

2024 AMC 8 Solutions

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1. What is the ones digit of

$$\begin{aligned} &222,222 - 22,222 - 2,222 \\ &\quad - 222 - 22 - 2? \end{aligned}$$

A 0

B 2

C 4

D 6

E 8

Solution:

Only the ones digits matter. Modulo 10, the expression is

$$\begin{aligned} 2 - 2 - 2 - 2 - 2 - 2 &= -8, \\ -8 &\equiv 2 \pmod{10}. \end{aligned}$$

Therefore its ones digit is 2.

Thus, **B** is the correct answer.

2. What is the value of this expression in decimal form?

$$\frac{44}{11} + \frac{110}{44} + \frac{44}{1100}$$

A 6.4

B 6.504

C 6.54

D 6.9

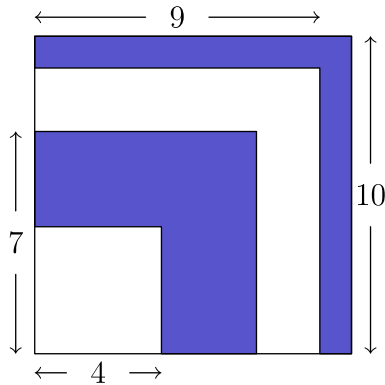
E 6.94

Solution:

We can simplify the fractions by taking out the common factor 11: $\frac{44}{11}$ simplifies to 4, $\frac{110}{44}$ simplifies to $\frac{10}{4} = \frac{5}{2} = 2.5$, and $\frac{44}{1100}$ simplifies to $\frac{4}{100} = 0.04$. Therefore, we have $4 + 2.5 + 0.04 = 6.54$.

Thus, **C** is the correct answer.

3. Four squares of side length 4, 7, 9, and 10 units are arranged in increasing size order so that their left edges and bottom edges align. The squares alternate shaded and unshaded, as shown in the figure. What is the area of the visible shaded region in square units?



- A 42
- B 45
- C 49
- D 50
- E 52**

Solution:

The visible shaded region is the part inside the 10 by 10 square but outside the 9 by 9 square, together with the part inside the 7 by 7 square but outside the 4 by 4 square. Its area is

$$\begin{aligned} &10^2 - 9^2 + 7^2 - 4^2 \\ &= 100 - 81 + 49 - 16 \\ &= 52. \end{aligned}$$

Thus, **E** is the correct answer.

4. When Yunji added all the integers from 1 to 9, she mistakenly left out a number. Her incorrect sum turned out to be a square number. Which number did Yunji leave out?

- ☐ A 5
- ☐ B 6
- ☐ C 7
- ☐ D 8
- ☒ E 9

Solution:

To find the number that Yunji left out, we need to find the sum of the integers from 1 to 9 and find its difference with the largest perfect square below the sum. We can calculate the sum of the integers from 1 to 9 as follows:

$$1 + \dots + 9 = \frac{9(9 + 1)}{2} = 45$$

The largest perfect square less than 45 would be 36 and $45 - 36 = 9$.

Thus, **E** is the correct answer.

5. Aaliyah rolls two standard 6-sided dice. She notices that the product of the two numbers rolled is a multiple of 6. Which of the following integers *cannot* be the sum of the two numbers?

A 5

B 6

C 7

D 8

E 9

Solution:

For the product to be a multiple of 6, the two dice together must supply a factor of 2 and a factor of 3. The possible sums among the answer choices can occur as follows:

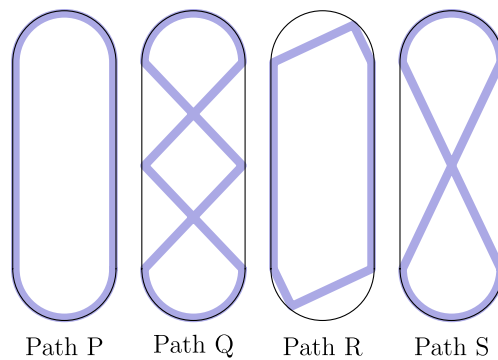
$$5 = 2 + 3, \quad 7 = 1 + 6,$$

$$8 = 2 + 6, \quad 9 = 3 + 6.$$

There is no way to get sum 6 while also having a product divisible by 6: the pairs with sum 6 are (1, 5), (2, 4), (3, 3), (4, 2), (5, 1), and none have product divisible by 6.

Thus, **B** is the correct answer.

6. Sergei skated around an ice rink, gliding along different paths. The marked lines in the figures below show four of the paths labeled P, Q, R, and S. What is the sorted order of the four paths from shortest to longest?



- ☐ A P, Q, R, S
- ☐ B P, R, S, Q
- ☐ C Q, S, P, R
- ☒ D R, P, S, Q
- ☐ E R, S, P, Q

Solution:

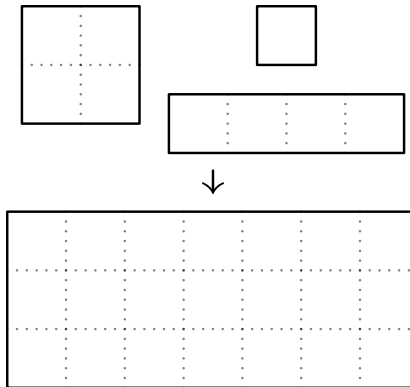
Path R is shortest because it replaces curved arc portions with straight-line chords. Path Q is longest because it includes the most interior crossing distance while still using the curved ends.

It remains to compare paths P and S. The relevant straight piece in path S is a diagonal across the rink, while the corresponding straight piece in path P is a side of the same right triangle. A diagonal is longer than a side, so path S is longer than path P.

The order from shortest to longest is R, P, S, Q.

Thus, **D** is the correct answer.

7. A 3×7 rectangle is covered without overlap by 3 shapes of tiles: 2×2 , 1×4 , and 1×1 , shown below. What is the minimum possible number of 1×1 tiles used?



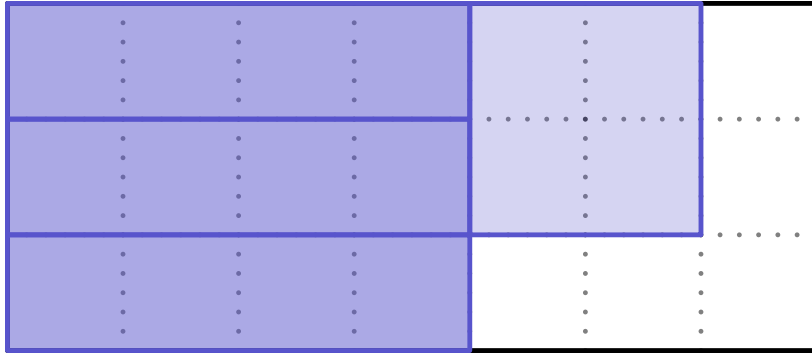
- A 1
- B 2
- C 3
- D 4
- E 5**

Solution:

The 2×2 and 1×4 tiles each have area 4. Since the rectangle has area 21, the number of 1×1 tiles must be congruent to 1 (mod 4). Among the choices, only 1 and 5 are possible by area.

It is impossible to use just one 1×1 tile. If the larger tiles covered the other 20 cells, then two rows would have all 7 cells covered by larger tiles. But each 2×2 tile covers 2 cells in any row it meets, and each 1×4 tile covers 4 cells in one row, so each row would have an even number of cells covered by larger tiles. A row cannot have 7 such cells.

The following tiling shows that 5 unit tiles are possible.



Thus, **E** is the correct answer.

8. On Monday Taye has \$2. Every day, he either gains \$3 or doubles the amount of money he had on the previous day. How many different dollar amounts could Taye have on Thursday, 3 days later?

- ☐ A 3
- ☐ B 4
- ☐ C 5
- ☒ D 6
- ☐ E 7

Solution:

After Tuesday, Taye could have 5 or 4 dollars. After Wednesday, the possible amounts are

8, 10, 7, 8,

so the distinct amounts are 7, 8, 10.

After Thursday, these can become

10, 14, 11, 16, 13, 20.

These are 6 distinct dollar amounts.

Thus, **D** is the correct answer.

9. All of the marbles in Maria's collection are red, green, or blue. Maria has half as many red marbles as green marbles and twice as many blue marbles as green marbles. Which of the following could be the total number of marbles in Maria's collection?

- A 24
- B 25
- C 26
- D 27
- E 28**

Solution:

We can let r be the number of red marbles that Maria has. Since Maria has half as many red marbles as green, then we know that she has $2r$ green marbles. Moreover, since she has twice as many blue marbles as green, then she will have $2(2r) = 4r$ blue marbles. Adding these together gives us $r + 2r + 4r = 7r$, and so the answer must be a multiple of 7. Among the answer choices, only 28 is a multiple of 7.

Thus, **E** is the correct answer.

10. In January 1980 the Mauna Loa Observatory recorded carbon dioxide CO₂ levels of 338 ppm (parts per million). Over the years the average CO₂ reading has increased by about 1.515 ppm each year. What is the expected CO₂ level in ppm in January 2030? Round your answer to the nearest integer.

A 399

B 414

C 420

D 444

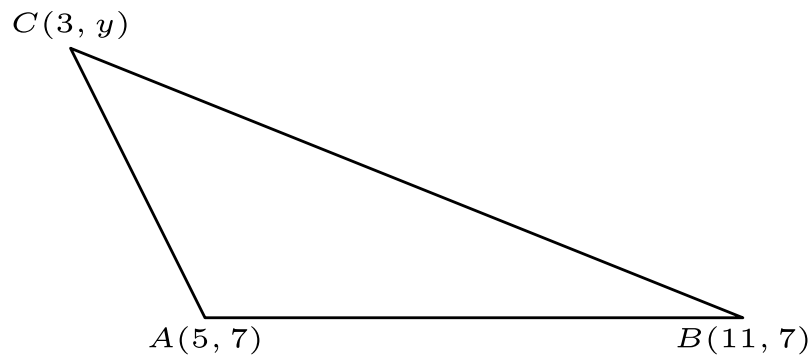
E 459

Solution:

There are 50 years between 1980 and 2030, so we can expect the CO₂ reading to increase by $50 \times 1.515 = 75.75 \approx 76$ ppm by 2030. Since the CO₂ reading in 1980 was 338 ppm, then we will have $338 + 76 = 414$ ppm by 2030.

Thus, **B** is the correct answer.

11. The coordinates of $\triangle ABC$ are $A(5, 7)$, $B(11, 7)$, and $C(3, y)$, with $y > 7$. The area of $\triangle ABC$ is 12. What is the value of y ?



- ☐ A 8
- ☐ B 9
- ☐ C 10
- ☒ D 11
- ☐ E 12

Solution:

Consider the base of the triangle to be $\overline{AB}$ which has length $11 - 5 = 6$. Given that the area of the triangle is 12, its height must be of length $\frac{2(12)}{6} = 4$. Since $y > 7$, then y must be $7 + 4 = 11$.

Thus, **D** is the correct answer.

12. Rohan keeps a total of 90 guppies in 4 fish tanks.

- There is 1 more guppy in the 2nd tank than in the 1st tank.
- There are 2 more guppies in the 3rd tank than in the 2nd tank.
- There are 3 more guppies in the 4th tank than in the 3rd tank.

How many guppies are in the 4th tank?

- ☐ A 20
- ☐ B 21
- ☐ C 23
- ☐ D 24
- ☒ E 26

Solution:

Let x be the number of guppies in the 1st tank. Hence, there are $x + 1$ guppies in the 2nd tank, $x + 3$ guppies in the 3rd tank, and $x + 6$ guppies in the 4th tank. We then use the fact that there are a total of 90 guppies in the 4 tanks to find x :

$$x + x + 1 + x + 3 + x + 6 = 90$$

$$4x + 10 = 90$$

$$x = 20.$$

Note that we are **not** yet done since we are asked for the number of guppies in the 4th tank and not the 1st. There are $x + 6 = 20 + 6 = 26$ guppies in the 4th tank.

Thus, **E** is the correct answer.

13. Buzz Bunny is hopping up and down a set of stairs, one step at a time. In how many ways can Buzz start on the ground, make a sequence of 6 hops, and end up back on the ground? (For example, one sequence of hops is up-up-down-down-up-down.)

- A 4
- B 5**
- C 6
- D 8
- E 12

Solution:

We can deduce from the choices that it is possible to exhaust all possible cases for this problem. Note that all sequences must start with up (U) and end with down (D), and that it should not be possible to go down more times than Buzz has gone up so far. Keeping this in mind, we can arrive at the following possible cases:

$UUUDDD$

$UUDUDD$

$UUDDUD$

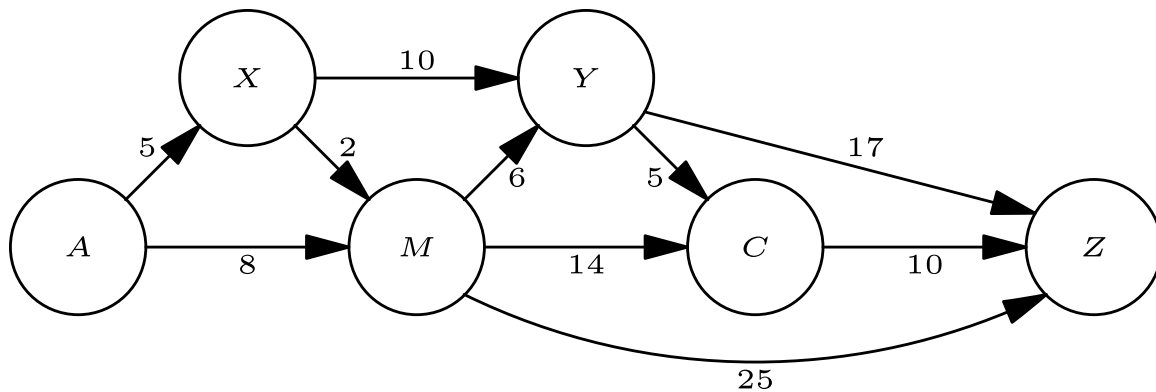
$UDUDUD$

$UDUDD$

which is a total of five possible sequences.

Thus, **B** is the correct answer.

14. The one-way routes connecting towns A , M , C , X , Y , and Z are shown in the figure below (not drawn to scale). The distances in kilometers along each route are marked. Traveling along these routes, what is the shortest distance from A to Z in kilometers?



A 28

B 29

C 30

D 31

E 32

Solution:

A systematic way of tracking the shortest overall distance to Z is to consider the shortest distance to get to each town from A . For instance, the shortest distance to get to town X from A is 5 km, trivially.

Then, for town M , going to town X first will be shorter compared to going directly from A , so the shortest path to town M has a length of 7 km.

For town Y , it will take us 15 km if we come from town X and only 13 km coming from M , so 13 km is the length of shortest path to Y from A .

Doing the same for town C will give us 18 km as the shortest distance by coming from town Y .

Finally, for town Z , we can either come from town Y , C , or M . The total distance if we come from each three towns respectively would be 30, 28, and 32. Hence, 28 km is the shortest distance from A to Z .

Thus, **A** is the correct answer.

15. Let the letters F, L, Y, B, U, G represent distinct digits. Suppose FLYFLY is the greatest number that satisfies the equation

$$8 \cdot \text{FLYFLY} = \text{BUGBUG}.$$

What is the value of $\text{FLY} + \text{BUG}$?

- ☐ A 1089
- ☐ B 1098
- ☒ C 1107
- ☐ D 1116
- ☐ E 1125

Solution:

Firstly, note that

$$\text{FLYFLY} = 1001(\text{FLY})$$

and, similarly,

$$\text{BUGBUG} = 1001(\text{BUG})$$

so the equation can be simplified to $8 \cdot \text{FLY} = \text{BUG}$.

Because BUG has three digits, $\text{FLY} \leq 124$. The largest possible three-digit number at most 124 with distinct digits is 124, but $8 \cdot 124 = 992$ repeats a digit and also reuses the digit 2.

The next candidate is 123, and $8 \cdot 123 = 984$. The six digits 1, 2, 3, 9, 8, 4 are all distinct, so 123 is the greatest possible value of FLY .

Hence,

$$\begin{aligned}\text{FLY} + \text{BUG} &= 123 + 984 \\ &= 1107.\end{aligned}$$

Thus, **C** is the correct answer.

16. Minh enters the numbers 1 through 81 into the cells of a 9×9 grid in some order. She calculates the product of the numbers in each row and column. What is the least number of rows and columns that could have a product divisible by 3?

- A 8
- B 9
- C 10
- D 11**
- E 12

Solution:

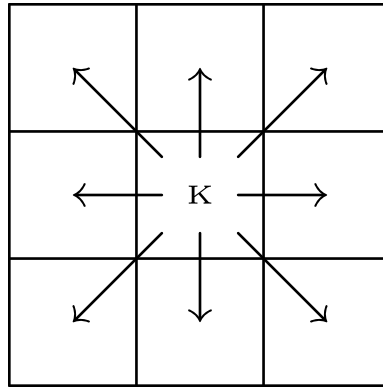
There are 27 multiples of 3 from 1 through 81. A row or column has product divisible by 3 exactly when it contains at least one of these multiples.

Suppose r rows and c columns have products divisible by 3. Every multiple of 3 must lie in one of those r rows and also in one of those c columns, or else it would create another marked row or column. Thus the 27 multiples must fit in the rc intersection cells. If $r + c \leq 10$, then $rc \leq 25$, which is too small. So at least 11 rows and columns are needed.

This can be done by placing 25 multiples of 3 in a 5×5 block, then placing the remaining 2 multiples in a sixth column within two of those same rows. Then exactly 5 rows and 6 columns are marked, for a total of 11.

Thus, **D** is the correct answer.

17. A chess king is said to *attack* all the squares one step away from it, horizontally, vertically, or diagonally. For instance, a king on the center square of a 3×3 grid attacks all 8 other squares, as shown below. Suppose a white king and a black king are placed on different squares of a 3×3 grid so that they do not attack each other. In how many ways can this be done?



- A 20
- B 24
- C 27
- D 28
- E 32**

Solution:

Count ordered placements by first choosing the square for the white king. If the white king is in the center, it attacks every other square, so there are 0 choices for the black king.

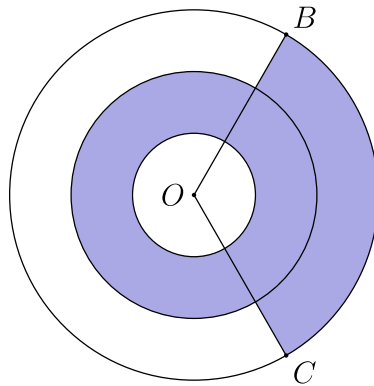
If the white king is in a corner, it attacks 3 squares, so the black king has $9 - 1 - 3 = 5$ safe squares. There are 4 corner choices, giving $4 \cdot 5 = 20$ placements.

If the white king is on an edge but not a corner, it attacks 5 squares, so the black king has $9 - 1 - 5 = 3$ safe squares. There are 4 such edge choices, giving $4 \cdot 3 = 12$ placements.

The total number of placements is $20 + 12 = 32$.

Thus, **E** is the correct answer.

18. Three concentric circles centered at O have radii of 1, 2, and 3. Points B and C lie on the largest circle. The region between the two smaller circles is shaded, as is the portion of the region between the two larger circles bounded by central angle BOC , as shown in the figure below. Suppose the shaded and unshaded regions are equal in area. What is the measure of $\angle BOC$ in degrees?



A 108

B 120

C 135

D 144

E 150

Solution:

Let θ be the measure of $\angle BOC$.

One component of the shaded region is the area of the circle with radius 2 minus the area of the circle with radius 1. This part has area $4\pi - \pi = 3\pi$. The remaining area is a sector of the biggest circle minus the area of the circle with radius 2. This has area $\frac{\theta}{360}(9\pi - 4\pi) = \frac{\theta}{360}(5\pi)$. Hence, the total area of the shaded region is $3\pi + \frac{\theta}{360}(5\pi)$.

Next, we note that the unshaded region is composed of the smallest circle and the unshaded portion of the outer ring. This will have a total area of $\pi + \frac{360 - \theta}{360}(5\pi)$

Lastly, we equate the area of both regions and solve for θ :

$$\begin{aligned}3\pi + \frac{\theta}{360}(5\pi) \\&= \pi + \frac{360 - \theta}{360}(5\pi) \\2\pi &= \frac{360 - \theta - \theta}{360}(5\pi) \\\frac{2}{5} &= 1 - \frac{2\theta}{360} \\2\theta &= \frac{3}{5}(360) \\\theta &= 108.\end{aligned}$$

Thus, **A** is the correct answer.

19. Jordan owns 15 pairs of sneakers. Three fifths of the pairs are red and the rest are white. Two thirds of the pairs are high-top and the rest are low-top. The red high-top sneakers make up a fraction of the collection. What is the least possible value of this fraction?

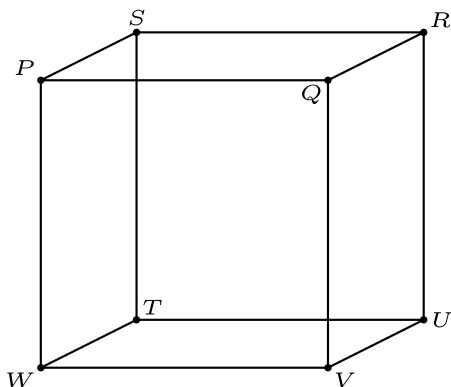
- A 0
- B $\frac{1}{5}$
- C $\frac{4}{15}$**
- D $\frac{1}{3}$
- E $\frac{2}{5}$

Solution:

Jordan has $\frac{3}{5} \times 15 = 9$ pairs of red sneakers and 6 pairs of white sneakers. Moreover, $\frac{2}{3} \times 15 = 10$ are high-top and 5 are low-top. If we want to minimize the number of red high-top sneakers, then we can set all 6 white sneakers to be high-top, leaving $10 - 6 = 4$ red sneakers as high-top. Hence, the fraction of red high-top sneakers would be $\frac{4}{15}$.

Thus, **C** is the correct answer.

20. Any three vertices of the cube $PQRSTUW$, shown in the figure below, can be connected to form a triangle. (For example, vertices P , Q , and R can be connected to form isosceles $\triangle PQR$.) How many of these triangles are equilateral and contain P as a vertex?



- A 0
- B 1
- C 2
- D 3**
- E 6

Solution:

We first note that we can only form equilateral triangles if we go through the diagonals of the square faces, otherwise at least one angle of the triangle will be different. Afterwards, it is easy to exhaust all possible equilateral triangles that can be formed: $\triangle PVT$, $\triangle PRT$, and $\triangle PRV$.

Thus, **D** is the correct answer.

21. A group of frogs (called an *army*) is living in a tree. A frog turns green when in the shade and turns yellow when in the sun. Initially, the ratio of green to yellow frogs was $3 : 1$. Then 3 green frogs moved to the sunny side and 5 yellow frogs moved to the shady side. Now the ratio is $4 : 1$. What is the difference between the number of green frogs and yellow frogs now?

A 10

B 12

C 16

D 20

E 24

Solution:

We can let g be the number of green frogs and y be the number of yellow frogs. Initially, we have $g = 3y$. Then, after some frogs moved, we have the following proportion:

$$\frac{g + 5 - 3}{y + 3 - 5} = \frac{4}{1}.$$

Substituting $3y$ for g will allow us to determine the number of yellow frogs originally (y):

$$\frac{3y + 2}{y - 2} = 4$$

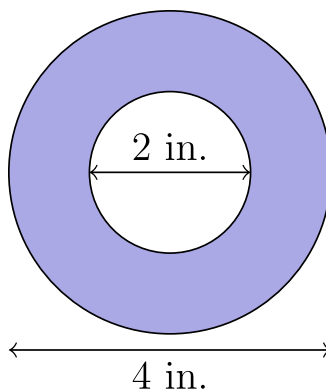
$$3y + 2 = 4(y - 2) = 4y - 8$$

$$y = 10$$

Hence, there were 10 yellow frogs and $3 \times 10 = 30$ green frogs initially. After some frogs moved, we now have $10 + 3 - 5 = 8$ yellow frogs and $30 + 5 - 3 = 32$ green frogs, giving us a difference of $32 - 8 = 24$ between the number of green and yellow frogs.

Thus, **E** is the correct answer.

22. A roll of tape is 4 inches in diameter and is wrapped around a ring that is 2 inches in diameter. A cross section of the tape is shown in the figure below. The tape is 0.015 inches thick. If the tape is completely unrolled, approximately how long would it be? Round your answer to the nearest 100 inches.



- A 300
- B 600**
- C 1200
- D 1500
- E 1800

Solution:

The tape occupies an annulus with outer radius 2 inches and inner radius 1 inch, so its cross-sectional area is

$$\pi(2^2 - 1^2) = 3\pi.$$

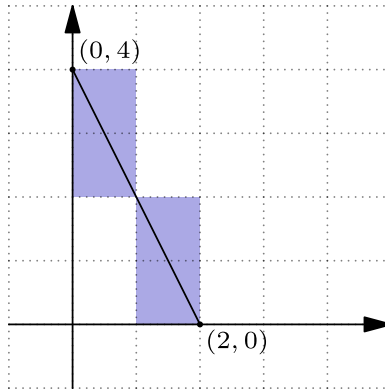
When unrolled, this same cross section is a rectangle of thickness 0.015 inches and length L . Thus

$$0.015L = 3\pi,$$

so $L = 200\pi \approx 628$ inches. Rounded to the nearest 100 inches, this is 600 inches.

Thus, **B** is the correct answer.

23. Rodrigo has a very large piece of graph paper. First he draws a line segment connecting point $(0, 4)$ to point $(2, 0)$ and colors the 4 cells whose interiors intersect the segment, as shown below. Next, Rodrigo draws a line segment connecting point $(2000, 3000)$ to point $(5000, 8000)$. Again he colors the cells whose interiors intersect the segment. How many cells will he color this time?



- A 6000
- B 6500
- C 7000**
- D 7500
- E 8000

Solution:

For a segment whose endpoint differences are a horizontally and b vertically, the segment crosses a vertical grid lines and b horizontal grid lines, but crossings at lattice points are counted twice. Therefore the number of cells whose interiors are intersected is

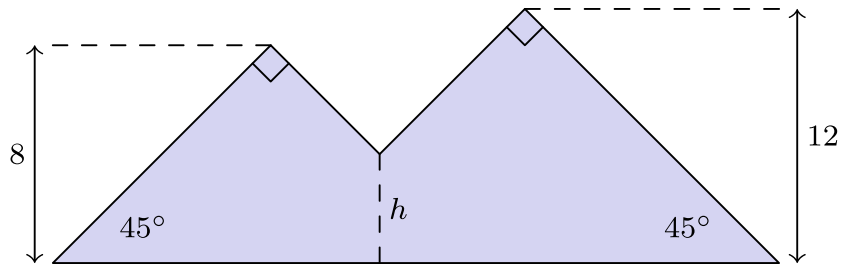
$$a + b - \gcd(a, b).$$

Here the endpoint differences are 3000 and 5000, and $\gcd(3000, 5000) = 1000$. Thus the number of cells colored is

$$3000 + 5000 - 1000 = 7000.$$

Thus, **C** is the correct answer.

24. Jean made a piece of stained glass art in the shape of two mountains, as shown in the figure below. One mountain peak is 8 feet high and the other peak is 12 feet high. Each peak forms a 90° angle, and the straight sides of the mountains form 45° angles with the ground. The artwork has an area of 183 square feet. The sides of the mountains meet at an intersection point near the center of the artwork, h feet above the ground. What is the value of h ?



- A 4
- B 5**
- C $4\sqrt{2}$
- D 6
- E $5\sqrt{2}$

Solution:

Each mountain is a 45° - 45° - 90° triangle. A right isosceles triangle with height x has area x^2 , since its two perpendicular sides each have length $x\sqrt{2}$.

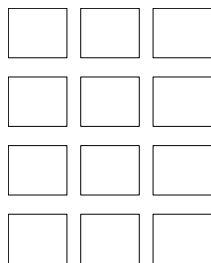
The two large mountains have areas $8^2 = 64$ and $12^2 = 144$. Their overlap is also a 45° - 45° - 90° triangle with height h , so its area is h^2 . Thus

$$64 + 144 - h^2 = 183,$$

so $h^2 = 25$, and $h = 5$.

Thus, **B** is the correct answer.

25. A small airplane has 4 rows of seats with 3 seats in each row. Eight passengers have boarded the plane and are distributed randomly among the seats. A married couple is next to board. What is the probability there will be 2 adjacent seats in the same row for the couple?



- A $\frac{8}{15}$
- B $\frac{32}{55}$
- C $\frac{20}{33}$
- D $\frac{34}{55}$
- E $\frac{8}{11}$

Solution:

After the first 8 passengers sit, there are 4 empty seats among the 12 seats, so there are $\binom{12}{4} = 495$ equally likely sets of empty seats.

Count the complement, where no row has two adjacent empty seats. In a row of three seats, the possible empty-seat patterns with no adjacent empty seats have sizes 0, 1, 2, with 1, 3, 1 choices respectively. So we need the coefficient of x^4 in

$$(1 + 3x + x^2)^4.$$

This coefficient is

$$\begin{aligned} 3^4 + 4 \cdot 3 \cdot 3^2 + \binom{4}{2} \\ = 81 + 108 + 6 \\ = 195. \end{aligned}$$

Therefore the probability that at least one row has adjacent empty seats is

$$\frac{495 - 195}{495} = \frac{20}{33}.$$

Thus, **C** is the correct answer.

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