

2022 AMC 8 Solutions

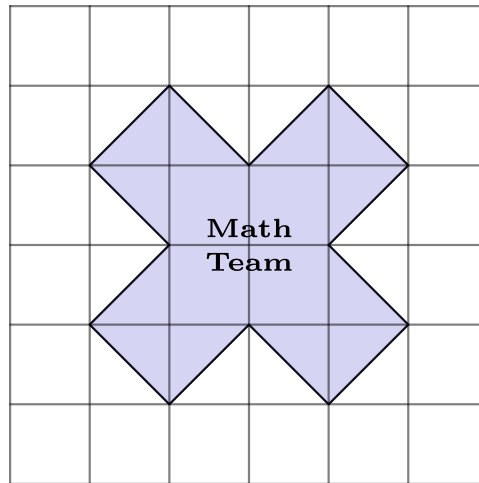
Typeset by: LIVE by Po-Shen Loh

<https://live.poshenloh.com/past-contests/amc8/2022/solutions>



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1. The Math Team designed a logo shaped like a multiplication symbol, shown below on a grid of 1-inch squares. What is the area of the logo in square inches?



- ☒ A 10
- ☐ B 12
- ☐ C 13
- ☐ D 14
- ☐ E 15

Solution:

The logo is made from 5 congruent tilted squares. Each tilted square has both diagonals of length 2, so its area is $\frac{2 \cdot 2}{2} = 2$ square inches.

The total area is $5 \cdot 2 = 10$.

Thus, the correct answer is **A**.

2. Consider these two operations:

$$a \blacklozenge b = a^2 - b^2$$

$$a \star b = (a - b)^2$$

Compute the value:

$$(5 \blacklozenge 3) \star 6$$

A -20

B 4

C 16

D 100

E 220

Solution:

Using the definitions,

$$\begin{aligned}(5 \blacklozenge 3) \star 6 &= (5^2 - 3^2) \star 6 \\ &= 16 \star 6 \\ &= (16 - 6)^2 \\ &= 100\end{aligned}$$

Thus, the correct answer is **D**.

3. When three positive integers a , b , and c are multiplied together, their product is 100. Suppose $a < b < c$. In how many ways can the numbers be chosen?

- ☐ A 0
- ☐ B 1
- ☐ C 2
- ☐ D 3
- ☒ E 4

Solution:

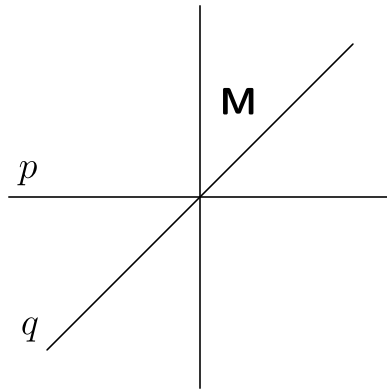
Since $a < b < c$ and $abc = 100$, we have $a^3 < 100$, so $a \leq 4$. Also a must divide 100, so $a \in \{1, 2, 4\}$.

If $a = 1$, the pairs (b, c) are $(2, 50)$, $(4, 25)$, $(5, 20)$. If $a = 2$, the only possible pair is $(5, 10)$. If $a = 4$, then $bc = 25$, and there is no integer b with $4 < b < c$.

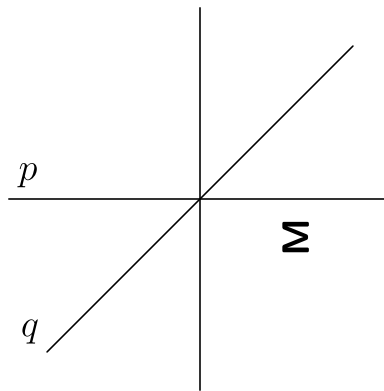
There are 4 possible triples.

Thus, the correct answer is **E**.

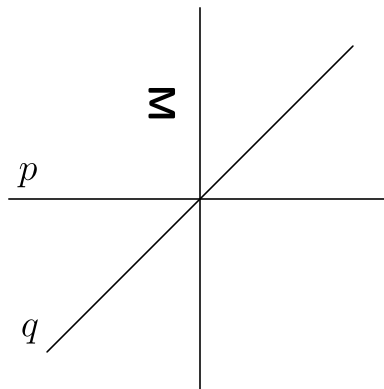
4. The letter **M** in the figure below is first reflected over the line q and then reflected over the line p . What is the resulting image?



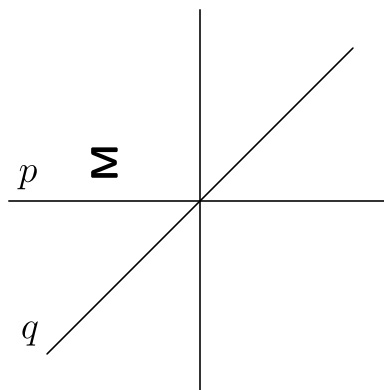
A



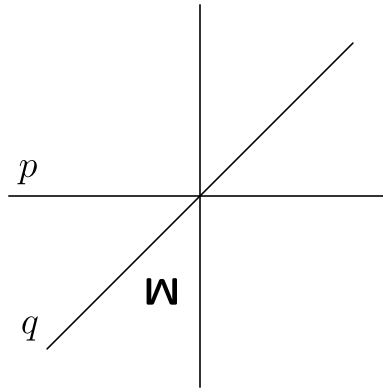
B



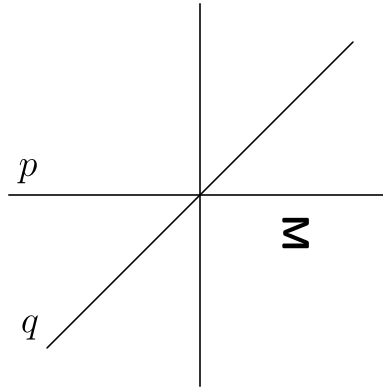
C



D



E



Solution:

Reflecting over the diagonal line q changes the orientation of the letter as if the diagonal were a mirror. Reflecting that image over the horizontal line p then places the final letter below p , to the right of the vertical line, with the orientation shown in choice E.

Thus, the correct answer is **E**.

5. Anna and Bella are celebrating their birthdays together. Five years ago, when Bella turned 6 years old, she received a newborn kitten as a birthday present. Today the sum of the ages of the two children and the kitten is 30 years. How many years older than Bella is Anna?

- ☐ A 1
- ☐ B 2
- ☒ C 3
- ☐ D 4
- ☐ E 5

Solution:

If Bella was 6 five years ago, then she is 11 right now.

If the kitten was a newborn five years ago, then it is 5 right now.

Since the sum of all 3 ages is 30, Anna's age is

$$30 - 5 - 11 = 14$$

Since Anna is 14 and Bella is 11, she is 3 years older than Bella.

Thus, the correct answer is **C**.

6. Three positive integers are equally spaced on a number line. The middle number is 15 and the largest number is 4 times the smallest number. What is the smallest of these three numbers?

- A 4
- B 5
- C 6**
- D 7
- E 8

Solution:

Let the smallest number be x . Then the largest is $4x$. Since the three numbers are equally spaced, the middle number is the average of the smallest and largest:

$$\frac{x + 4x}{2} = 15.$$

Thus $5x = 30$, so $x = 6$.

Thus, the correct answer is **C**.

7. When the World Wide Web first became popular in the 1990s, download speeds reached a maximum of about 56 kilobits per second. Approximately how many minutes would the download of a 4.2-megabyte song have taken at that speed? (Note that there are 8000 kilobits in a megabyte.)

A 0.6

B 10

C 1800

D 7200

E 36000

Solution:

The song has $4.2 \cdot 8000 = 33,600$ kilobits. At 56 kilobits per second, the download takes

$$\frac{33,600}{56} = 600$$

seconds, which is 10 minutes.

Thus, the correct answer is **B**.

8. What is the value of:

$$\frac{1}{3} \cdot \frac{2}{4} \cdot \frac{3}{5} \cdots \frac{18}{20} \cdot \frac{19}{21} \cdot \frac{20}{22}$$

A $\frac{1}{462}$

B $\frac{1}{231}$

C $\frac{1}{132}$

D $\frac{2}{213}$

E $\frac{1}{22}$

Solution:

Since every integer from **3** to **20** occurs once as a denominator and once as a numerator, they cancel each other out.

After canceling every number out, we have only **1** and **2** left as numerators and **21** and **22** left as denominators.

The remaining fraction is $\frac{1 \cdot 2}{21 \cdot 22}$. This simplifies to $\frac{2}{462} = \frac{1}{231}$

Thus, the correct answer is **B**.

9. A cup of boiling water (212°F) is placed to cool in a room whose temperature remains constant at 68°F . Suppose the difference between the water temperature and the room temperature is halved every 5 minutes. What is the water temperature, in degrees Fahrenheit, after 15 minutes?

A 77

B 86

C 92

D 98

E 104

Solution:

The current difference is $212 - 68 = 144$.

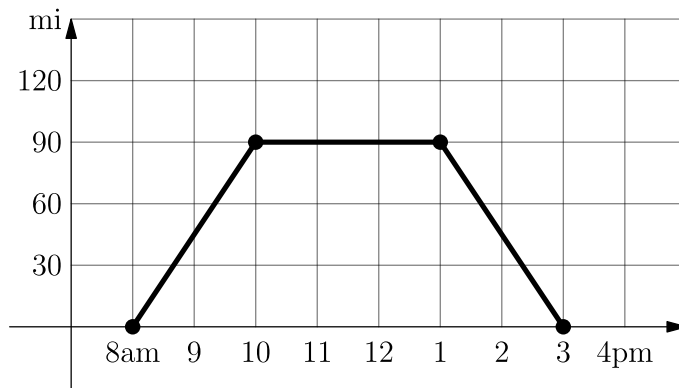
Since we have 15 minutes while halving every 5 minutes, we halve the difference 3 times.

This means the difference is multiplied by $(\frac{1}{2})^3 = \frac{1}{8}$, so our new difference is $\frac{1}{8} \cdot 144 = 18$. This makes our final temperature $68 + 18 = 86$.

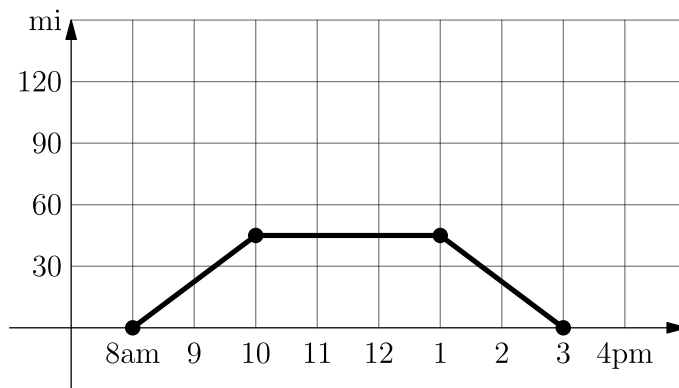
Thus, the correct answer is **B**.

10. One sunny day, Ling decided to take a hike in the mountains. She left her house at 8 a.m., drove at a constant speed of 45 miles per hour, and arrived at the hiking trail at 10 a.m. After hiking for 3 hours, Ling drove home at a constant speed of 60 miles per hour. Which of the following graphs best illustrates the distance between Ling's car and her house over the course of her trip?

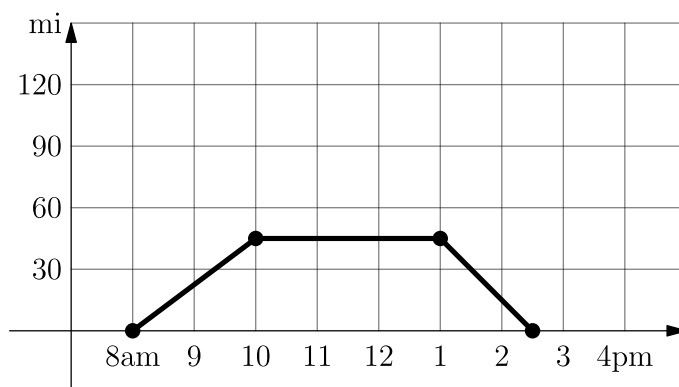
A



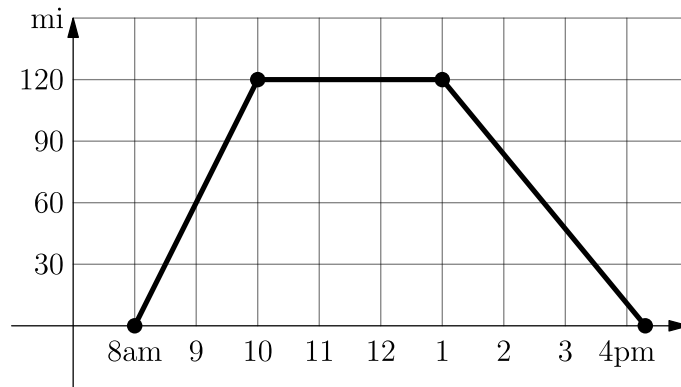
B



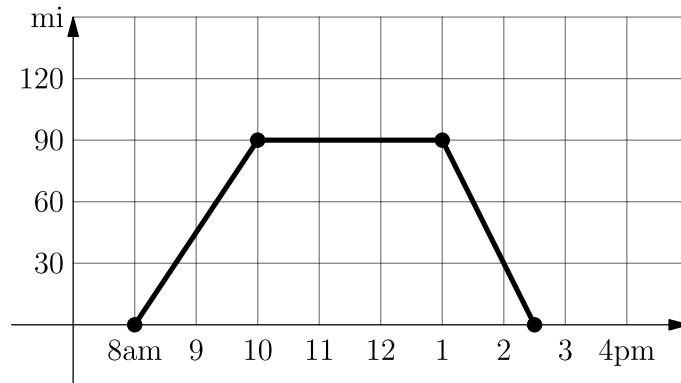
C



D



E



Solution:

She drives 45 miles per hour for 2 hours, so the trail is 90 miles from her house. The graph must rise from 0 to 90 miles between 8 AM and 10 AM, then stay flat for the 3-hour hike.

She starts home at 1 PM. Driving 90 miles at 60 miles per hour takes 1.5 hours, so she gets home at 2:30 PM. This matches choice E.

Thus, the correct answer is **E**.

11. Henry the donkey has a very long piece of pasta. He takes a number of bites of pasta, each time eating 3 inches of pasta from the middle of one piece. In the end, he has 10 pieces of pasta whose total length is 17 inches. How long, in inches, was the piece of pasta he started with?

A 34

B 38

C 41

D 44

E 47

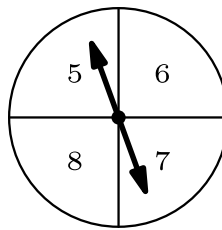
Solution:

Since there are 10 pieces, there were 9 locations where a bite was made. Since we have 9 bites and 3 inches are removed per bite, a total of 27 inches were removed.

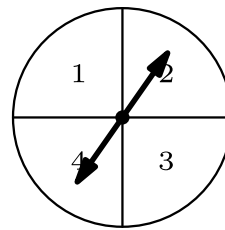
With 27 inches removed and 17 inches remaining, we know we started with $27 + 17 = 44$ inches.

Thus, the correct answer is **D**.

12. The arrows on the two spinners shown below are spun. Let the number N equal 10 times the number on Spinner A, added to the number on Spinner B. What is the probability that N is a perfect square number?



Spinner A



Spinner B

- A $\frac{1}{16}$
- B $\frac{1}{8}$**
- C $\frac{1}{4}$
- D $\frac{3}{8}$
- E $\frac{1}{2}$

Solution:

Spinner A gives the tens digit and Spinner B gives the ones digit. There are $4 \cdot 4 = 16$ equally likely two-digit numbers.

The possible perfect squares between 51 and 84 are 64 and 81, and both can occur. Thus the probability is

$$\frac{2}{16} = \frac{1}{8}.$$

Thus, the correct answer is **B**.

13. How many positive integers can fill the blank in the sentence below?

“One positive integer is ___ more than twice another, and the sum of the two numbers is 28”

- A 6
- B 7
- C 8
- D 9**
- E 10

Solution:

Let the smaller number be x , and let the blank be c . Then the other number is $2x + c$, where both x and c are positive integers.

The sum condition gives $x + (2x + c) = 28$, so $c = 28 - 3x$. For c to be positive, $28 - 3x > 0$, so $x < \frac{28}{3}$.

Thus x can be any integer from 1 through 9, giving 9 possible values of the blank.

Thus, the correct answer is **D**.

14. In how many ways can the letters in **BEEKEEPER** be rearranged so that two or more **E**'s do not appear together?

- ☐ A 1
- ☐ B 4
- ☐ C 12
- ☒ D 24
- ☐ E 120

Solution:

The word has 5 E's and 4 other letters: B, K, P, and R. To keep no two E's adjacent, the only possible pattern is

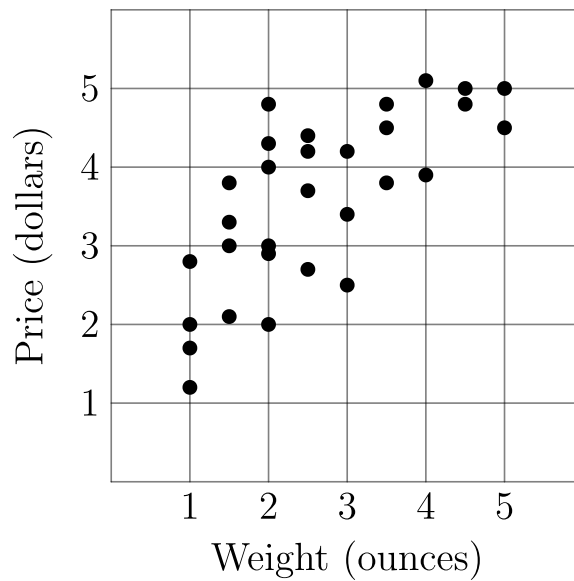
$$E_E_E_E_E,$$

with the four non-E letters in the four gaps.

The letters B, K, P, and R can be arranged in those gaps in $4! = 24$ ways.

Thus, the correct answer is **D**.

15. Laszlo went online to shop for black pepper and found thirty different black pepper options varying in weight and price, shown in the scatter plot below. In ounces, what is the weight of the pepper that offers the lowest price per ounce?



- A 1
- B 2
- C 3**
- D 4
- E 5

Solution:

For each weight, only the lowest-price point at that weight can give the lowest price per ounce.

At 1 ounce, the lowest price is more than 1 dollar, so the price per ounce is more than 1.

At 2 ounces, the lowest price is 2 dollars, so the price per ounce is 1.

At 3 ounces, the lowest price is about 2.5 dollars, so the price per ounce is about $\frac{2.5}{3}$, less than 1.

At 4 ounces, the lowest price is close to 3.9 dollars, so the price per ounce is close to $\frac{3.9}{4}$, still larger than the 3-ounce option.

At 5 ounces, the lowest price is close to 4.5 dollars, so the price per ounce is close to $\frac{4.5}{5}$, also larger than the 3-ounce option.

The price per ounce is lowest at 3 ounces.

Thus, the correct answer is **C**.

16. Four numbers are written in a row. The average of the first two is 21, the average of the middle two is 26, and the average of the last two is 30. What is the average of the first and last of the numbers?

A 24

B 25

C 26

D 27

E 28

Solution:

Let the numbers be a, b, c, d in order.

Since the average of a and b is 21, we know their sum is $21 \cdot 2 = 42$.

Since the average of c and d is 30, we know their sum is $30 \cdot 2 = 60$.

Since $a + b = 42$ and $c + d = 60$, we know

$$\begin{aligned}a + b + c + d &= 42 + 60 \\ &= 102\end{aligned}$$

Now, with the average of b and c being 26, we know their sum is $26 \cdot 2 = 52$. This means $b + c = 52$.

Subtracting this result from the sum of all the terms yields $a + d = 50$.

Since $\frac{a + d}{2} = 25$, our answer is 25.

Thus, the correct answer is **B**.

17. If n is an even positive integer, the double factorial notation $n!!$ represents the product of all the even integers from 2 to n . For example:

$$8!! = 2 \times 4 \times 6 \times 8$$

What is the units digit of the following sum?

$$2!! + 4!! + \cdots + 2022!!$$

- ☐ A 0
- ☒ B 2
- ☐ C 4
- ☐ D 6
- ☐ E 8

Solution:

For every even $n \geq 10$, the product $n!!$ contains a factor of 10, so that term has units digit 0. Only the first four terms can affect the units digit.

They sum to

$$\begin{aligned} &2!! + 4!! + 6!! + 8!! \\ &= 2 + 8 + 48 + 384 \\ &= 442. \end{aligned}$$

Therefore the full sum has units digit 2.

Thus, the correct answer is **B**.

18. The midpoints of the four sides of a rectangle are $(-3, 0)$, $(2, 0)$, $(5, 4)$, and $(0, 4)$. What is the area of the rectangle?

- ☐ A 20
- ☐ B 25
- ☒ C 40
- ☐ D 50
- ☐ E 80

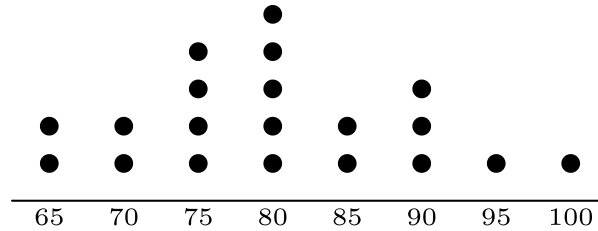
Solution:

The four given midpoints form a parallelogram. Its horizontal base has length $2 - (-3) = 5$, and its height is 4, so its area is $5 \cdot 4 = 20$.

For any rectangle, the parallelogram formed by joining the side midpoints has half the area of the rectangle. Therefore the rectangle's area is $2 \cdot 20 = 40$.

Thus, the correct answer is **C**.

19. Mr. Ramos gave a test to his class of 20 students. The dot plot below shows the distribution of test scores.



Later Mr. Ramos discovered that there was a scoring error on one of the questions. He regraded the tests, awarding some of the students 5 extra points, which increased the median test score to 85. What is the minimum number of students who received extra points?

(Note that the median test score equals the average of the 2 scores in the middle if the 20 test scores are arranged in increasing order.)

- ☐ A 2
- ☐ B 3
- ☒ C 4
- ☐ D 5
- ☐ E 6

Solution:

All scores are multiples of 5, and adding 5 preserves that fact. If the 10th and 11th scores average 85 but are not both 85, then the 11th score must be at least 90. That would require at least 10 scores of 90 or more.

However, even after regrading, only the original scores of 85 or more can reach 90. The plot has only 7 such scores, so the two middle scores must both be 85.

The plot initially has 7 scores of at least 85. To make the 10th and 11th scores both 85, there must be at least 11 scores of at least 85. Raising four of the 80's to 85 is both necessary and sufficient.

Thus, the correct answer is **C**.

20. The grid below is to be filled with integers in such a way that the sum of the numbers in each row and the sum of the numbers in each column are the same. Four numbers are missing. The number x in the lower left corner is larger than the other three missing numbers. What is the smallest possible value of x ?

-2	9	5
		-1
x		8

- A -1
- B 5
- C 6
- D 8**
- E 9

Solution:

Adding the numbers in the top row shows that every row and column must have sum 12.

In the first column, the missing number above x is $14 - x$. In the bottom row, the missing number to the right of x is $4 - x$. In the middle row, the remaining missing number is $x - 1$.

Since x is larger than the other missing numbers, we need $x > 14 - x$, $x > 4 - x$, and $x > x - 1$. The strongest condition is $x > 7$, so the smallest possible integer value is 8.

Thus, the correct answer is **D**.

21. Steph scored 15 baskets out of 20 attempts in the first half of a game, and 10 baskets out of 10 attempts in the second half. Candace took 12 attempts in the first half and 18 attempts in the second. In each half, Steph scored a higher percentage of baskets than Candace. Surprisingly they ended with the same overall percentage of baskets scored. How many more baskets did Candace score in the second half than in the first?

	First Half	Second Half
Steph	$\frac{15}{20}$	$\frac{10}{10}$
Candace	$\frac{\boxed{}}{12}$	$\frac{\boxed{}}{18}$

- A 7
- B 8
- C 9**
- D 10
- E 11

Solution:

Steph made 25 baskets in 30 attempts. Candace also took $12 + 18 = 30$ attempts, and their overall percentages were the same, so Candace also made 25 baskets.

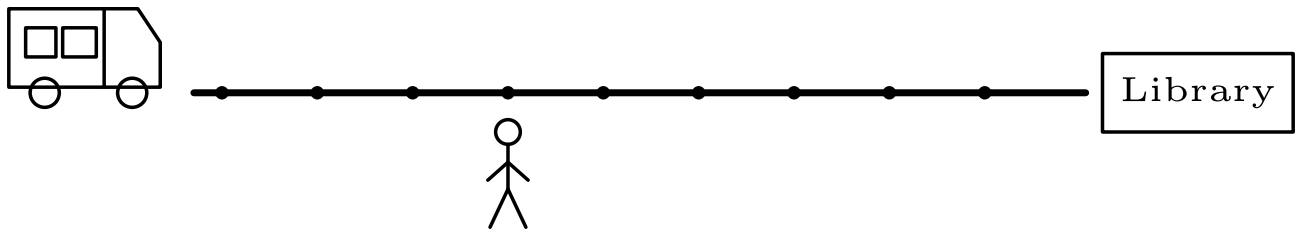
Let f be Candace's first-half baskets and s be her second-half baskets. Then $f + s = 25$.

Steph's first-half percentage was $\frac{15}{20} = \frac{3}{4}$, so $\frac{f}{12} < \frac{3}{4}$, giving $f < 9$. Steph's second-half percentage was 1, so $s < 18$.

Thus $f \leq 8$ and $s \leq 17$. Since $f + s = 25$, the only possibility is $f = 8$ and $s = 17$, so $s - f = 9$.

Thus, the correct answer is **C**.

22. A bus takes 2 minutes to drive from one stop to the next, and waits 1 minute at each stop to let passengers board. Zia takes 5 minutes to walk from one bus stop to the next. As Zia reaches a bus stop, if the bus is at the previous stop or has already left the previous stop, then she will wait for the bus. Otherwise she will start walking toward the next stop. Suppose the bus and Zia start at the same time toward the library, with the bus 3 stops behind. After how many minutes will Zia board the bus?



- ☒ A 17
- ☐ B 19
- ☐ C 20
- ☐ D 21
- ☐ E 23

Solution:

The bus takes 2 minutes to drive to the next stop and then waits 1 minute, so it moves through one-stop cycles every 3 minutes.

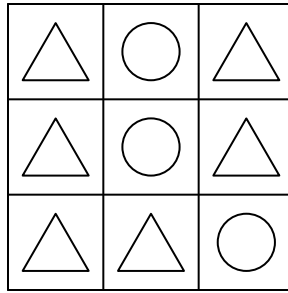
Zia makes a decision only when she reaches a stop, every 5 minutes. Measure stops from the bus's starting stop. Zia starts at stop 3.

After 5 minutes, Zia is at stop 4, while the bus is waiting at stop 2, so she keeps walking. After 10 minutes, Zia is at stop 5, while the bus is between stops 3 and 4, so she keeps walking. After 15 minutes, Zia is at stop 6, while the bus is at stop 5, the previous stop, so she waits.

The bus then takes 2 more minutes to reach her stop, so she boards after 17 minutes.

Thus, the correct answer is **A**.

23. A $\triangle$ or $\bigcirc$ is placed in each of the nine squares in a 3×3 grid. Shown below is a sample configuration with three $\triangle$'s in a line.



How many configurations will have three $\triangle$'s in a line and three $\bigcirc$'s in a line?

- A 39
- B 42
- C 78
- D 84**
- E 96

Solution:

Let k be the number of triangles. To have both a triangle line and a circle line, we need $3 \leq k \leq 6$.

If $k = 3$, the three triangles must form a line. A row or column works, because the remaining six circles contain a full circle row or column. A diagonal does not work, because its complement contains no full line of circles. Thus there are 6 configurations for $k = 3$. By symmetry, there are also 6 configurations for $k = 6$.

If $k = 4$, choose the triangle line and then the extra triangle. If the line is a row or column, all 6 possible extra positions work, because one parallel row or column remains all circles. This gives $6 \cdot 6 = 36$ configurations. If the triangle line is a diagonal, no extra position leaves a full circle line. By symmetry, $k = 5$ also gives 36 configurations.

The total number of configurations is $6 + 36 + 36 + 6 = 84$.

Thus, the correct answer is **D**.

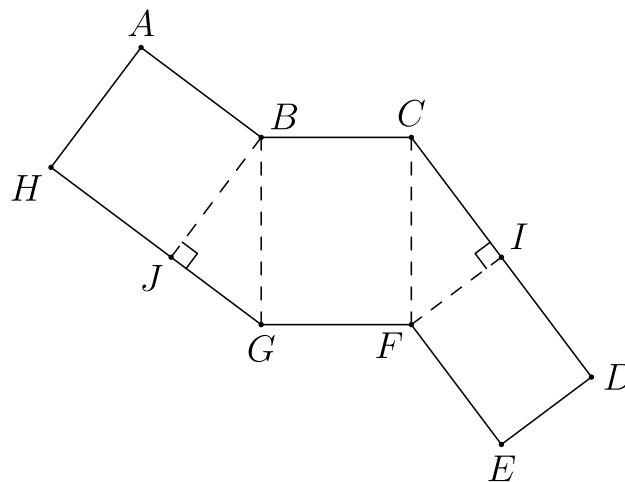
24. The figure below shows a polygon $ABCDEFGH$, consisting of rectangles and right triangles. When cut out and folded on the dotted lines, the polygon forms a triangular prism. Suppose that:

$$AH = EF = 8$$

and

$$GH = 14.$$

What is the volume of the prism?



- ☐ A 112
- ☐ B 128
- ☒ C 192
- ☐ D 240
- ☐ E 288

Solution:

When the net is folded, the matching side lengths give $GF = EF = 8$. Since $GFCB$ is a rectangle, $BC = 8$. The fold identifies AB with BC , so $AB = 8$. In rectangle

$HJBA$, this gives $HJ = 8$, and the opposite side BJ equals $AH = 8$.

Since $GH = 14$, we have $GJ = 14 - 8 = 6$. Thus one triangular base of the prism is right triangle BJG , with area

$$\frac{6 \cdot 8}{2} = 24.$$

The prism length is $GF = 8$, so the volume is $24 \cdot 8 = 192$.

Thus, the correct answer is **C**.

25. A cricket randomly hops between 4 leaves, on each turn hopping to one of the other 3 leaves with equal probability. After 4 hops, what is the probability that the cricket has returned to the leaf where it started?

A $\frac{2}{9}$

B $\frac{19}{80}$

C $\frac{20}{81}$

D $\frac{1}{4}$

E $\frac{7}{27}$

Solution:

Let p_n be the probability that the cricket is on its starting leaf after n hops. We have $p_0 = 1$.

If the cricket is on the starting leaf, the next hop must leave it. If the cricket is not on the starting leaf, exactly one of the 3 possible hops returns to the start. Therefore

$$p_{n+1} = \frac{1 - p_n}{3}.$$

Thus

$$p_1 = 0, \quad p_2 = \frac{1}{3}, \quad p_3 = \frac{2}{9},$$
$$p_4 = \frac{1 - \frac{2}{9}}{3} = \frac{7}{27}.$$

Thus, the correct answer is **E**.

Problems: <https://live.poshenloh.com/past-contests/amc8/2022>

