pm\infty$.

(B) $+y$-direction, because B_z is negative for positive values of x.

(C) $-y$-direction, because B_z approaches zero as x approaches $\pm\infty$.

(D) $-y$-direction, because B_z is negative for positive values of x.

(E) $-y$-direction, because B_z has both positive and negative values.

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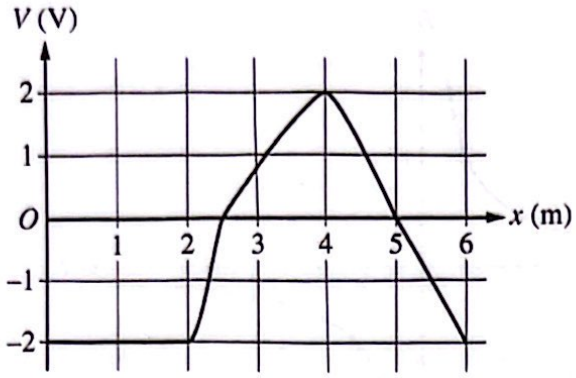

29. The graph shows the electric potential V in a region of space as a function of position x. The electric field is parallel to the x-axis. Which of the following claims about the electric field is correct?

(A) The electric field is zero at $x = 1.0$ m.

(B) The electric field changes direction between $x = 2.0$ m and $x = 4.0$ m.

(C) The magnitude of the electric field is a maximum at $x = 4.0$ m.

(D) The electric field is zero at $x = 5.0$ m.

(E) The magnitude of the electric field is a maximum at $x = 5.0$ m.

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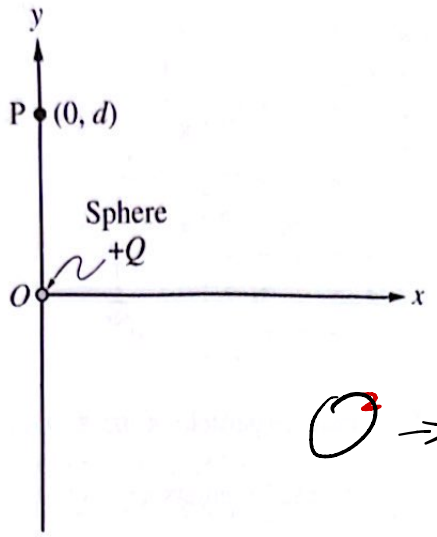

Figure 1

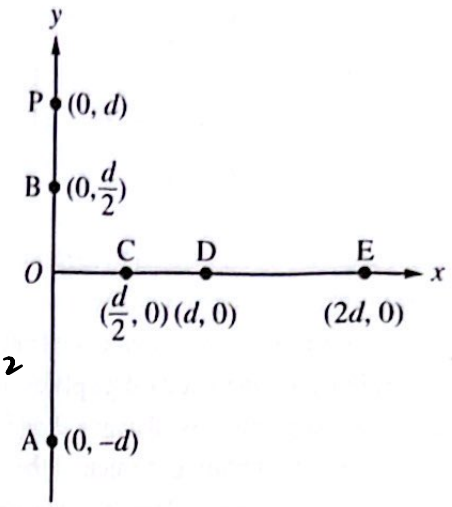

Figure 2

30. An isolated sphere with positive charge $+Q$ is positioned at the origin of an xy-coordinate system. Point P has coordinates $(0, d)$, as shown in Figure 1. The magnitude of the electric field at Point P due to the sphere is E_0. Additional points A, B, C, D, and E and their respective coordinates are labeled in Figure 2. At which of the following labeled points should the sphere be placed so the magnitude of the electric field at Point P is $\frac{E_0}{2}$?

(A) A
(B) B
(C) C
(D) D
(E) E

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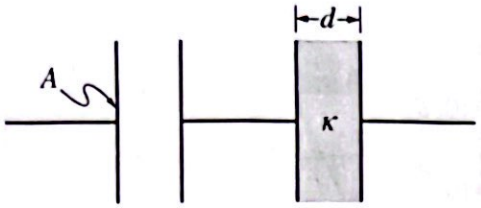


31. An air-filled capacitor is connected in series to another capacitor that contains a dielectric material of dielectric constant κ that completely fills the region between the plates. The plates of each capacitor are separated by distance d and each plate has area A, as shown. Which of the following is equal to the equivalent capacitance of the capacitors?

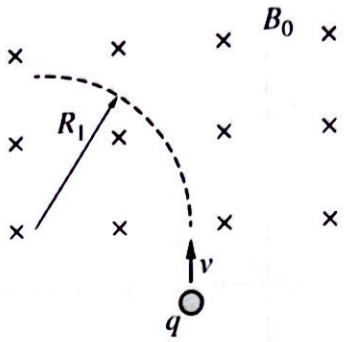
(A) $\frac{1}{2}\frac{\kappa\varepsilon_0 A}{d}$

(B) $\frac{\varepsilon_0 A}{d}(1+\kappa)$

(C) $\frac{1}{2}\frac{\varepsilon_0 A}{d}\left(\frac{\kappa}{\kappa+1}\right)$

(D) $\frac{\varepsilon_0 A}{d}\left(\frac{\kappa}{\kappa+1}\right)$

(E) $\frac{\varepsilon_0 A}{d}\left(\frac{\kappa+1}{\kappa}\right)$



32. A charged particle of mass m, charge q, and with speed v enters a region of uniform magnetic field of magnitude B_0 that is directed into the page. The particle travels in a circular path of radius R_1, as shown. A second particle of mass m, charge $3q$, and with speed $\frac{v}{2}$ then enters the field along the same direction as the first particle. The second particle moves in a circular path of radius R_2. Which of the following correctly relates R_2 and R_1 ?

(A) $R_2 = 6R_1$

(B) $R_2 = \frac{3}{2}R_1$

(C) $R_2 = \frac{2}{3}R_1$

(D) $R_2 = \frac{1}{5}R_1$

(E) $R_2 = \frac{1}{6}R_1$

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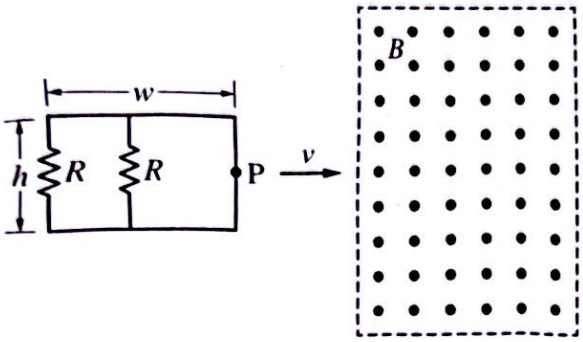

33. Two resistors, each of resistance R, are connected to a rectangular loop of wire that has height h and width w. The loop moves with speed v toward a region of external uniform magnetic field of magnitude B that is directed out of the page, as shown. Point P is located on the side of the loop that is closest to the magnetic field, as shown. A magnetic force of magnitude F_M is exerted on the loop immediately after Point P enters the magnetic field. Which of the following expressions is equal to F_M ?

(A) $\frac{1}{2}\frac{B^2w^2v}{R}$

(B) $2\frac{B^2w^2v}{R}$

(C) $\frac{1}{2}\frac{B^2h^2v}{R}$

(D) $2\frac{B^2h^2v}{R}$

(E) $2\frac{B^2whv}{R}$

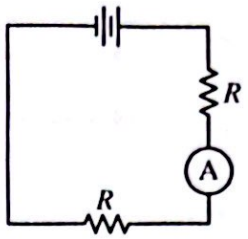

Figure 1

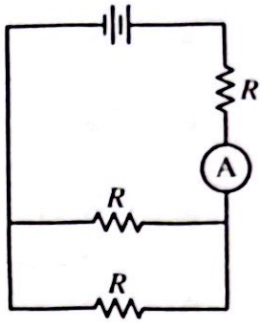

Figure 2

34. The circuit in Figure 1 consists of an ideal battery, two resistors, each of resistance R, and ammeter A that measures current I_1. The circuit in Figure 2 consists of the battery, three resistors, each of resistance R, and ammeter A that measures current I_2. Which of the following is equal to I_2 ?

(A) $\frac{1}{3}I_1$

(B) $\frac{1}{2}I_1$

(C) $\frac{3}{4}I_1$

(D) I_1

(E) $\frac{4}{3}I_1$

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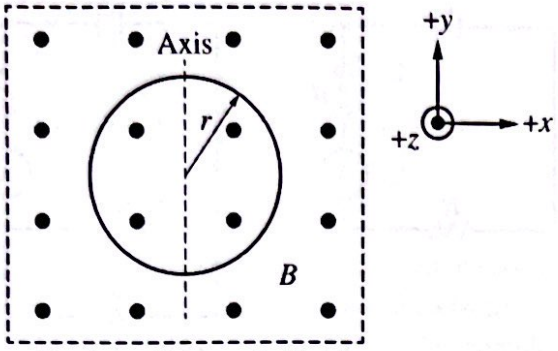

Region of Magnetic Field

35. A circular conducting loop of radius r rotates with angular frequency ω about a central axis that is parallel to the y-axis. The loop is in a region of external uniform magnetic field of magnitude B that is in the $+z$-direction, as shown. Which of the following expressions is equal to the maximum induced emf in the loop?

(A) $\frac{1}{4}B\pi r^2\omega$

(B) $\frac{1}{2}B\pi r^2\omega$

(C) $B\pi r^2\omega$

(D) $2B\pi r^2\omega$

(E) $4B\pi r^2\omega$

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Begin your response to **QUESTION 1** on this page.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

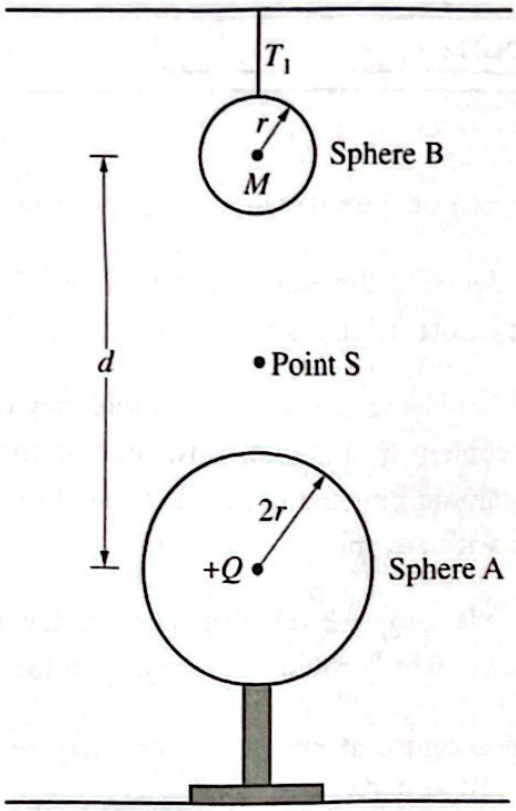

Figure 1

Note: Figure not drawn to scale.

1. Conducting Sphere A of radius $2r$ and positive net charge $+Q$ is attached to an insulated stand. Neutral conducting Sphere B of radius r and mass M is suspended by an insulated string so that Sphere B is at rest directly above Sphere A. The tension force exerted on Sphere B by the string has magnitude T_1. The centers of the spheres are separated by distance d, as shown in Figure 1, where $r \ll d$. Point S is midway between the centers of the spheres.

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(a) The dots in Figure 2 represent the center of Sphere A, Point S, and the center of Sphere B. In Figure 2, **draw** a single arrow on each dot to represent the direction of the net electric field at the location of the dot. Each arrow must start on and point away from the dot. If the net electric field is zero, write "$E = 0$" next to the dot.

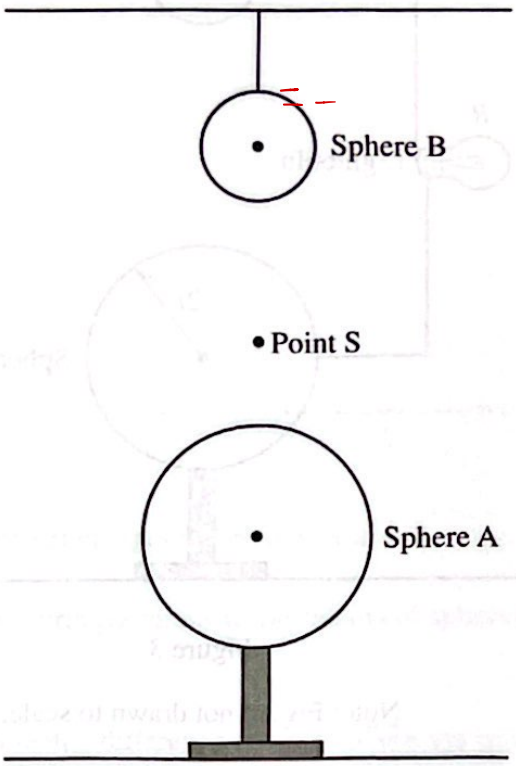

Figure 2

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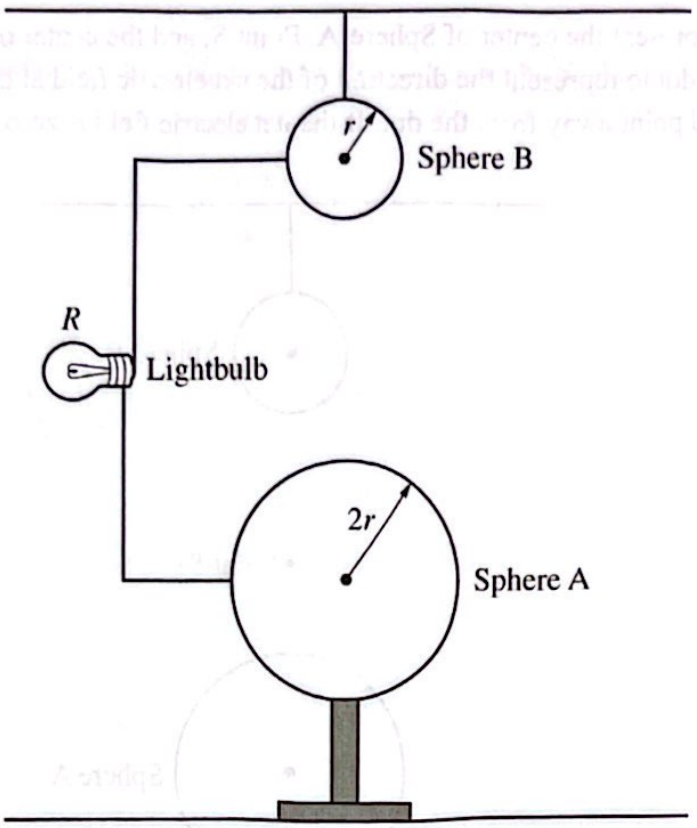

Figure 3

Note: Figure not drawn to scale.

At time $t = 0$, a lightbulb of resistance R and conducting wires are connected between the spheres, as shown in Figure 3. The lightbulb remains connected between the spheres for a long time t_1.

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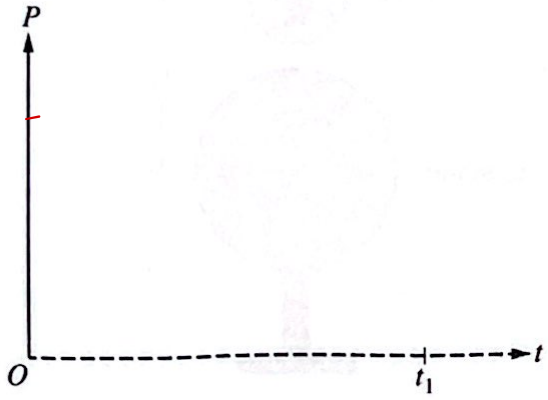
(b) Consider time interval $0 \le t \le t_1$.

i. On the axes shown, **sketch** a graph of the electrical power P dissipated by the lightbulb as a function of t during $0 \le t \le t_1$. Explicitly label the vertical intercept in terms of r, Q, M, t_1, d, R, and physical constants, as appropriate.



ii. $Q_A(t)$ and $Q_B(t)$ represent the net charges on spheres A and B, respectively, as functions of t.

$V_A(t)$ and $V_B(t)$ represent the electric potentials at the centers of spheres A and B, respectively, as functions of t.

Derive an expression for the potential difference $\Delta V(t)$ between the spheres as a function of t,

where $\Delta V(t) = V_A(t) - V_B(t)$, to show that $\Delta V(t) = k\left(\frac{3Q_A(t)}{2r} - \frac{Q}{r}\right)$, where $k = \frac{1}{4\pi\varepsilon_0}$.

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(c) After t_1, the lightbulb and wires are removed without disturbing the charges on spheres A and B. Q_A is the net charge on Sphere A, and Q_B is the net charge on Sphere B.

i. **Calculate** the ratio $\frac{Q_B}{Q_A}$.

ii. The magnitude of the net electric field at Point S is now E_{new}. **Derive** an expression for E_{new} in terms of r, Q, M, t_1, d, R, and physical constants, as appropriate.

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Continue your response to **QUESTION 1** on this page.

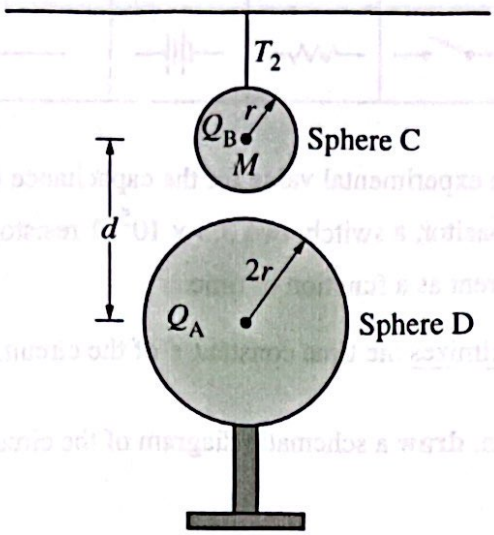

Figure 4

(d) Sphere B is replaced with nonconducting Sphere C of the same radius r, the same mass M, and the same charge Q_B as those of Sphere B in part (c). Sphere A is replaced by nonconducting Sphere D of the same radius $2r$ and the same charge Q_A as those of Sphere A in part (c). The charge is distributed uniformly throughout the volume of each sphere. Sphere D is moved close to Sphere C. Sphere D is then held at rest directly underneath Sphere C such that $r \ll d$ is no longer assumed, as shown in Figure 4.

i. **Describe** the changes, if any, in the distribution of charge on Sphere D as Sphere D is moved closer to Sphere C.

ii. The tension force exerted on Sphere C by the string has magnitude T_2 after Sphere D is brought to rest. **Indicate** whether T_2 is greater than, less than, or equal to T_1, the magnitude of the tension force exerted on Sphere B in Figure 1. Briefly **justify** your answer.

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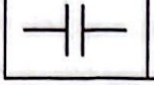
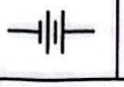
Begin your response to **QUESTION 2** on this page.

Capacitor	Switch	Resistor	Battery	Connecting Wires	Ammeter
					

2. Students are asked to determine an experimental value for the capacitance C_{EXP} of an initially uncharged capacitor. The students use the capacitor, a switch, two $3.3 \times 10^5\ \Omega$ resistors, a 9.0 V battery, connecting wires, and an ammeter that measures current as a function of time.

(a) A circuit is assembled that maximizes the time constant τ of the circuit. The circuit can be used to determine C_{EXP}.

i. Using the symbols shown, **draw** a schematic diagram of the circuit that includes both resistors.

ii. **Describe** a procedure for collecting data as the capacitor is being charged that would allow the students to determine C_{EXP}. Provide enough detail so that another student could replicate the experiment.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Continue your response to **QUESTION 2** on this page.

(b) It is determined that $\tau = 3.1$ s. **Calculate** C_{EXP}.

At time $t = 3.1$ s, the charge on the positive plate of the capacitor was Q_1. After the experiment, it is determined that the ammeter that was used was nonideal. The experiment is repeated with an ideal ammeter. At time $t = 3.1$ s, the charge on the positive plate of the capacitor is Q_2.

(c) **Indicate** whether Q_2 is greater than, less than, or equal to Q_1.

_____ $Q_2 > Q_1$ _____ $Q_2 < Q_1$ _____ $Q_2 = Q_1$

Justify your answer.

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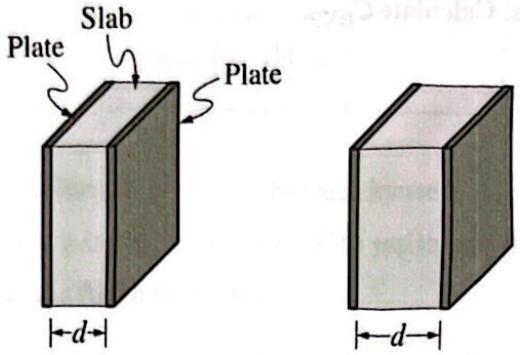
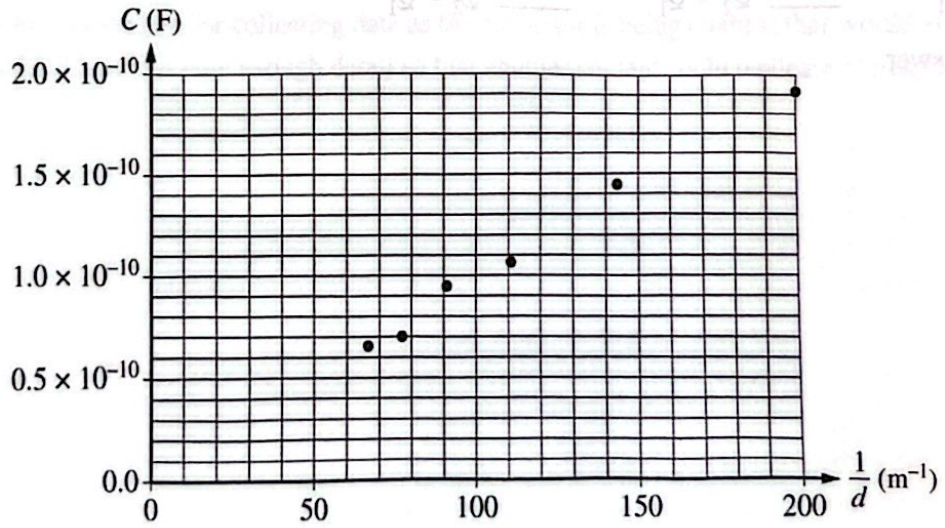


Trial 1 Trial 2

In a different experiment, students are asked to determine an experimental value for the dielectric constant κ of a type of plastic. The students have two parallel conducting plates and six different slabs made of the plastic. The slabs differ in thickness. For each trial, one slab is inserted between the plates to make a capacitor. The plate separation d is adjusted so that d is the slab thickness. The capacitance C of the capacitor is then determined. The capacitor for two trials is shown.

Collected data are used to plot the following graph of C as a function of $\frac{1}{d}$.



(d)

i. **Draw** the best-fit line for the data.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

Continue your response to **QUESTION 2** on this page.

ii. Using the best-fit line, **calculate** the experimental value for κ. Each plate has area $5.0 \times 10^{-2}\,\mathrm{m}^2$.

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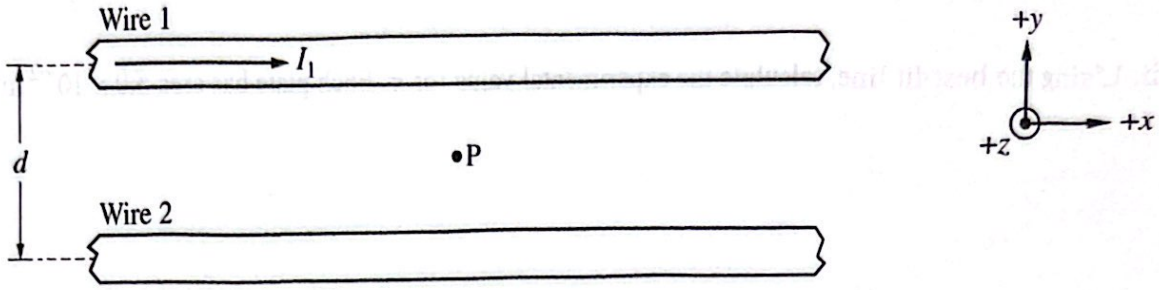

Figure 1. Side View

3. Long parallel wires 1 and 2 are positioned horizontally and are separated by vertical distance d. Wire 1 is fixed in place and has current I_1 in the $+x$-direction. Wire 2 is initially held in place but can be released. Each wire has linear mass density λ and resistance per length $\frac{R}{\ell}$. Point P is midway between wires 1 and 2, as shown in the side view in Figure 1.

(a) **Draw** your answers to parts (a)(i) and (a)(ii) on the cross-sectional view shown in Figure 2.

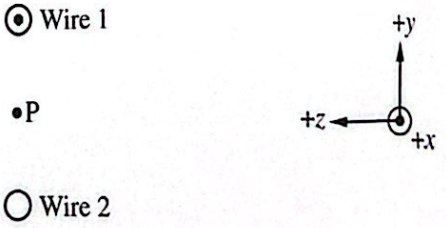

Figure 2. Cross-Sectional View

i. On Figure 2, **draw** an appropriate Amperian loop that can be used to determine the magnitude of the magnetic field at the location of Wire 2 due to Wire 1.

ii. On Figure 2, **draw** an arrow starting on, and pointing away from, the dot representing Point P to show the direction of the magnetic field at Point P due to Wire 1.

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Continue your response to **QUESTION 3** on this page.

iii. Using Ampere's law, **derive** an expression for the magnitude of the magnetic field B at the location of Wire 2 due to Wire 1. Express your answer in terms of d, I_1, and physical constants, as appropriate.

Consider the situation in which Wire 2 has current I_2 and there is a gravitational force exerted on Wire 2 in the $-y$-direction. Wire 2 is then released but remains at rest.

(b) **Derive** an expression for I_2 in terms of d, I_1, λ, and physical constants, as appropriate.

Consider the situation in which Wire 1 has these properties: $I_1 = 5.0\,\text{A}$ and $\frac{R}{\ell} = 0.12\ \frac{\Omega}{\text{m}}$.

(c) **Calculate** the total energy ΔE dissipated in a section of Wire 1 of length $L = 0.40\,\text{m}$ for a time interval of $\Delta t = 3.0\ \text{s}$.

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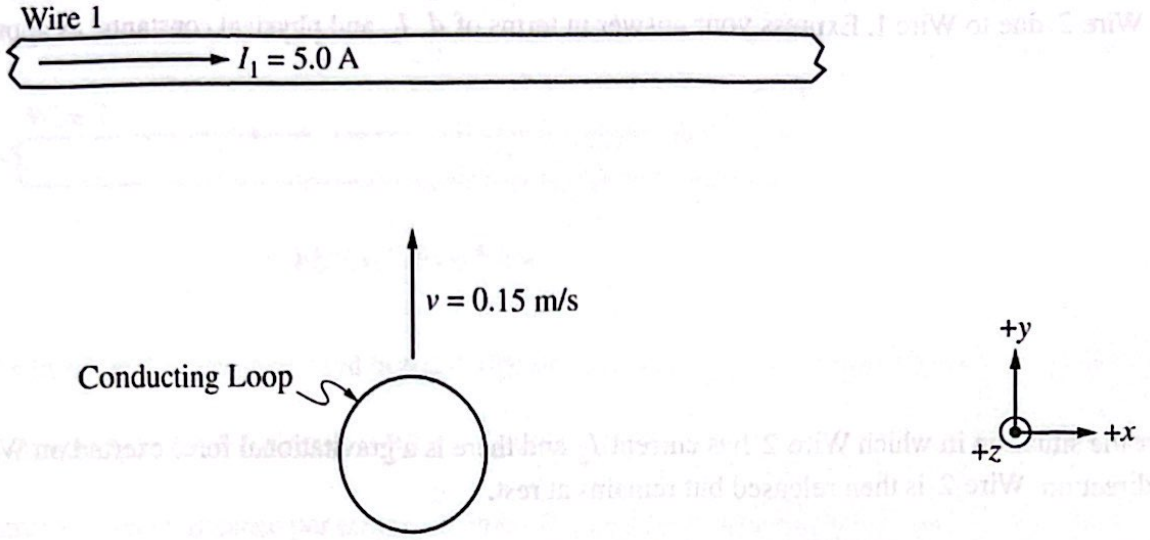

Figure 3. Side View

Wire 2 is removed. Wire 1 has current $I_1 = 5.0$ A in the $+x$-direction. A circular conducting loop is placed directly below Wire 1. The loop is moved toward Wire 1 in the $+y$-direction at constant speed $v = 0.15$ m/s, as shown in the side view in Figure 3.

(d) **Indicate** whether the direction of the induced current in the loop is clockwise, counterclockwise, or zero as the loop is moved toward Wire 1.

_____ Clockwise _____ Counterclockwise _____ Zero

Briefly **justify** your answer.

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Continue your response to **QUESTION 3** on this page.

The rate of change of the magnitude of the magnetic field at the center of the moving loop due to Wire 1 is $\frac{dB}{dt}$.

The distance between the center of Wire 1 and the center of the moving loop is h, which varies as a function of time.

(e) **Derive** the relationship between the magnitude of $\frac{dB}{dt}$ and h to verify that $\left|\frac{dB}{dt}\right| = \left(1.5 \times 10^{-7} \frac{\text{T}\cdot\text{m}^2}{\text{s}}\right)\left(\frac{1}{h^2}\right)$.

Write your expression in terms of numerical values and h only.

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

AP Physics C Electricity and Magnetism

2024

APPENDIX - ANSWERS

Matching multiple-choice answers and free-response scoring follow.

Supplied handwritten solutions; nonofficial.

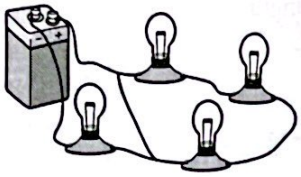
PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION I

Time—45 minutes

35 Questions

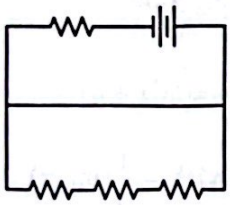
Directions: Each of the questions or incomplete statements below is followed by five suggested answers or completions. Select the one that is best in each case and then fill in the corresponding circle on the answer sheet.



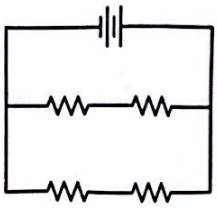
A

1. The circuit shown consists of four identical lightbulbs and a battery. Which of the following diagrams best represents the circuit?

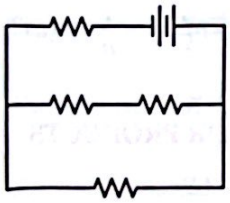
(A)



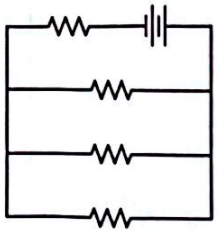
(B)



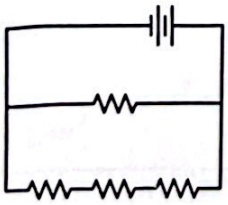
(C)



(D)



(E)



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B

2. A solid, nonconducting sphere of radius R has charge Q distributed uniformly throughout its volume. Which of the following is equal to the magnitude of the electric field at distance $\frac{R}{2}$ from the center of the sphere?

(A) Zero

(B) $\frac{1}{2}\frac{kQ}{R^2}$

(C) $\frac{kQ}{R^2}$

(D) $2\frac{kQ}{R^2}$

(E) $4\frac{kQ}{R^2}$

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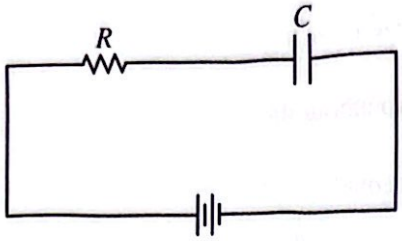
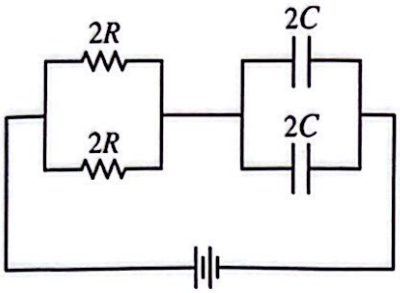


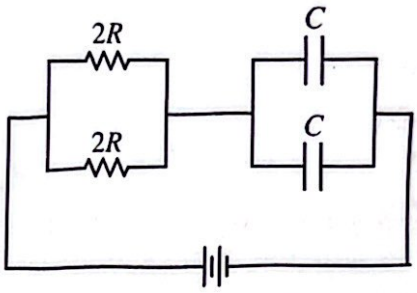
C

3. A resistor of resistance R and a capacitor of capacitance C are connected to an ideal battery, as shown. This circuit has time constant τ. Which of the following circuits also has time constant τ ?

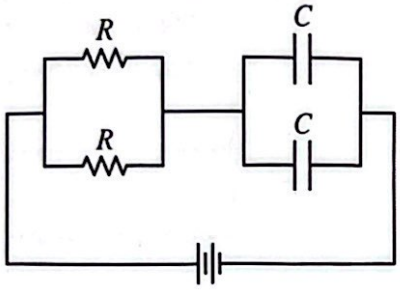
(A)



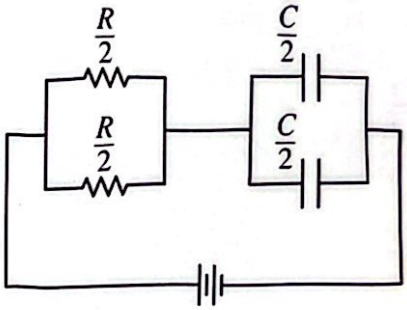
(B)



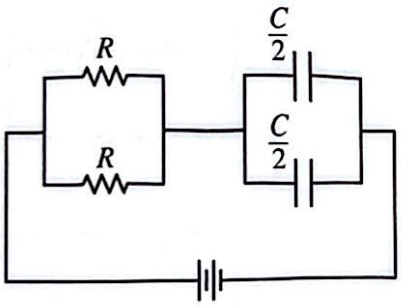
(C)



(D)



(E)



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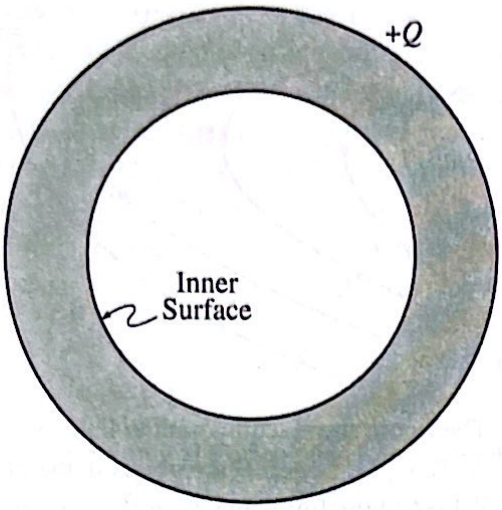

E

4. A conducting spherical shell has a net positive charge $+Q$, as shown. Which of the following indicates the net charge on the inner surface of the shell and the net charge inside of the shell (shaded region)?

	Inner Surface	Inside Shell
A	Zero	Zero
B	$-Q$	$-Q$
C	Zero	$-Q$
D	$-Q$	Zero
E	$+Q$	Zero

(A) A
(B) B
(C) C
(D) D
(E) E

C

5. An air-filled parallel plate capacitor stores 8.85×10^{-10} J of energy when the potential difference across the plates is 20.0 V and the plates are separated by a distance of 1.00×10^{-3} m. What is the area of each plate of the capacitor?

(A) $1.25 \times 10^{-4}\ m^2$
(B) $2.50 \times 10^{-4}\ m^2$
(C) $5.00 \times 10^{-4}\ m^2$
(D) $1.00 \times 10^{-3}\ m^2$
(E) $1.50 \times 10^{-3}\ m^2$

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d (m)	a (m/s^2)
0.1	90
0.2	90
0.3	90
0.4	90

B

6. Identical nonconducting spheres 1 and 2, each with equal amounts of positive net charge, are initially held at rest near each other so that the force between the spheres is nonnegligible. Sphere 2 is then released while Sphere 1 remains at rest. The table shows the acceleration a of Sphere 2 along a straight path as a function of the separation distance d between the centers of the spheres. Which of the following claims can be supported by data in the table?

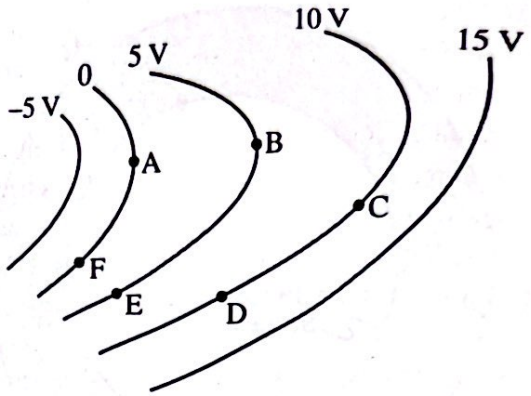
(A) The spheres are isolated from other objects and fields.

(B) The spheres are in a region of external uniform electric field only.

(C) The spheres are in a region of external nonuniform electric field only.

(D) The spheres are in a region of external uniform magnetic field only.

(E) The spheres are in a region of external uniform electric and magnetic fields only.



D

7. The figure shows equipotential lines in a plane. A, B, C, D, E, and F are points in the plane. Which of the following claims is correct?

(A) The electric field at Point A is directed toward Point B.

(B) The electric field at Point C is directed toward Point D.

(C) The magnitude of the electric field is greater at Point B than at Point E.

(D) The magnitude of the average electric field is greater between points F and E than between points A and B.

(E) The magnitude of the average electric field is greater between points B and C than between points E and D.

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A

8. Nonconducting Sphere 1 has a positive net charge and is held at rest. Conducting Sphere 2, which is near Sphere 1, has no net charge and is given an initial speed moving away from Sphere 1. All forces other than the electric force are negligible. Which of the following claims is correct for when Sphere 2 moves away from Sphere 1 ?

(A) The acceleration of Sphere 2 is toward Sphere 1.

(B) The momentum of Sphere 2 is increasing.

(C) The net force exerted on Sphere 2 is zero.

(D) The kinetic energy of Sphere 2 is constant.

(E) The electric potential energy of the Sphere 1-Sphere 2 system is decreasing.

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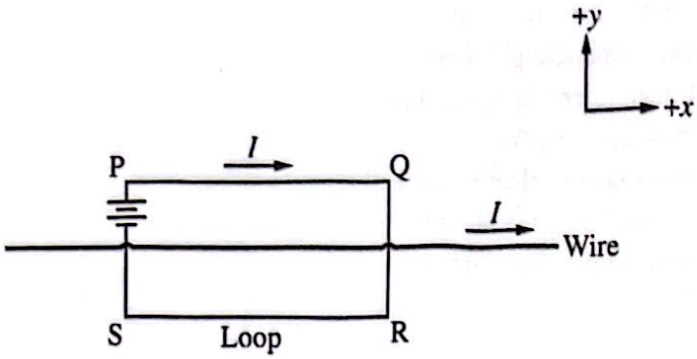

Top View

D

9. A conducting loop is beneath a long, straight, insulated wire. The loop and the wire are in the plane of the page. At the instant shown, the current in the wire and the current in Segment PQ of the loop are I in the $+x$-direction. Which of the following indicates the direction, if any, of the net magnetic force $\vec{F}_M$ exerted on Segment RS of the loop and provides a valid justification?

(A) $+y$, because the currents in Segment RS and the wire create magnetic fields that produce an attractive force.

(B) $+y$, because the currents in segments RS and PQ create magnetic fields that produce an attractive force.

(C) No direction, because the currents in Segment RS and the wire are constant.

(D) $-y$, because the currents in Segment RS and the wire create magnetic fields that produce a repulsive force.

(E) $-y$, because in the region between Segment RS and the wire, the magnetic field from Segment RS is directed in the opposite direction from the magnetic field from the wire.

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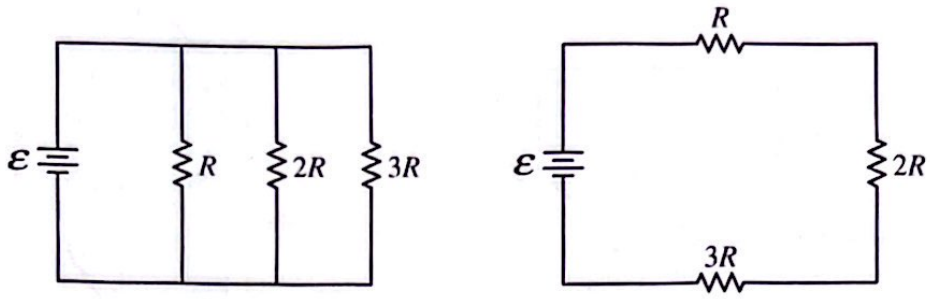

Circuit 1 Circuit 2

E

10. Circuits 1 and 2 shown each consist of an ideal battery of emf $\mathcal{E}$ and resistors with resistances of R, $2R$, and $3R$. The rates at which thermal energy is dissipated by the resistor with resistance R in circuits 1 and 2 are P_1 and P_2, respectively. Which of the following correctly compares P_2 to P_1 and provides a valid justification?

(A) $P_2 = P_1$, because the resistance of the resistor is the same in each circuit.

(B) $P_2 > P_1$, because the equivalent resistance of Circuit 1 is smaller than the equivalent resistance of Circuit 2.

(C) $P_2 > P_1$, because the current in the resistor for Circuit 1 is smaller than the current in the resistor for Circuit 2.

(D) $P_2 < P_1$, because the equivalent resistance of Circuit 1 is larger than the equivalent resistance of Circuit 2.

(E) $P_2 < P_1$, because the current in the resistor for Circuit 1 is larger than the current in the resistor for Circuit 2.

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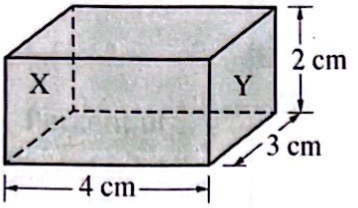
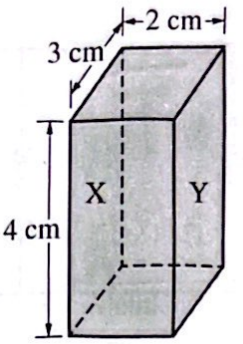


Piece 1



Piece 2

A

11. The resistance between the left side X and right side Y of two separate pieces, 1 and 2, of the same resistive material is measured. The dimensions of the pieces are shown. The resistances of pieces 1 and 2 are R_1 and R_2, respectively. Which of the following correctly relates R_2 and R_1 ?

(A) $R_2 = \frac{1}{4} R_1$

(B) $R_2 = \frac{1}{2} R_1$

(C) $R_2 = R_1$

(D) $R_2 = 2R_1$

(E) $R_2 = 4R_1$

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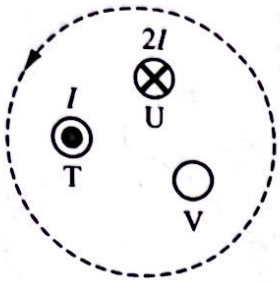
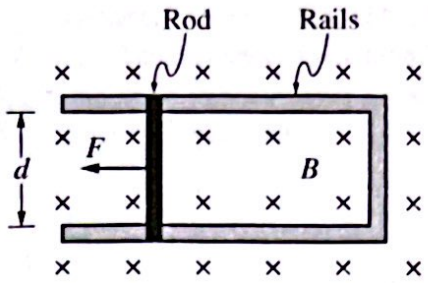


D

12. Wires T, U, and V each carry current and are oriented perpendicular to the plane of the page. Wire T carries current I out of the page. Wire U carries current $2I$ into the page. The value and direction of the current in Wire V are unknown. The line integral of the magnetic field $\vec{B}$ taken counterclockwise around the dashed path shown, $\oint \vec{B} \cdot d\vec{\ell}$, is equal to $2\mu_0 I$. Which of the following indicates the value and direction of the current in Wire V ?

(A) I out of the page
(B) $2I$ out of the page
(C) $2I$ into the page
(D) $3I$ out of the page
(E) $3I$ into the page



Top View

D

13. A metal rod of mass m and resistance R is in contact with the rails of a U-shaped conductor of width d and negligible resistance. The rails rest on a horizontal surface. The coefficient of kinetic friction between the rod and rails is fi. The rod and rails are in a uniform magnetic field of magnitude B that is directed into the surface, as shown. An external force of magnitude F is exerted on the rod so that the rod has constant speed v to the left. Which of the following expressions is equal to the power delivered to the rod by the force?

(A) $\frac{Bdv}{R}$
(B) $\frac{Bdv}{R} + \mu mgv$
(C) $\frac{B^2d^2v^2}{R}$
(D) $\frac{B^2d^2v^2}{R} + \mu mgv$
(E) $\frac{B^2d^2v^2}{R} - \mu mgv$

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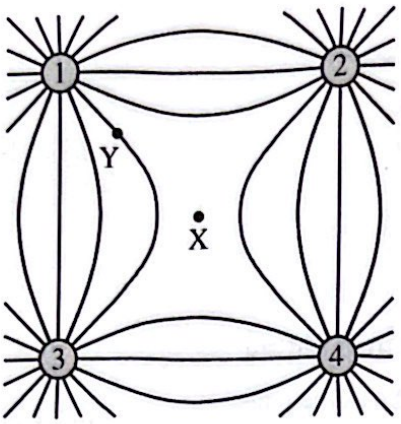

Point	Direction of $\vec{E}$
X	No direction
Y	Toward Sphere 4

C

14. The figure shown represents the electric field resulting from the square arrangement of charged spheres 1, 2, 3, and 4. The directions, if any, of the electric field $\vec{E}$ at points X and Y are given in the table. Which of the following identifies the signs of the charges on the spheres, consistent with the collected data?

	Positively Charged	Negatively Charged
A	1, 2, 3, and 4	None
B	1, 2, and 3	4
C	1 and 4	2 and 3
D	2 and 3	1 and 4
E	None	1, 2, 3, and 4

(A) A
(B) B
(C) C
(D) D
(E) E

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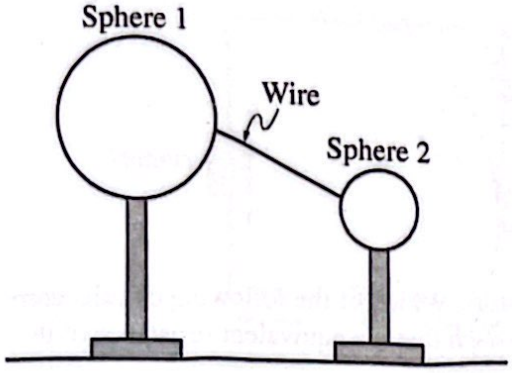
$L = \frac{\mu_0 N^2 \cdot A}{\ell} = \frac{1}{4}\frac{\mu_0 N^2 \pi d^2}{\ell}$



Note: Figure not drawn to scale.

C

15. Conducting spheres 1 and 2 are each on an insulating stand. The distance between the spheres is large compared to their diameters. Sphere 1 has a known amount of charge on it, and Sphere 2 is initially neutral. A wire is then connected between the spheres for a long time, as shown. Which of the following indicates the fewest measurable quantities necessary to calculate the electric potential at the surface of each sphere?

(A) The diameter of Sphere 1
(B) The diameter of Sphere 2
(C) The diameters of both spheres
(D) The diameters of both spheres and the length of the wire
(E) The diameters of both spheres, the length of the wire, and the heights of both stands

C

16. A long, cylindrical-shaped solenoid has length ℓ and N loops. The diameter of one loop is d. Which of the following expressions is equal to the inductance of the solenoid?

(A) $\frac{1}{4}\frac{N\mu_0\pi d^2}{\ell}$
(B) $\frac{1}{2}\frac{N\mu_0\pi d^2}{\ell}$
(C) $\frac{1}{4}\frac{N^2\mu_0\pi d^2}{\ell}$
(D) $\frac{1}{2}\frac{N^2\mu_0\pi d^2}{\ell}$
(E) $\frac{N^2\mu_0\pi d^2}{\ell}$

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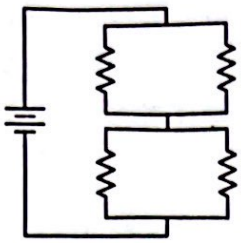
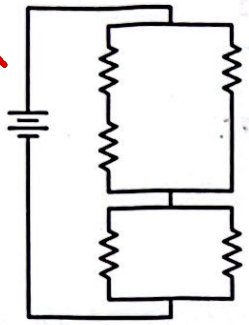


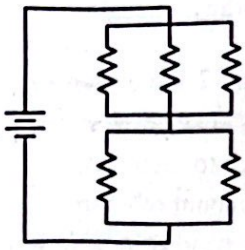
R

D 17. The circuit shown consists of a battery and four identical resistors. Which of the following circuits correctly shows how a fifth identical resistor can be added to the circuit such that the equivalent resistance of the circuit is doubled?

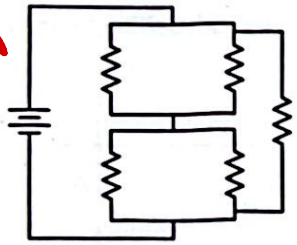
(A)



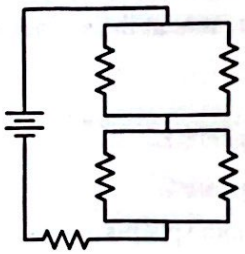
(B)



(C)

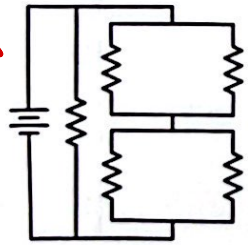


(D)



2R

(E)



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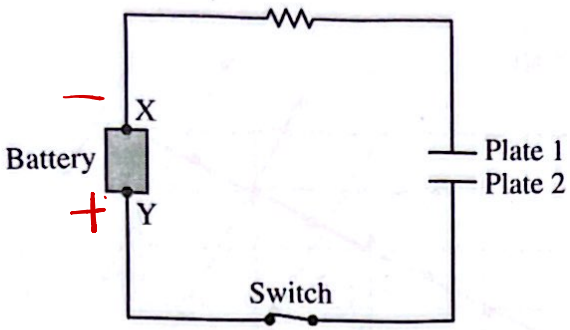

Figure 1

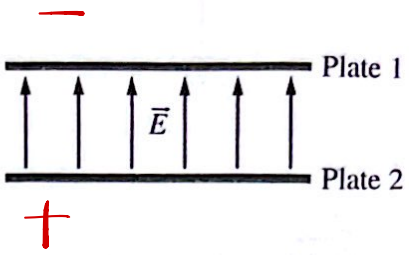

Figure 2

D

18. The circuit represented in Figure 1 consists of a battery, a resistor, a switch that has been closed for a long time, and a capacitor. Electric potentials V_X and V_Y are measured at positions X and Y, respectively, at the terminals of the battery. Figure 2 shows the direction of the electric field $\vec{E}$ between plates 1 and 2 of the capacitor. Which of the following correctly indicates whether Plate 1 or Plate 2 contains positive charge and indicates the correct sign, if any, of the potential difference $\Delta V_{XY} = V_X - V_Y$?

	Plate Containing Positive Charge	Sign of ΔV_{XY}
A	1	+
B	1	−
C	2	None
D	2	+
E	2	−

(A) A
(B) B
(C) C
(D) D
(E) E

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$F = qvB$

$qB = 10^{-8}$

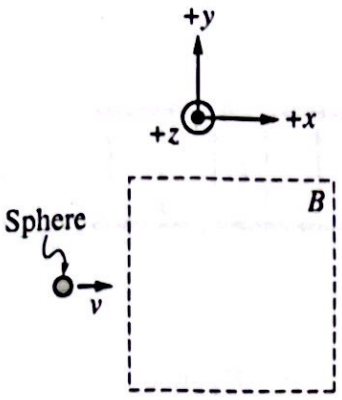

Figure 1

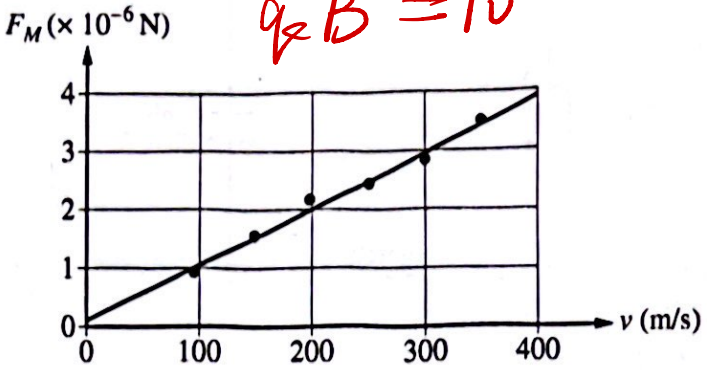

Figure 2

B

19. A sphere of charge $+2.0 \times 10^{-8}$ C is launched with different speeds v in the $+x$-direction toward a region of external uniform magnetic field of magnitude B represented by the dashed square in Figure 1. The magnetic field is perpendicular to the velocity of the sphere but not shown. The resulting magnetic force exerted on the sphere has magnitude F_M and is directed in the $+y$-direction immediately after the sphere enters the region. The trend line in Figure 2 represents the best-fit line to the graph of F_M as a function of v immediately after the sphere enters the region. Which of the following best approximates B and indicates the direction of the magnetic field?

$B = \frac{10^{-8}}{2 \times 10^{-8}} = 0.5$

	B (T)	Direction
A	0.5	$+z$
B	0.5	$-z$
C	1.0	$+z$
D	1.0	$-z$
E	1.0	$+y$

(A) A
(B) B
(C) C
(D) D
(E) E

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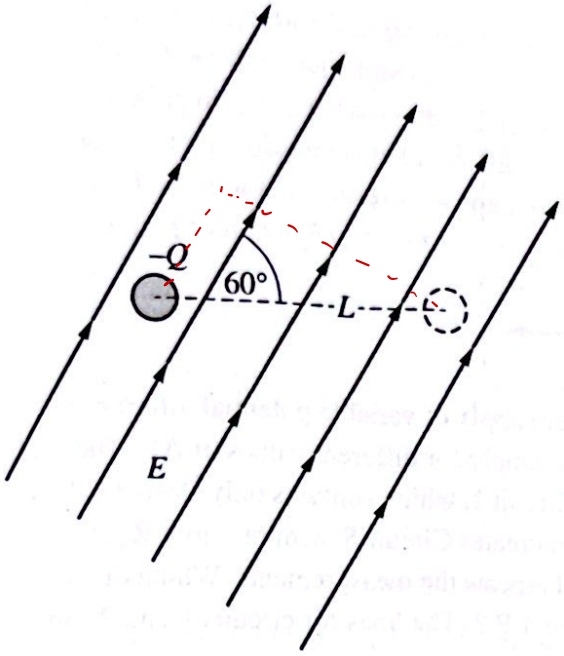

D

20. An external force moves a sphere with negative charge $-Q$ at a constant speed through a uniform electric field of magnitude E that is directed 60° above the horizontal, as shown. Which of the following represents the work done by the external force to move the sphere a horizontal distance L ?

(A) $-\frac{\sqrt{3}}{2}QEL$

(B) $-QEL$

(C) $-\frac{1}{2}QEL$

(D) $\frac{1}{2}QEL$

(E) $\frac{\sqrt{3}}{2}QEL$

B

21. Spheres 1, 2, and 3 each have net positive charge $+Q$ and radius r. Sphere 1 is a thin conducting shell. Sphere 2 is a solid conductor. Sphere 3 is a solid nonconductor with uniform charge distribution throughout its volume. Each sphere is isolated from the other spheres. The magnitudes of the electric fields at a distance $\frac{r}{5}$ from the centers of the spheres are E_1, E_2, and E_3 for spheres 1, 2, and 3, respectively. Which of the following correctly compares E_1, E_2, and E_3 ? $E_1 = E_2 = 0$

(A) $E_1 = E_2 = E_3$

(B) $(E_1 = E_2) < E_3$ $E_3 > 0$

(C) $E_1 < (E_2 = E_3)$

(D) $E_1 < E_2 < E_3$

(E) $E_1 > (E_2 = E_3)$

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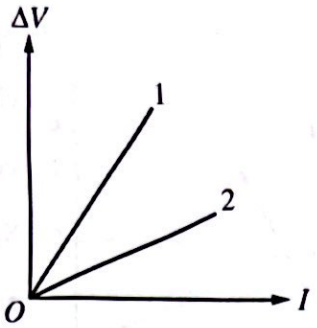
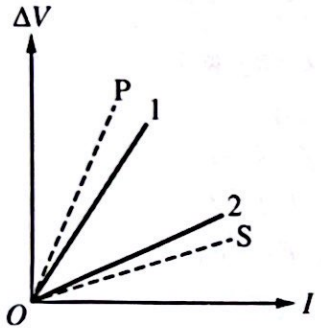
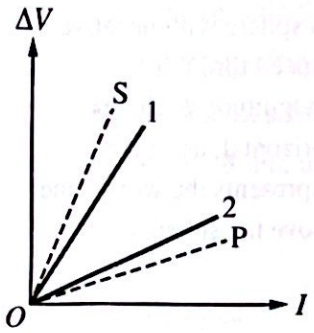
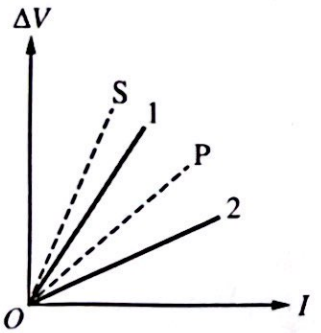


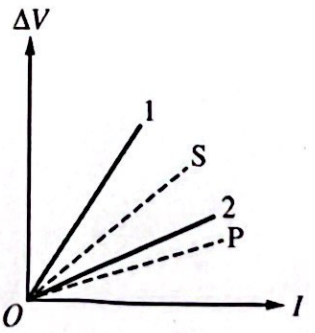
B

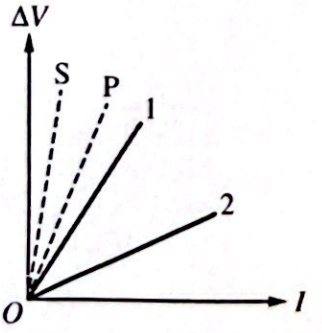
22. A student connects resistors R_1 and R_2 separately to a power supply of variable potential difference ΔV. For each circuit, the student measures the current I in the power supply for different values of ΔV. The student then creates the graph shown of ΔV as a function of I for Circuit 1, which contains only Resistor R_1, and Circuit 2, which contains only Resistor R_2. The student then creates Circuit S with resistors R_1 and R_2 in series and Circuit P with resistors R_1 and R_2 in parallel and repeats the measurements. Which of the following graphs best represents ΔV as a function of I for circuits S and P ? (The lines for circuits 1 and 2 are included for reference.)

(A)


(B)


(C)


(D)


(E)


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23. A parallel plate capacitor is connected to a battery for a long time and stores 5 J of energy. The separation distance between the plates is d. The battery is then disconnected from the capacitor. Which of the following is equal to the work required to move the plates to separation distance $2d$?

(A) −10 J
(B) −5 J
(C) 0 J
(D) +5 J
(E) +10 J

D

Q 不變

$U = \frac{1}{2} \cdot \frac{Q^2}{C}$

$C' = \frac{C}{2}$

$U' = 2U = 10\,J$

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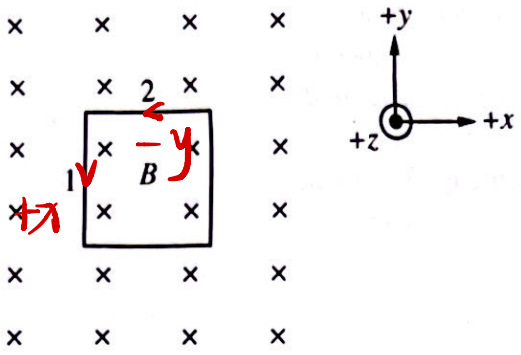

D

24. A conducting loop with labeled sides 1 and 2 is held at rest in a region of an external uniform magnetic field of magnitude B that is directed in the $-z$-direction, as shown. Starting at time $t = 0$, B is increased linearly as a function of t. The magnetic forces exerted on sides 1 and 2 are $\vec{F}_1$ and $\vec{F}_2$, respectively. Which of the following best describes $\vec{F}_1$ and $\vec{F}_2$ immediately after $t = 0$?

	$\vec{F}_1$	$\vec{F}_2$
A	In the $+x$-direction and increasing	In the $-y$-direction and increasing
B	In the $+y$-direction and increasing	In the $+x$-direction and increasing
C	In the $-y$-direction and increasing	In the $-x$-direction and increasing
D	In the $+x$-direction and decreasing	In the $-y$-direction and decreasing
E	In the $-y$-direction and decreasing	In the $-x$-direction and decreasing

(A) A
(B) B
(C) C
(D) D
(E) E

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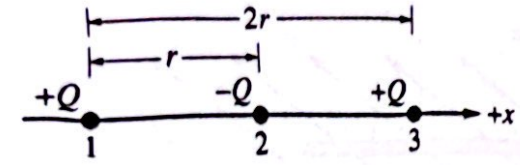

Figure 1

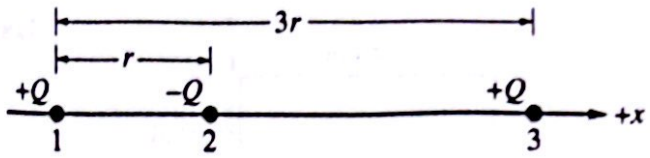

Figure 2

D

25. Small spheres 1, 2, and 3 of charges $+Q$, $-Q$, and $+Q$, respectively, are held at rest on the x-axis. When the centers of spheres 1 and 2 are distance r apart and the centers of spheres 1 and 3 are $2r$ apart, as shown in Figure 1, the magnitude of the net electric force exerted on Sphere 1 is F. Sphere 3 is then moved in the $+x$-direction and held at rest so that the centers of spheres 1 and 3 are $3r$ apart, as shown in Figure 2. Which of the following is equal to the magnitude of the resulting net electric force exerted on Sphere 1 in Figure 2 ?

(A) $\frac{2}{3}F$

(B) $\frac{8}{9}F$

(C) F

(D) $\frac{32}{27}F$

(E) $3F$

$$F = \frac{kQ^2}{r^2} - \frac{kQ^2}{4r^2} = \frac{3kQ^2}{4r^2}$$

$$F' = \frac{kQ^2}{r^2} - \frac{kQ^2}{9r^2} = \frac{8kQ^2}{9r^2}$$

$$\frac{F'}{F} = \frac{8}{9} \times \frac{4}{3} = \frac{32}{27}$$

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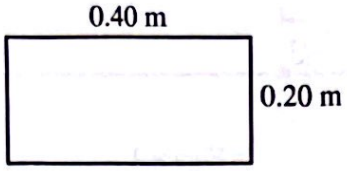

Figure 1. Top View

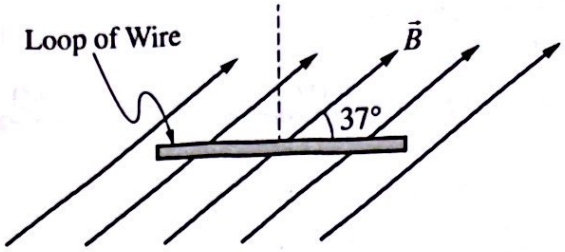

Figure 2. Edge View

A

26. A rectangular loop of wire has side lengths of 0.40 m and 0.20 m, as shown in Figure 1. The loop is placed in a uniform magnetic field $\vec{B}$ of magnitude 10.0 mT that is directed at an angle of 37° to the plane of and through the loop, as shown in Figure 2. What is the magnetic flux through the loop?

$0.08 \times 10 \times 10^{-3} \times \sin 37°$ 3/5

(A) 0.48 mT · m^2

(B) 0.60 mT · m^2

(C) 0.64 mT · m^2

(D) 0.80 mT · m^2

(E) 1.39 mT · m^2

E

27. Parallel-plate capacitors 1 and 2 each have identical plate areas. Capacitor 1 is filled with air, has a plate separation of 4.0×10^{-3} m, and has a capacitance of 2.0 μF. Capacitor 2 is filled with a material of dielectric constant $\kappa = 3$, and the plates are separated by a distance of 2.0×10^{-3} m. Which of the following is equal to the capacitance of Capacitor 2 ?

$C = \frac{3\,\varepsilon_r \cdot \varepsilon_0 \cdot A}{d\,\frac{1}{2}}$

(A) 0.20 μF

(B) 0.30 μF

(C) 2.0 μF

(D) 6.0 μF

(E) 12 μF

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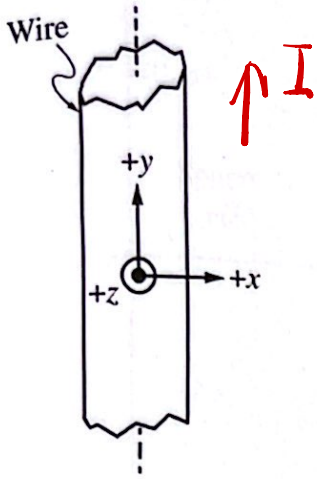

Figure 1

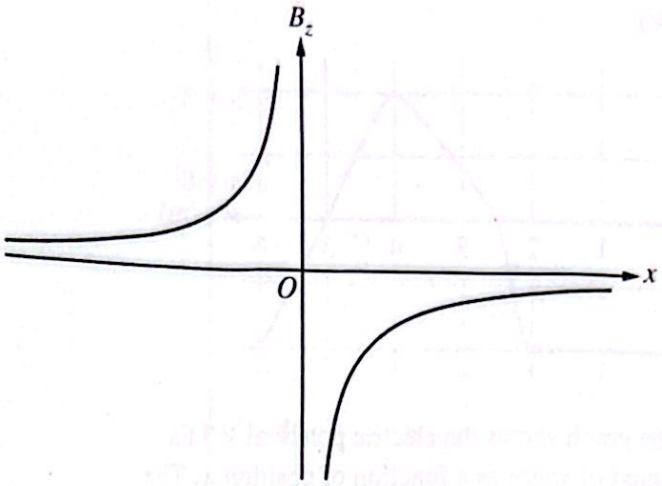

Figure 2

28. A long, straight wire is parallel to the y-axis, as shown in Figure 1. The wire carries current I. A graph of the z-component B_z of the magnetic field as a function of position x outside of the wire is shown in Figure 2. Which of the following correctly indicates the direction of I and provides a valid justification?

(A) $+y$-direction, because B_z approaches zero as x approaches $\pm\infty$.

(B) $+y$-direction, because B_z is negative for positive values of x.

(C) $-y$-direction, because B_z approaches zero as x approaches $\pm\infty$.

(D) $-y$-direction, because B_z is negative for positive values of x.

(E) $-y$-direction, because B_z has both positive and negative values.

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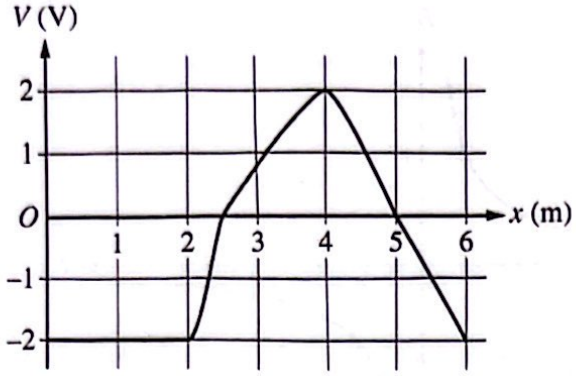
A



$E = \frac{dV}{dx}$

29. The graph shows the electric potential V in a region of space as a function of position x. The electric field is parallel to the x-axis. Which of the following claims about the electric field is correct?

(A) The electric field is zero at $x = 1.0$ m.

(B) The electric field changes direction between $x = 2.0$ m and $x = 4.0$ m.

(C) The magnitude of the electric field is a maximum at $x = 4.0$ m.

(D) The electric field is zero at $x = 5.0$ m.

(E) The magnitude of the electric field is a maximum at $x = 5.0$ m.

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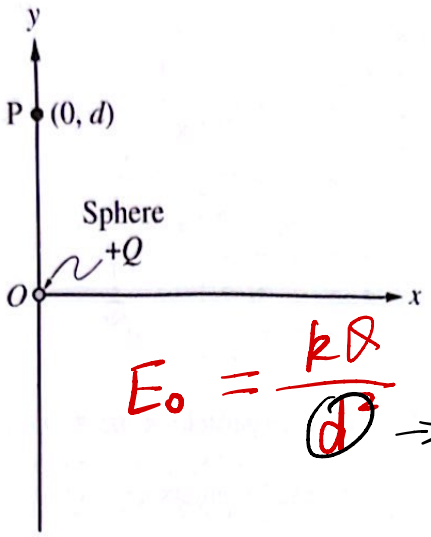

$E_0 = \frac{kQ}{d^2} \rightarrow 2d^2$

Figure 1

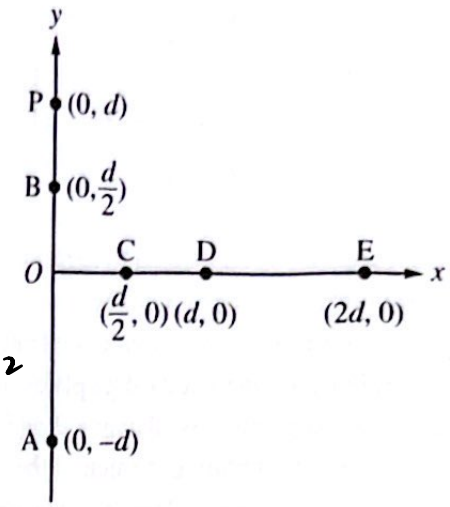

Figure 2

D

30. An isolated sphere with positive charge $+Q$ is positioned at the origin of an xy-coordinate system. Point P has coordinates $(0, d)$, as shown in Figure 1. The magnitude of the electric field at Point P due to the sphere is E_0. Additional points A, B, C, D, and E and their respective coordinates are labeled in Figure 2. At which of the following labeled points should the sphere be placed so the magnitude of the electric field at Point P is $\frac{E_0}{2}$?

(A) A
(B) B
(C) C
(D) D
(E) E

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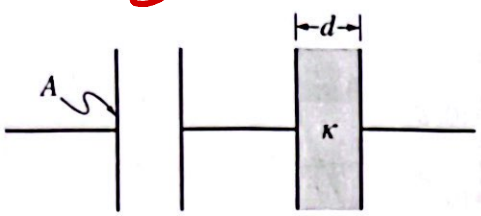
C kC



D

31. An air-filled capacitor is connected in series to another capacitor that contains a dielectric material of dielectric constant κ that completely fills the region between the plates. The plates of each capacitor are separated by distance d and each plate has area A, as shown. Which of the following is equal to the equivalent capacitance of the capacitors?

(A) $\frac{1}{2}\frac{\kappa\varepsilon_0 A}{d}$

(B) $\frac{\varepsilon_0 A}{d}(1+\kappa)$

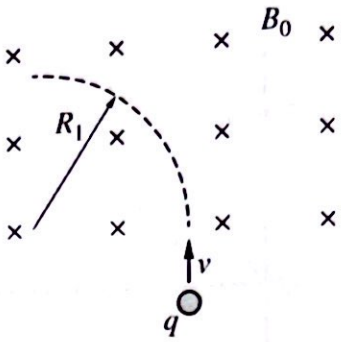
(C) $\frac{1}{2}\frac{\varepsilon_0 A}{d}\left(\frac{\kappa}{\kappa+1}\right)$

(D) $\frac{\varepsilon_0 A}{d}\left(\frac{\kappa}{\kappa+1}\right)$

(E) $\frac{\varepsilon_0 A}{d}\left(\frac{\kappa+1}{\kappa}\right)$

$\frac{K}{K+1}C$

$R = \frac{mv}{qB}$ $\frac{1}{2}$ 3



E

32. A charged particle of mass m, charge q, and with speed v enters a region of uniform magnetic field of magnitude B_0 that is directed into the page. The particle travels in a circular path of radius R_1, as shown. A second particle of mass m, charge $3q$, and with speed $\frac{v}{2}$ then enters the field along the same direction as the first particle. The second particle moves in a circular path of radius R_2. Which of the following correctly relates R_2 and R_1 ?

(A) $R_2 = 6R_1$

(B) $R_2 = \frac{3}{2}R_1$

(C) $R_2 = \frac{2}{3}R_1$

(D) $R_2 = \frac{1}{5}R_1$

(E) $R_2 = \frac{1}{6}R_1$

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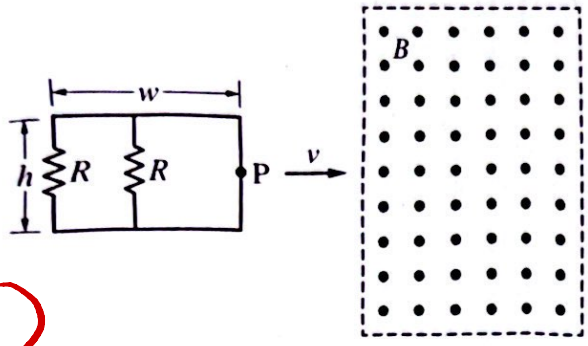

D

33. Two resistors, each of resistance R, are connected to a rectangular loop of wire that has height h and width w. The loop moves with speed v toward a region of external uniform magnetic field of magnitude B that is directed out of the page, as shown. Point P is located on the side of the loop that is closest to the magnetic field, as shown. A magnetic force of magnitude F_M is exerted on the loop immediately after Point P enters the magnetic field. Which of the following expressions is equal to F_M ?

$\mathcal{E} = Bhv$

(A) $\frac{1}{2}\frac{B^2w^2v}{R}$

(B) $2\frac{B^2w^2v}{R}$

$I = \frac{2Bhv}{R}$

(C) $\frac{1}{2}\frac{B^2h^2v}{R}$

(D) $2\frac{B^2h^2v}{R}$

$F = BIh$

(E) $2\frac{B^2whv}{R}$

$= \frac{2B^2h^2v}{R}$

E

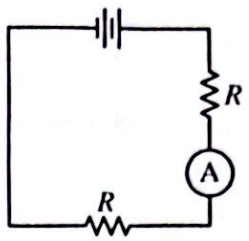

Figure 1

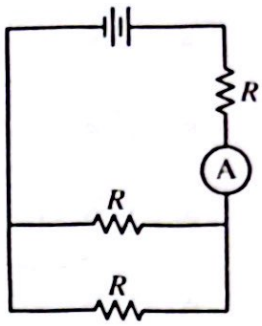

Figure 2

$I_1 = \frac{\mathcal{E}}{2R}$

$I_2 = \frac{\mathcal{E}}{\frac{3}{2}R} = \frac{2\mathcal{E}}{3R}$

34. The circuit in Figure 1 consists of an ideal battery, two resistors, each of resistance R, and ammeter A that measures current I_1. The circuit in Figure 2 consists of the battery, three resistors, each of resistance R, and ammeter A that measures current I_2. Which of the following is equal to I_2 ?

(A) $\frac{1}{3}I_1$

(B) $\frac{1}{2}I_1$

(C) $\frac{3}{4}I_1$

(D) I_1

(E) $\frac{4}{3}I_1$

$\frac{I_2}{I_1} = \frac{2}{3} \times 2 = \frac{4}{3}$

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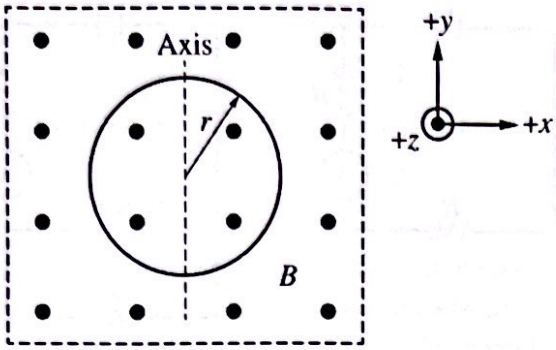

Region of Magnetic Field

$\phi = \pi r^2 \cdot B \cdot \sin \omega t$

$\frac{d\phi}{dt} = \omega \pi r^2 B \cos \omega t$

C 35. A circular conducting loop of radius r rotates with angular frequency about a central axis that is parallel to the y-axis. The loop is in a region of external uniform magnetic field of magnitude B that is in the $+z$-direction, as shown. Which of the following expressions is equal to the maximum induced emf in the loop?

(A) $\frac{1}{4} B\pi r^2 \omega$

(B) $\frac{1}{2} B\pi r^2 \omega$

(C) $B\pi r^2 \omega$

(D) $2B\pi r^2 \omega$

(E) $4B\pi r^2 \omega$

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Begin your response to **QUESTION 1** on this page.

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

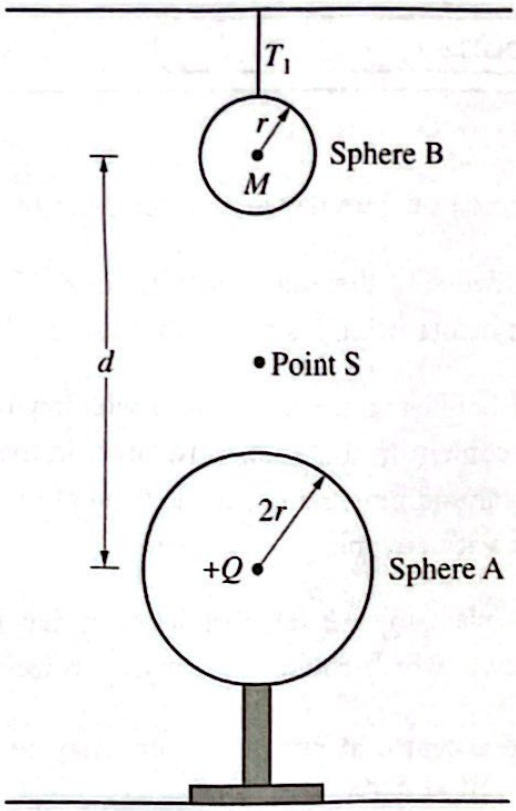

Figure 1

Note: Figure not drawn to scale.

1. Conducting Sphere A of radius $2r$ and positive net charge $+Q$ is attached to an insulated stand. Neutral conducting Sphere B of radius r and mass M is suspended by an insulated string so that Sphere B is at rest directly above Sphere A. The tension force exerted on Sphere B by the string has magnitude T_1. The centers of the spheres are separated by distance d, as shown in Figure 1, where $r \ll d$. Point S is midway between the centers of the spheres.

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(a) The dots in Figure 2 represent the center of Sphere A, Point S, and the center of Sphere B. In Figure 2, **draw** a single arrow on each dot to represent the direction of the net electric field at the location of the dot. Each arrow must start on and point away from the dot. If the net electric field is zero, write "$E = 0$" next to the dot.

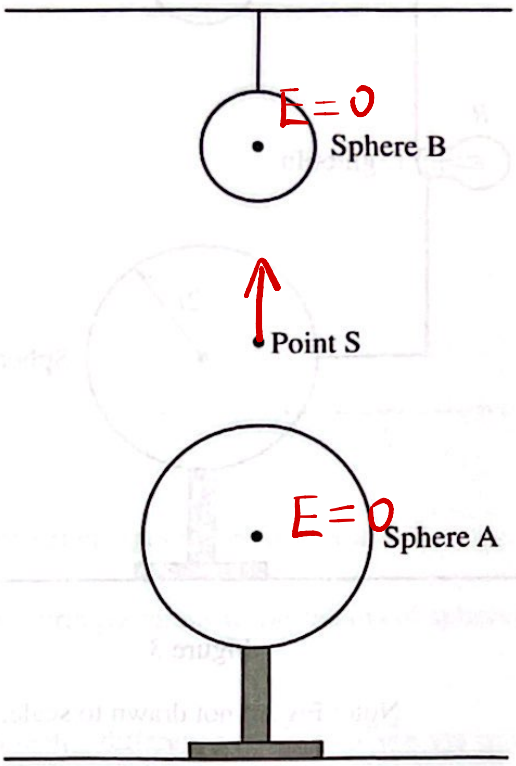

Figure 2

Note: Figure not drawn to scale.

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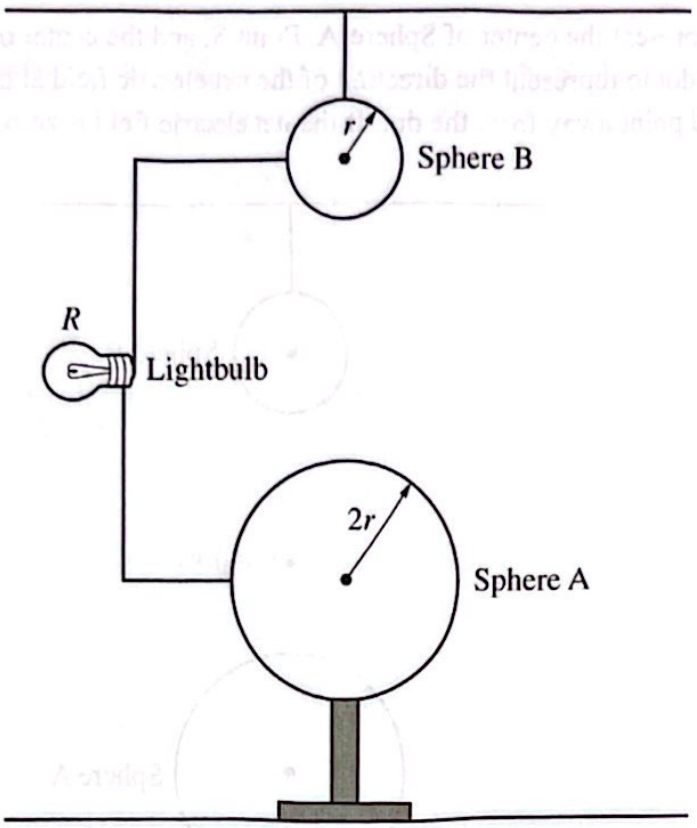

Figure 3

Note: Figure not drawn to scale.

At time $t = 0$, a lightbulb of resistance R and conducting wires are connected between the spheres, as shown in Figure 3. The lightbulb remains connected between the spheres for a long time t_1.

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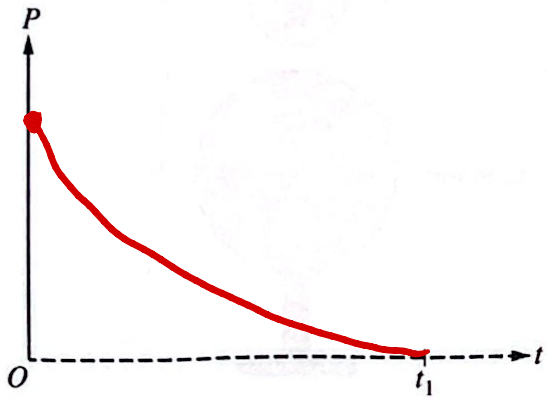
(b) Consider time interval $0 \le t \le t_1$.

i. On the axes shown, **sketch** a graph of the electrical power P dissipated by the lightbulb as a function of t during $0 \le t \le t_1$. Explicitly label the vertical intercept in terms of r, Q, M, t_1, d, R, and physical constants, as appropriate.



ii. $Q_A(t)$ and $Q_B(t)$ represent the net charges on spheres A and B, respectively, as functions of t.

$V_A(t)$ and $V_B(t)$ represent the electric potentials at the centers of spheres A and B, respectively, as functions of t.

Derive an expression for the potential difference $\Delta V(t)$ between the spheres as a function of t,

where $\Delta V(t) = V_A(t) - V_B(t)$, to show that $\Delta V(t) = k\left(\frac{3Q_A(t)}{2r} - \frac{Q}{r}\right)$, where $k = \frac{1}{4\pi\varepsilon_0}$.

$V_A(t) = \frac{k \cdot Q_A(t)}{2r}$, $V_B(t) = \frac{k(Q - Q_A(t))}{r}$

$\Delta V(t) = V_A(t) - V_B(t) = \frac{k}{r} \cdot \left(\frac{3}{2} Q_A(t) - Q\right)$

$= k \cdot \left(\frac{3Q_A(t)}{2r} - \frac{Q}{r}\right)$

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(c) After t_1, the lightbulb and wires are removed without disturbing the charges on spheres A and B. Q_A is the net charge on Sphere A, and Q_B is the net charge on Sphere B.

i. **Calculate** the ratio $\frac{Q_B}{Q_A}$.

$$\frac{kQ_A}{2r} = \frac{kQ_B}{r}$$

$$\frac{Q_B}{Q_A} = \frac{1}{2}$$

ii. The magnitude of the net electric field at Point S is now E_{new}. **Derive** an expression for E_{new} in terms of r, Q, M, t_1, d, R, and physical constants, as appropriate.

$$Q_A + Q_B = Q$$

$$Q_A = \frac{2}{3}Q, \quad Q_B = \frac{1}{3}Q$$

$$E = \frac{k\cdot\frac{2}{3}Q}{\left(\frac{d}{2}\right)^2} - \frac{k\cdot\frac{1}{3}Q}{\left(\frac{d}{2}\right)^2}$$

$$= \frac{4kQ}{3d^2}$$

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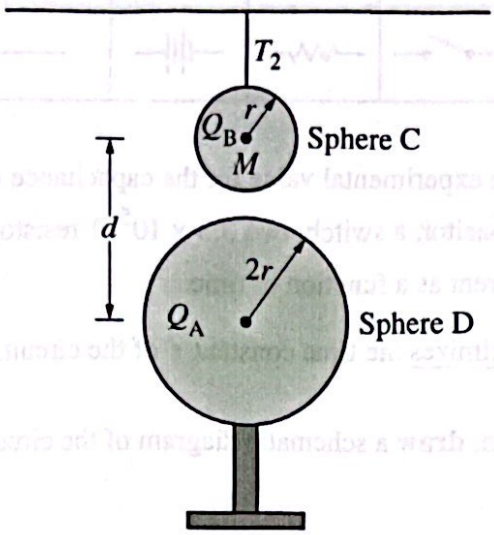

Figure 4

(d) Sphere B is replaced with nonconducting Sphere C of the same radius r, the same mass M, and the same charge Q_B as those of Sphere B in part (c). Sphere A is replaced by nonconducting Sphere D of the same radius $2r$ and the same charge Q_A as those of Sphere A in part (c). The charge is distributed uniformly throughout the volume of each sphere. Sphere D is moved close to Sphere C. Sphere D is then held at rest directly underneath Sphere C such that $r \ll d$ is no longer assumed, as shown in Figure 4.

i. **Describe** the changes, if any, in the distribution of charge on Sphere D as Sphere D is moved closer to Sphere C.

negative charge will gather to the top.

ii. The tension force exerted on Sphere C by the string has magnitude T_2 after Sphere D is brought to rest. **Indicate** whether T_2 is greater than, less than, or equal to T_1, the magnitude of the tension force exerted on Sphere B in Figure 1. Briefly **justify** your answer.

$T_2 < T_1$, because the electrical force to B is upward.

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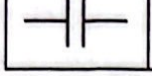
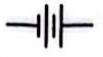
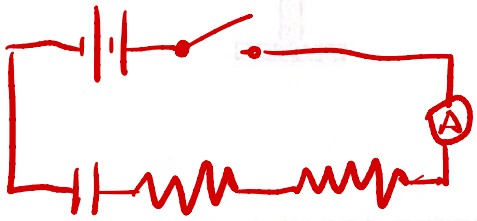
Capacitor	Switch	Resistor	Battery	Connecting Wires	Ammeter
					

2. Students are asked to determine an experimental value for the capacitance C_{EXP} of an initially uncharged capacitor. The students use the capacitor, a switch, two $3.3 \times 10^5\ \Omega$ resistors, a 9.0 V battery, connecting wires, and an ammeter that measures current as a function of time.

(a) A circuit is assembled that maximizes the time constant τ of the circuit. The circuit can be used to determine C_{EXP}.

i. Using the symbols shown, **draw** a schematic diagram of the circuit that includes both resistors.



ii. Describe a procedure for collecting data as the capacitor is being charged that would allow the students to determine Cexp. Provide enough detail so that another student could replicate the experiment.

① close the switch and measure the current corresponding to different times.

② $I = \frac{\varepsilon}{2R} \cdot e^{-\frac{1}{RC}t}$, then plot $\ln I - t$, draw the best line, slope $= -\frac{1}{RC}$, so get C_{exp}

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(b) It is determined that $\tau = 3.1$ s. **Calculate** C_{EXP}.

$\tau = RC$

$C_{EXP} = \frac{\tau}{R} = \frac{3.1}{3.3 \times 10^{5} \times 2} = 4.70 \times 10^{-6}\ F$

At time $t = 3.1$ s, the charge on the positive plate of the capacitor was Q_1. After the experiment, it is determined that the ammeter that was used was nonideal. The experiment is repeated with an ideal ammeter. At time $t = 3.1$ s, the charge on the positive plate of the capacitor is Q_2.

(c) **Indicate** whether Q_2 is greater than, less than, or equal to Q_1.

____ $Q_2 > Q_1$ ✓ $Q_2 < Q_1$ ____ $Q_2 = Q_1$

Justify your answer.

$Q = V \cdot C$

the capacitance is same, the nonideal ammeter have part of the voltage so the voltage of the capacitor will be smaller, so $Q_1 < Q_2$

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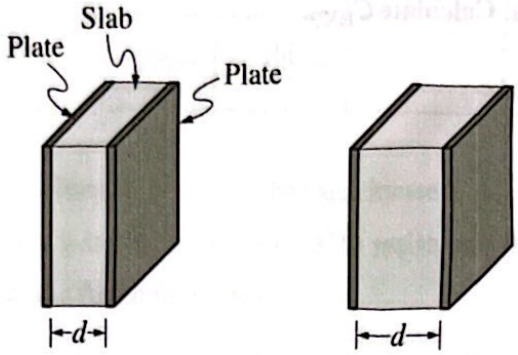
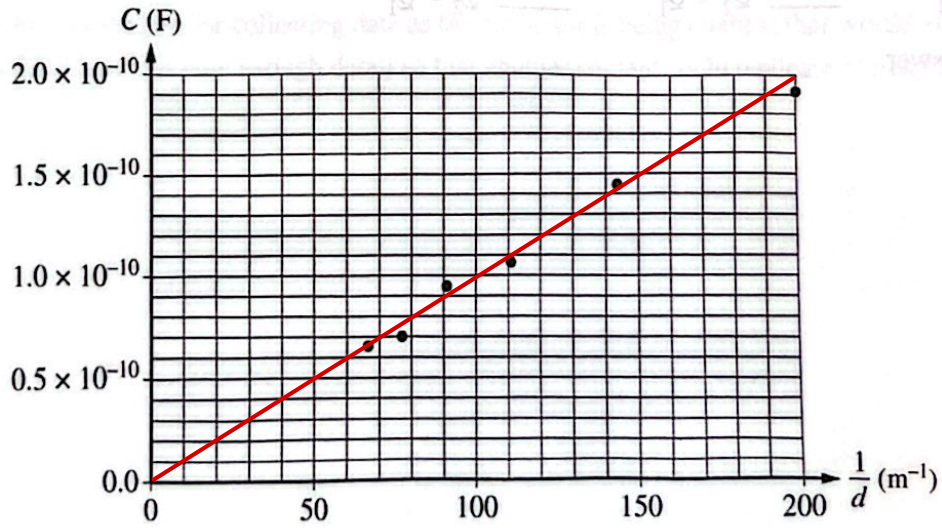


Trial 1 Trial 2

In a different experiment, students are asked to determine an experimental value for the dielectric constant κ of a type of plastic. The students have two parallel conducting plates and six different slabs made of the plastic. The slabs differ in thickness. For each trial, one slab is inserted between the plates to make a capacitor. The plate separation d is adjusted so that d is the slab thickness. The capacitance C of the capacitor is then determined. The capacitor for two trials is shown.

Collected data are used to plot the following graph of C as a function of $\frac{1}{d}$.



(d)

i. **Draw** the best-fit line for the data.

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ii. Using the best-fit line, **calculate** the experimental value for κ. Each plate has area $5.0 \times 10^{-2}\,\text{m}^2$.

$$C = \frac{\kappa \varepsilon_0 A}{d}$$

$$\text{slope} = \frac{2 \times 10^{-10}}{200} = 10^{-12}$$

$$\kappa \varepsilon_0 A = 10^{-12}$$

$$\kappa = \frac{10^{-12}}{8.85 \times 10^{-12} \times 5 \times 10^{-2}} = 2.26$$

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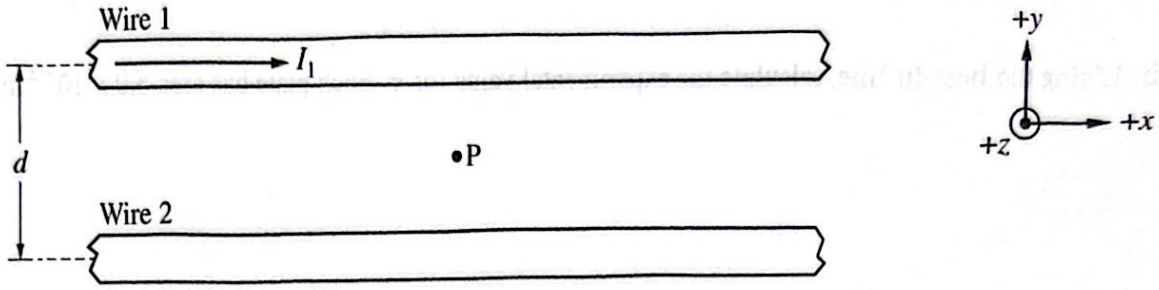

Figure 1. Side View

3. Long parallel wires 1 and 2 are positioned horizontally and are separated by vertical distance d. Wire 1 is fixed in place and has current I_1 in the $+x$-direction. Wire 2 is initially held in place but can be released. Each wire has linear mass density λ and resistance per length $\frac{R}{\ell}$. Point P is midway between wires 1 and 2, as shown in the side view in Figure 1.

(a) **Draw** your answers to parts (a)(i) and (a)(ii) on the cross-sectional view shown in Figure 2.

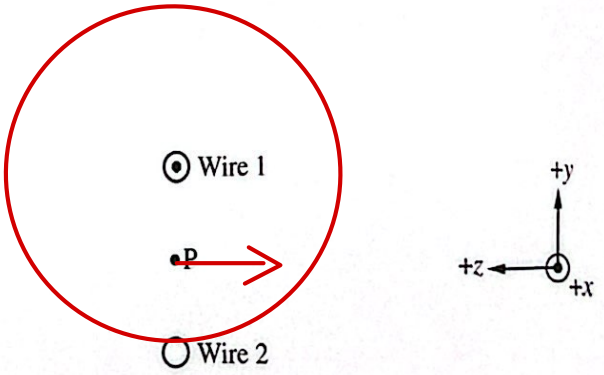

Figure 2. Cross-Sectional View

i. On Figure 2, **draw** an appropriate Amperian loop that can be used to determine the magnitude of the magnetic field at the location of Wire 2 due to Wire 1.

ii. On Figure 2, **draw** an arrow starting on, and pointing away from, the dot representing Point P to show the direction of the magnetic field at Point P due to Wire 1.

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iii. Using Ampere's law, **derive** an expression for the magnitude of the magnetic field B at the location of Wire 2 due to Wire 1. Express your answer in terms of d, I_1, and physical constants, as appropriate.

$$B \cdot 2\pi d = \mu_0 \cdot I_1$$

$$B = \frac{\mu_0 I_1}{2\pi d}$$

Consider the situation in which Wire 2 has current I_2 and there is a gravitational force exerted on Wire 2 in the $-y$-direction. Wire 2 is then released but remains at rest.

(b) **Derive** an expression for I_2 in terms of d, I_1, λ, and physical constants, as appropriate.

$$F = mg = B \cdot I_2 L$$

$$I_2 = \frac{mg}{BL} = \lambda g \times \frac{2\pi d}{\mu_0 I_1} = \frac{2\pi \lambda g d}{\mu_0 I_1}$$

Consider the situation in which Wire 1 has these properties: $I_1 = 5.0\,\text{A}$ and $\frac{R}{\ell} = 0.12\ \frac{\Omega}{\text{m}}$.

(c) **Calculate** the total energy ΔE dissipated in a section of Wire 1 of length $L = 0.40\,\text{m}$ for a time interval of $\Delta t = 3.0\,\text{s}$.

$$\Delta E = I^2 R t = 5^2 \times 0.12 \times 0.4 \times 3$$

$$= 3.6\,\text{W}$$

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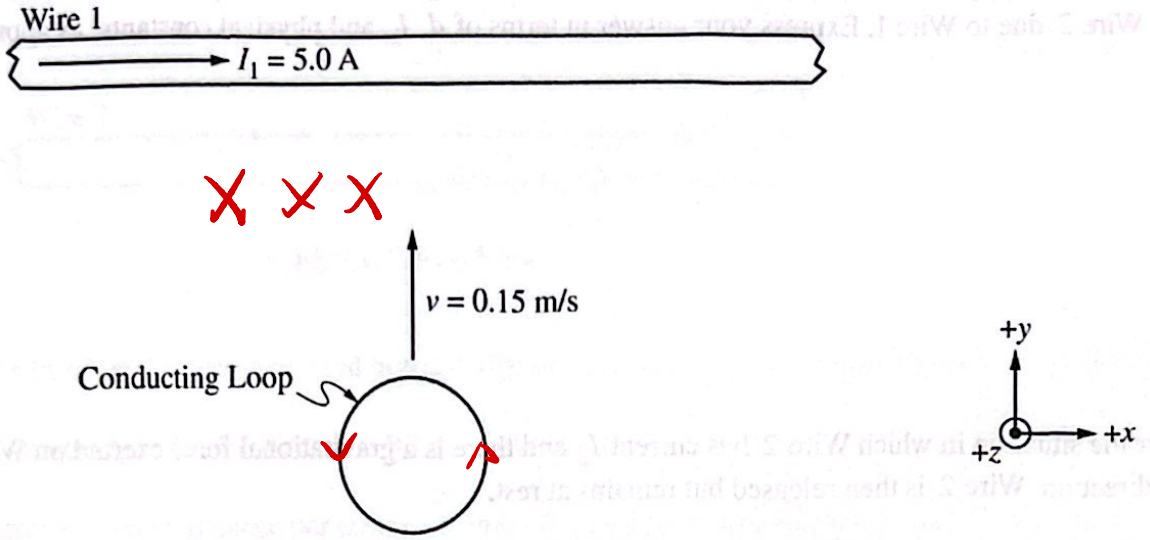

Figure 3. Side View

Wire 2 is removed. Wire 1 has current $I_1 = 5.0$ A in the $+x$-direction. A circular conducting loop is placed directly below Wire 1. The loop is moved toward Wire 1 in the $+y$-direction at constant speed $v = 0.15$ m/s, as shown in the side view in Figure 3.

(d) **Indicate** whether the direction of the induced current in the loop is clockwise, counterclockwise, or zero as the loop is moved toward Wire 1.

_____ Clockwise ___✓___ Counterclockwise _____ Zero

Briefly **justify** your answer.

the magnetic flux into page is increasing, according to Lenz's law, the induction current in the loop should be out of page, so the direction is counterclockwise

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The rate of change of the magnitude of the magnetic field at the center of the moving loop due to Wire 1 is $\frac{dB}{dt}$.

The distance between the center of Wire 1 and the center of the moving loop is h, which varies as a function of time.

(e) **Derive** the relationship between the magnitude of $\frac{dB}{dt}$ and h to verify that $\left|\frac{dB}{dt}\right| = \left(1.5 \times 10^{-7} \frac{\text{T}\cdot\text{m}^2}{\text{s}}\right)\left(\frac{1}{h^2}\right)$.

Write your expression in terms of numerical values and h only.

$h = vt$, $\frac{dh}{dt} = v = 0.15$

$B = \frac{\mu_0 \cdot I}{2\pi h}$

$\left|\frac{dB}{dt}\right| = \frac{\mu_0 I}{2\pi} \cdot \frac{1}{h^2} \cdot \frac{dh}{dt}$

$= \frac{4\pi \times 10^{-7} \times 5 \times 0.15}{2\pi} \cdot \frac{1}{h^2}$

$= 1.5 \times 10^{-7} \cdot \frac{1}{h^2}$

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