

2020 AMC 8 Solutions

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1. Luka is making lemonade to sell at a school fundraiser. His recipe requires 4 times as much water as sugar and twice as much sugar as lemon juice. He uses 3 cups of lemon juice. How many cups of water does he need?

A 6

B 8

C 12

D 18

E 24

Solution:

Since Luka needs twice as much sugar as lemon, he needs $2 \cdot 3 = 6$ cups of sugar.

Since Luka also needs 4 times as much water as sugar, he needs $4 \cdot 6 = 24$ cups of water.

Thus, the correct answer is **E**.

2. Four friends do yardwork for their neighbors over the weekend, earning \$15, \$20, \$25, and \$40 respectively. They decide to split their earnings equally among themselves. In total how many dollars will the friend who earned \$40 give to the others?

A 5

B 10

C 15

D 20

E 25

Solution:

First, the total amount of money that they make is $\$15 + \$20 + \$25 + \$40 = \$100$.

Since they divide this equally, they each get $\frac{100}{4} = 25$ dollars.

The friend who earned \$40 therefore gives away $\$40 - \$25 = \$15$.

Thus, the correct answer is **C**.

3. Carrie has a rectangular garden that measures 6 feet by 8 feet. She plants the entire garden with strawberry plants. Carrie is able to plant 4 strawberry plants per square foot, and she harvests an average of 10 strawberries per plant. How many strawberries can she expect to harvest?

A 560

B 960

C 1120

D 1920

E 3840

Solution:

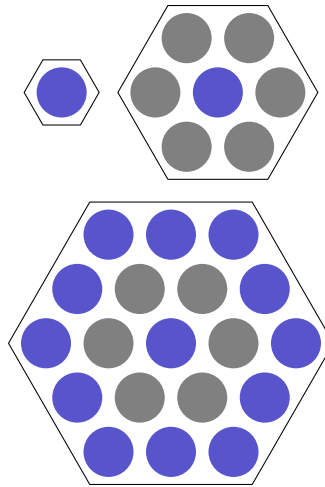
First, the size of the garden is $6 \cdot 8 = 48$ square feet.

Next, since there are 4 plants per square foot, Carrie can plant $4 \cdot 48 = 192$ plants total.

Finally, since there are 10 strawberries per plant, Carrie can harvest $10 \cdot 192 = 1920$ strawberries total.

Thus, the correct answer is **D**.

4. Three hexagons of increasing size are shown below. Suppose the dot pattern continues so that each successive hexagon contains one more band of dots. How many dots are in the next hexagon?



- A 35
- B 37**
- C 39
- D 43
- E 49

Solution:

The first three hexagons contain 1, 7, and 19 dots. Each new band adds 6 more dots than the previous band.

The fourth hexagon adds 18 dots around the third hexagon, so it contains $19 + 18 = 37$ dots.

Thus, the correct answer is **B**.

5. Three fourths of a pitcher is filled with pineapple juice. The pitcher is emptied by pouring an equal amount of juice into each of 5 cups. What percent of the total capacity of the pitcher did each cup receive?

A 5

B 10

C 15

D 20

E 25

Solution:

Each cup receives one fifth of the $\frac{3}{4}$ of a pitcher, or

$$\frac{1}{5} \cdot \frac{3}{4} = \frac{3}{20} = 15\%.$$

Thus, the correct answer is **C**.

6. Aaron, Darren, Karen, Maren, and Sharon rode on a small train that has five cars that seat one person each. Maren sat in the last car. Aaron sat directly behind Sharon. Darren sat in one of the cars in front of Aaron. At least one person sat between Karen and Darren. Who sat in the middle car?

- ☒ A Aaron
- ☐ B Darren
- ☐ C Karen
- ☐ D Maren
- ☐ E Sharon

Solution:

Maren is in the last car. Since Aaron is directly behind Sharon and Darren is in front of Aaron, the possible placements of Darren, Sharon, and Aaron before Maren are limited.

If Sharon and Aaron were in cars 1 and 2, Darren could not be in front of Aaron. If they were in cars 3 and 4, then Darren and Karen would have to occupy cars 1 and 2, which are adjacent.

Thus Sharon and Aaron must be in cars 2 and 3. Darren is then in car 1, Karen is in car 4, and Aaron is in the middle car.

Thus, the correct answer is **A**.

7. How many integers between 2020 and 2400 have four distinct digits arranged in increasing order? (For example, 2357 is one integer.)

- ☐ A 9
- ☐ B 10
- ☒ C 15
- ☐ D 21
- ☐ E 28

Solution:

The thousands digit must be 2. Since the digits are distinct and increasing, the hundreds digit must be 3.

The last two digits must be chosen from 4, 5, 6, 7, 8, 9. Once the two digits are chosen, their order is forced.

There are $\binom{6}{2} = 15$ such integers.

Thus, the correct answer is **C**.

8. Ricardo has 2020 coins, some of which are pennies (1-cent coins) and the rest of which are nickels (5-cent coins). He has at least one penny and at least one nickel. What is the difference in cents between the greatest possible and least possible amounts of money that Ricardo can have?

A 8062

B 8068

C 8072

D 8076

E 8082

Solution:

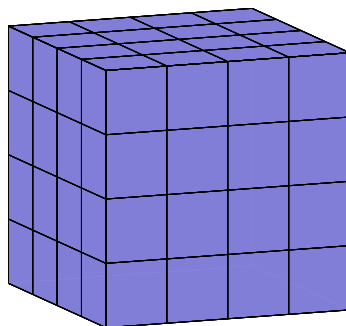
The greatest value occurs with 2019 nickels and 1 penny, while the least occurs with 1 nickel and 2019 pennies.

Between these two cases, 2018 pennies have been replaced by nickels. Each replacement adds $5 - 1 = 4$ cents, so the difference is

$$2018 \cdot 4 = 8072 \text{ cents.}$$

Thus, the correct answer is **C**.

9. Akash's birthday cake is in the form of a $4 \times 4 \times 4$ inch cube. The cake has icing on the top and the four side faces, and no icing on the bottom. Suppose the cake is cut into 64 smaller cubes, each measuring $1 \times 1 \times 1$ inch, as shown below. How many small pieces will have icing on exactly two sides?



- ☐ A 12
- ☐ B 16
- ☐ C 18
- ☒ D 20
- ☐ E 24

Solution:

In the top layer, the non-corner edge pieces have icing on exactly two sides. There are 4 edges with 2 such pieces each, for 8 pieces.

In each of the other 3 layers, the only pieces with exactly two iced sides are the 4 corner pieces. This adds $3 \cdot 4 = 12$ pieces.

The total is $8 + 12 = 20$.

Thus, the correct answer is **D**.

10. Zara has a collection of 4 marbles: an Aggie, a Bumblebee, a Steelie, and a Tiger. She wants to display them in a row on a shelf, but does not want to put the Steelie and the Tiger next to one another. In how many ways can she do this?

- A 6
- B 8
- C 12**
- D 18
- E 24

Solution:

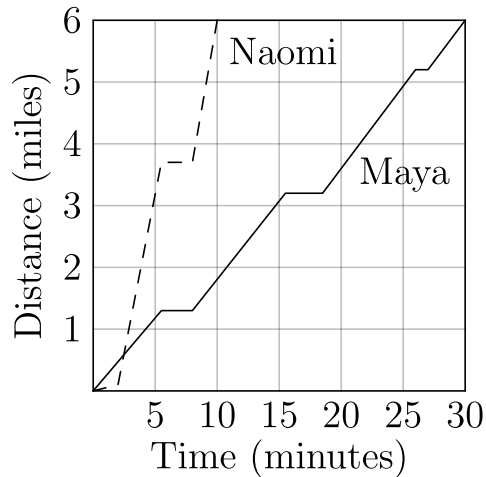
There are $4! = 24$ total arrangements of the marbles.

If the Steelie and Tiger are adjacent, treat them as one block. Then there are $3!$ ways to arrange the block with the other two marbles, and 2 orders inside the block, for $3! \cdot 2 = 12$ adjacent arrangements.

Therefore, $24 - 12 = 12$ arrangements keep the Steelie and Tiger separated.

Thus, the correct answer is **C**.

11. After school, Maya and Naomi headed to the beach, 6 miles away. Maya decided to bike while Naomi took a bus. The graph below shows their journeys, indicating the time and distance traveled. What was the difference, in miles per hour, between Naomi's and Maya's average speeds?



- A 6
- B 12
- C 18
- D 20
- E 24**

Solution:

Naomi traveled 6 miles in 10 minutes, so her average speed was

$$\frac{6}{\frac{10}{60}} = 36$$

miles per hour.

Maya traveled 6 miles in 30 minutes, so her average speed was

$$\frac{6}{\frac{30}{60}} = 12$$

miles per hour.

This difference is $36 - 12 = 24$.

Thus, the correct answer is **E**.

- 12.** For a positive integer n , the factorial notation $n!$ represents the product of the integers from n to 1. For example:

$$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

What value of N satisfies the following equation?

$$5! \times 9! = 12 \times N!$$

A 10

B 11

C 12

D 13

E 14

Solution:

Because $5! = 120 = 12 \cdot 10$, the equation becomes

$$12 \cdot 10 \cdot 9! = 12 \cdot N!.$$

Canceling 12 gives $N! = 10 \cdot 9! = 10!$, so $N = 10$.

Thus, the correct answer is **A**.

13. Jamal has a drawer containing 6 green socks, 18 purple socks, and 12 orange socks. After adding more purple socks, Jamal noticed that there is now a 60% chance that a sock randomly selected from the drawer is purple. How many purple socks did Jamal add?

- A 6
- B 9**
- C 12
- D 18
- E 24

Solution:

Suppose Jamal adds s purple socks. Then, there will be $s + 18$ purple socks.

Also, since there are 36 total socks to begin with, we have $s + 36$ socks after adding the socks.

Since we have a 60% chance of choosing a purple sock afterwards, we know

$$\frac{s + 18}{s + 36} = 0.6.$$

Solving for s yields:

$$s + 18 = 0.6s + 21.6$$

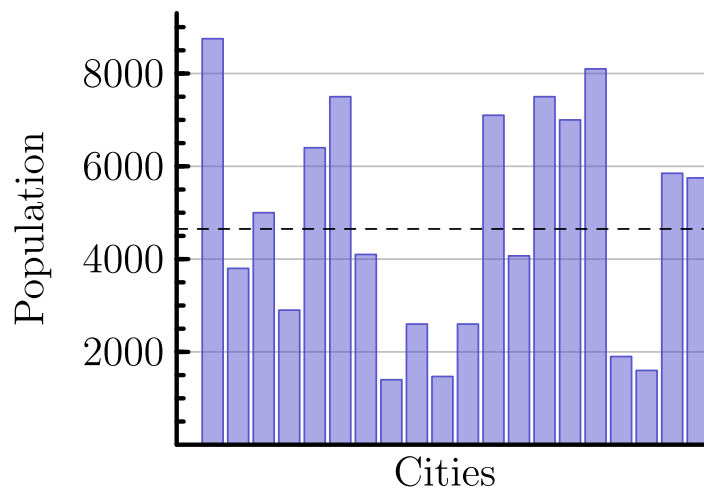
$$0.4s = 3.6$$

$$s = 9.$$

Therefore, 9 socks are added.

Thus, the correct answer is **B**.

14. There are 20 cities in the County of Newton. Their populations are shown in the bar chart below. The average population of all the cities is indicated by the horizontal dashed line. Which of the following is closest to the total population of all 20 cities?



- A 65,000
- B 75,000
- C 85,000
- D 95,000**
- E 105,000

Solution:

Looking at the horizontal dashed line, the average population is around 4750.

Since there are 20 cities, the total population is approximately

$$20 \cdot 4750 = 95,000.$$

Therefore, the total population is approximately 95,000.

Thus, the correct answer is **D**.

15. Suppose 15% of x equals 20% of y . What percentage of x is y ?

A 5

B 35

C 75

D $133\frac{1}{3}$

E 300

Solution:

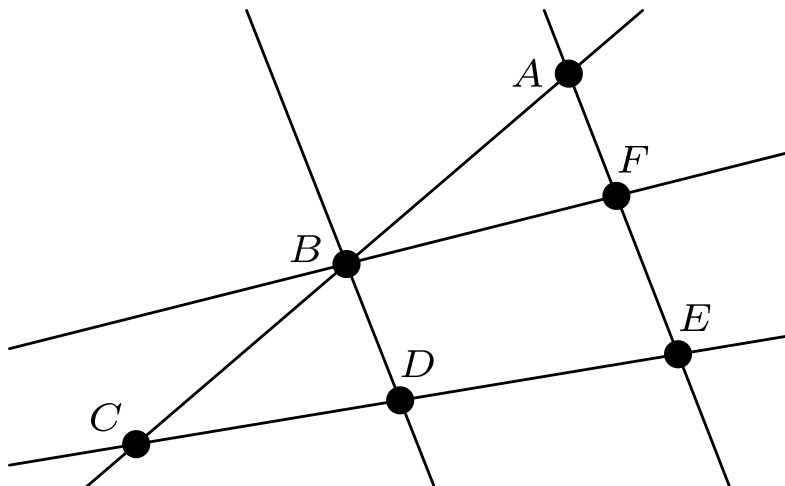
The statement gives $0.15x = 0.20y$.

Solving for y gives $y = \frac{0.15}{0.20}x = \frac{3}{4}x$.

Therefore, y is 75% of x .

Thus, the correct answer is **C**.

16. Each of the points A, B, C, D, E , and F in the figure below represents a different digit from 1 to 6. Each of the five lines shown passes through some of these points. The digits along each line are added to produce five sums, one for each line. The total of the five sums is 47. What is the digit represented by B ?



- ☐ A 1
- ☐ B 2
- ☐ C 3
- ☐ D 4
- ☒ E 5

Solution:

Every point is counted in two of the line sums except B , which is counted in three. Thus the total of the five line sums is twice the sum of all six digits, plus one extra B .

The digits are exactly 1, 2, 3, 4, 5, 6, whose sum is 21. Therefore

$$2 \cdot 21 + B = 47,$$

so $B = 5$.

Thus, the correct answer is **E**.

17. How many factors of 2020 have more than 3 factors? (As an example, 12 has 6 factors, namely 1, 2, 3, 4, 6, and 12.)

- ☐ A 6
- ☒ B 7
- ☐ C 8
- ☐ D 9
- ☐ E 10

Solution:

Since $2020 = 2^2 \cdot 5 \cdot 101$, it has

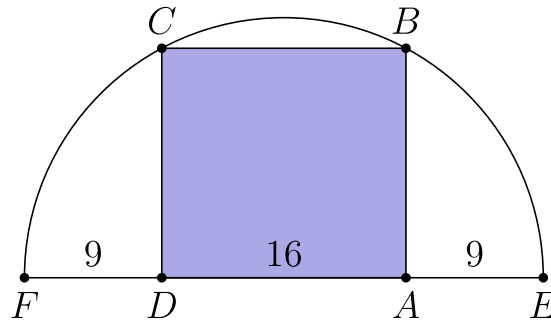
$$(2 + 1)(1 + 1)(1 + 1) = 12$$

positive factors.

Among those factors, only 1 and the primes 2, 5, 101 have fewer than three factors, while 4 has exactly three. The other $12 - 5 = 7$ factors therefore have more than three factors.

Thus, the correct answer is **B**.

18. Rectangle $ABCD$ is inscribed in a semicircle with diameter $\overline{FE}$, as shown in the figure. Let $DA = 16$, and let $FD = AE = 9$. What is the area of $ABCD$?



A 240

B 248

C 256

D 264

E 272

Solution:

Since FE is the diameter of the semicircle, we know the length of the diameter is 34, and so the radius is 17. Let O be the center of the diameter.

The length from OF therefore is 17.

Since D is on OF , we know

$$OD + FD = OF$$

$$OD + 9 = 17$$

$$OD = 8.$$

Also, since we have a semicircle, we know $OC = 17$.

Finally, since $ABCD$ is a rectangle, we know $\angle ODC$ is a right angle. This means we can find DC by the Pythagorean Theorem. We know

$$OD^2 + DC^2 = OC^2$$

$$8^2 + DC^2 = 17^2$$

$$DC = 15.$$

Thus, the area of the rectangle is $DC \cdot DA = 15 \cdot 16 = 240$.

Thus, the correct answer is **A**.

19. A number is called flippy if its digits alternate between two distinct digits. For example, 2020 and 37373 are flippy, but 3883 and 123123 are not. How many five-digit flippy numbers are divisible by 15?

- ☐ A 3
- ☒ B 4
- ☐ C 5
- ☐ D 6
- ☐ E 8

Solution:

A five-digit flippy number has the form $ABABA$, where $A \neq 0$ and $A \neq B$.

Divisibility by 5 requires the last digit A to be 0 or 5. Since $A \neq 0$, we must have $A = 5$.

The number therefore has the form $5B5B5$. Its digit sum is $15 + 2B$, so divisibility by 3 requires B to be a multiple of 3. The possibilities are $B = 0, 3, 6, 9$, giving 4 numbers.

Thus, the correct answer is **B**.

20. A scientist walking through a forest recorded as integers the heights of 5 trees standing in a row. She observed that each tree was either twice as tall or half as tall as the one to its right. Unfortunately some of her data was lost when rain fell on her notebook. Her notes are shown below, with blanks indicating the missing numbers. Based on her observations, the scientist was able to reconstruct the lost data. What was the average height of the trees, in meters?

T_1	__ m
T_2	11 m
T_3	__ m
T_4	__ m
T_5	__ m
$\overline{T}$	__0.2 m

- A 22.2
- B 24.2**
- C 33.2
- D 35.2
- E 37.2

Solution:

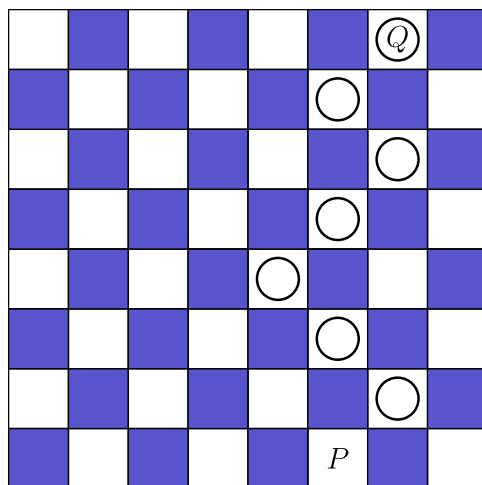
Tree 2 is 11 meters tall. Since all heights are integers and neighboring trees differ by a factor of 2, trees 1 and 3 must both be 22 meters tall.

The first three trees total $22 + 11 + 22 = 55$ meters. The possible integer pairs for trees 4 and 5 are (11, 22), (44, 22), and (44, 88).

These give averages 17.6, 24.2, and 37.4, respectively. The notebook shows that the average ends in 0.2, so it must be 24.2.

Thus, the correct answer is **B**.

21. A game board consists of 64 squares that alternate between shaded and unshaded. The figure below shows square P in the bottom row and square Q in the top row. A marker is placed at P . A step consists of moving the marker onto one of the adjoining unshaded squares in the row above. How many 7-step paths are there from P to Q ? (The figure shows a sample path.)



A 28

B 30

C 32

D 33

E 35

Solution:

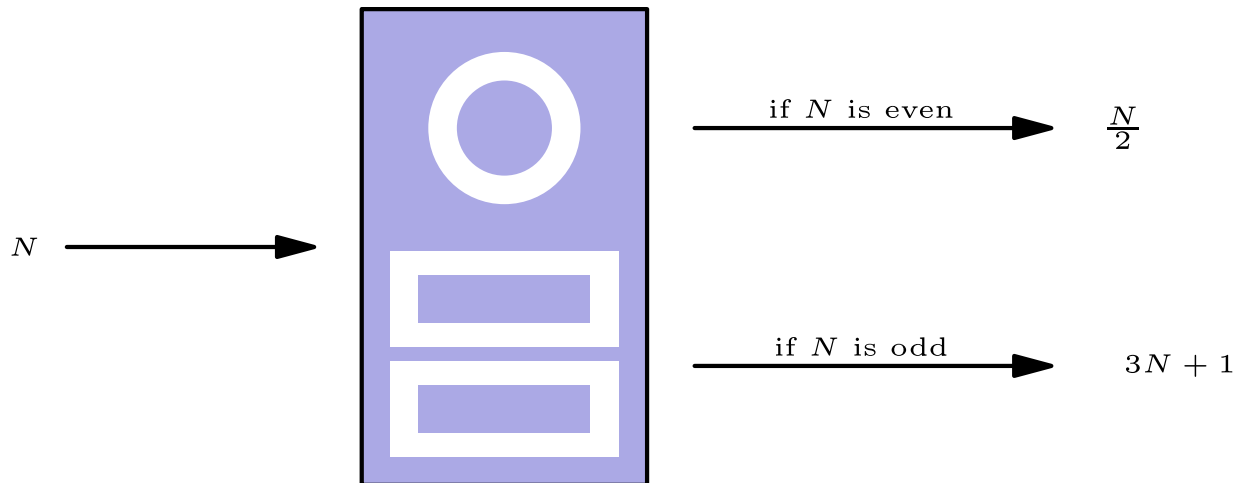
Each move must go up one row and either left or right. Counting row by row from P , each reachable square gets the sum of the counts from the two adjoining squares below it.

6		20		33		28	
	6		14		19		9
1		5		10		9	
	1		4		6		3
		1		3		3	
			1		2		1
				1		1	
				P			

The count at Q is 28, so there are 28 paths.

Thus, the correct answer is **A**.

22. When a positive integer N is fed into a machine, the output is a number calculated according to the rule shown below.



For example, starting with an input of $N = 7$, the machine will output $3 \cdot 7 + 1 = 22$. Then if the output is repeatedly inserted into the machine five more times, the final output is 26.

$$\begin{aligned} 7 &\rightarrow 22 \rightarrow 11 \rightarrow 34 \\ &\rightarrow 17 \rightarrow 52 \rightarrow 26 \end{aligned}$$

When the same 6-step process is applied to a different starting value of N , the final output is 1. What is the sum of all such integers N ?

$$\begin{aligned} N &\rightarrow _ \rightarrow _ \rightarrow _ \\ &\rightarrow _ \rightarrow _ \rightarrow 1 \end{aligned}$$

- A 73
- B 74
- C 75
- D 82
- E 83**

Solution:

Work backward from 1. A value m can always come from the even input $2m$. It can also come from the odd input $\frac{m-1}{3}$ when that expression is a positive odd integer.

Listing the possible values after each backward step gives

$$\begin{aligned}\{1\} &\leftarrow \{2\} \leftarrow \{4\} \leftarrow \{1, 8\} \\ &\leftarrow \{2, 16\} \leftarrow \{4, 5, 32\} \\ &\leftarrow \{1, 8, 10, 64\}.\end{aligned}$$

Thus the possible starting values are 1, 8, 10, 64, whose sum is 83.

Thus, the correct answer is **E**.

- 23.** Five different awards are to be given to three students. Each student will receive at least one award. In how many different ways can the awards be distributed?

A 120

B 150

C 180

D 210

E 240

Solution:

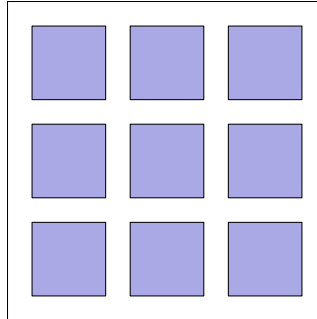
There are $3^5 = 243$ ways to give each of the 5 distinct awards to one of the 3 students.

Subtract the distributions in which at least one student receives no award. If a particular student receives none, the awards go to the other two students in 2^5 ways. This gives $3 \cdot 2^5$ counts, but the 3 cases in which one student receives all awards have each been subtracted twice.

By inclusion-exclusion, the desired number is $3^5 - 3 \cdot 2^5 + 3 = 150$.

Thus, the correct answer is **B**.

24. A large square region is paved with n^2 shaded square tiles, each measuring s inches on a side. A border d inches wide surrounds each tile. The figure below shows the case for $n = 3$. When $n = 24$, the 576 shaded tiles cover 64% of the area of the large square region. What is the ratio $\frac{d}{s}$ for this larger value of n ?



- A $\frac{6}{25}$
- B $\frac{1}{4}$
- C $\frac{9}{25}$
- D $\frac{7}{16}$
- E $\frac{9}{16}$

Solution:

For $n = 24$, the shaded tile area is $24^2 s^2$. Each side of the large square consists of 24 tiles and 25 borders, so its side length is $24s + 25d$.

The shaded tiles cover $64\% = \frac{16}{25}$ of the large square, so

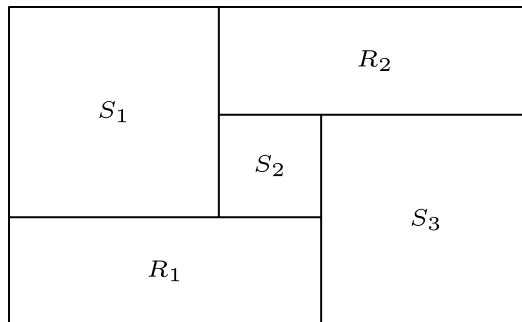
$$\frac{24^2 s^2}{(24s + 25d)^2} = \frac{16}{25}.$$

Taking positive square roots gives $\frac{24s}{24s + 25d} = \frac{4}{5}$.

Thus $120s = 96s + 100d$, so $24s = 100d$ and $\frac{d}{s} = \frac{6}{25}$.

Thus, the correct answer is **A**.

25. Rectangles R_1 and R_2 , and squares S_1 , S_2 , and S_3 , shown below, combine to form a rectangle that is 3322 units wide and 2020 units high. What is the side length of S_2 in units?



A 651

B 655

C 656

D 662

E 666

Solution:

Let s_1 , s_2 , s_3 be the side lengths of the three squares. Across the width of the large rectangle,

$$s_1 + s_2 + s_3 = 3322.$$

The height of R_2 is $s_1 - s_2$, so the total height is

$$s_1 - s_2 + s_3 = 2020.$$

Subtracting the height equation from the width equation gives $2s_2 = 1302$, so $s_2 = 651$.

Thus, the correct answer is **A**.

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