

AMC 12 PRACTICE ARCHIVE

2019

12A and 12B papers + answer keys

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Source and use note

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2019 AMC 12A

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Question 1

Not yet answered

Points out of 6

The area of a pizza with radius 4 is N percent larger than the area of a pizza with radius 3 inches. What is the integer closest to N ?

- (A) 25
- (B) 33
- (C) 44
- (D) 66
- (E) 78

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 2

Not yet answered

Points out of 6

Suppose a is 150% of b . What percent of a is $3b$?

- (A) 50
- (B) 66
- (C) 150
- (D) 200
- (E) 450

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 3

Not yet answered

Points out of 6

A box contains 28 red balls, 20 green balls, 19 yellow balls, 13 blue balls, 11 white balls, and 9 black balls. What is the minimum number of balls that must be drawn from the box without replacement to guarantee that at least 15 balls of a single color will be drawn?

(A) 75**(B) 76****(C) 79****(D) 84****(E) 91**

Select one:

☐ **A**☐ **B**☐ **C**☐ **D**☐ **E**☐ **Leave blank (1.5 points)****Question 4**

Not yet answered

Points out of 6

What is the greatest number of consecutive integers whose sum is 45?

(A) 9**(B) 25****(C) 45****(D) 90****(E) 120**

Select one:

☐ **A**☐ **B**☐ **C**☐ **D**☐ **E**☐ **Leave blank (1.5 points)**

Question 5

Not yet answered

Points out of 6

Two lines with slopes $\frac{1}{2}$ and 2 intersect at $(2, 2)$. What is the area of the triangle enclosed by these two lines and the line $x + y = 10$?

- (A) 4
- (B) $4\sqrt{2}$
- (C) 6
- (D) 8
- (E) $6\sqrt{2}$

Select one:

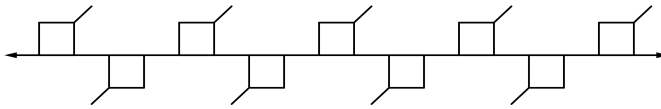
- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 6

Not yet answered

Points out of 6

The figure below shows line ℓ with a regular, infinite, recurring pattern of squares and line segments.



How many of the following four kinds of rigid motion transformations of the plane in which this figure is drawn, other than the identity transformation, will transform this figure into itself?

- some rotation around a point of line ℓ
- some translation in the direction parallel to line ℓ
- the reflection across line ℓ
- some reflection across a line perpendicular to line ℓ

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

Select one:

☐ A

☐ B

☐ C

☐ D

☐ E

☐ Leave blank (1.5 points)

Question 7

Not yet answered

Points out of 6

Melanie computes the mean μ , the median M , and the modes of the 365 values that are the dates in the months of 2019. Thus her data consists of 12 1s, 12 2s, ..., 12 28s, 11 29s, 11 30s, and 7 31s. Let d be the median of the modes. Which of the following statements is true?

(A) $\mu < d < M$

(B) $M < d < \mu$

(C) $d = M = \mu$

(D) $d < M < \mu$

(E) $d < \mu < M$

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 8

Not yet answered

Points out of 6

For a set of four distinct lines in a plane, there are exactly N distinct points that lie on two or more of the lines. What is the sum of all possible values of N ?

(A) 14

(B) 16

(C) 18

(D) 19

(E) 21

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 9

Not yet answered

Points out of 6

A sequence of numbers is defined recursively by $a_1 = 1$, $a_2 = \frac{3}{7}$, and

$$a_n = \frac{a_{n-2} \cdot a_{n-1}}{2a_{n-2} - a_{n-1}}$$

for all $n \geq 3$. Then a_{2019} can be written as $\frac{p}{q}$, where p and q are relatively prime positive integers. What is $p + q$?

- (A) 2020
- (B) 4039
- (C) 6057
- (D) 6061
- (E) 8078

Select one:

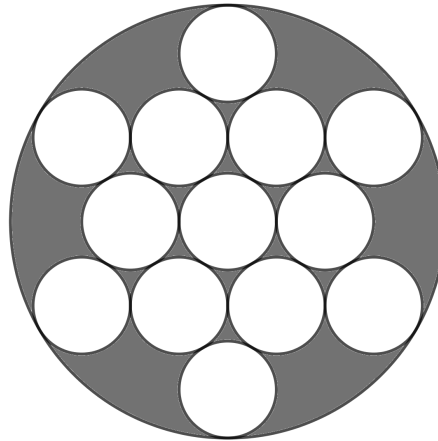
- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 10

Not yet answered

Points out of 6

The figure below shows 13 circles of radius 1 within a larger circle. All the intersections occur at points of tangency. What is the area of the region, shaded in the figure, inside the larger circle but outside all the circles of radius 1?



- (A) $4\pi\sqrt{3}$
- (B) 7π
- (C) $\pi(3\sqrt{3} + 2)$
- (D) $10\pi(\sqrt{3} - 1)$
- (E) $\pi(\sqrt{3} + 6)$

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 11

Not yet answered

Points out of 6

For some positive integer k , the repeating base- k representation of the (base-ten) fraction $\frac{7}{51}$ is $0.\overline{23}_k = 0.232323 \dots_k$. What is k ?

- (A) 13
- (B) 14
- (C) 15
- (D) 16
- (E) 17

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 12

Not yet answered

Points out of 6

Positive real numbers $x \neq 1$ and $y \neq 1$ satisfy $\log_2 x = \log_y 16$ and $xy = 64$. What is $(\log_2 \frac{x}{y})^2$?

- (A) $\frac{25}{2}$
- (B) 20
- (C) $\frac{45}{2}$
- (D) 25
- (E) 32

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 13

Not yet answered

Points out of 6

How many ways are there to paint each of the integers $2, 3, \dots, 9$ either red, green, or blue so that each number has a different color from each of its proper divisors?

- (A) 144
- (B) 216
- (C) 256
- (D) 384
- (E) 432

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 14

Not yet answered

Points out of 6

For a certain complex number c , the polynomial

$$P(x) = (x^2 - 2x + 2)(x^2 - cx + 4)(x^2 - 4x + 8)$$

has exactly 4 distinct roots. What is $|c|$?

- (A) 2
- (B) $\sqrt{6}$
- (C) $2\sqrt{2}$
- (D) 3
- (E) $\sqrt{10}$

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 15

Not yet answered

Points out of 6

Positive real numbers a and b have the property that

$$\sqrt{\log a} + \sqrt{\log b} + \log \sqrt{a} + \log \sqrt{b} = 100$$

and all four terms on the left are positive integers, where $\log$ denotes the base 10 logarithm. What is ab ?

(A) 10^{52}

(B) 10^{100}

(C) 10^{144}

(D) 10^{164}

(E) 10^{200}

Select one:

☐ A

☐ B

☐ C

☐ D

☐ E

☐ Leave blank (1.5 points)

Question 16

Not yet answered

Points out of 6

The numbers $1, 2, \dots, 9$ are randomly placed into the 9 squares of a 3×3 grid. Each square gets one number, and each of the numbers is used once. What is the probability that the sum of the numbers in each row and each column is odd?

(A) $\frac{1}{21}$

(B) $\frac{1}{14}$

(C) $\frac{5}{63}$

(D) $\frac{2}{21}$

(E) $\frac{1}{7}$

Select one:

☐ A

☐ B

☐ C

☐ D

☐ E

☐ Leave blank (1.5 points)

Question 17

Not yet answered

Points out of 6

Let s_k denote the sum of the k th powers of the roots of the polynomial $x^3 - 5x^2 + 8x - 13$. In particular, $s_0 = 3$, $s_1 = 5$, and $s_2 = 9$. Let a , b , and c be real numbers such that $s_{k+1} = a s_k + b s_{k-1} + c s_{k-2}$ for $k = 2, 3, \dots$. What is $a + b + c$?

(A) -6

(B) 0

(C) 6

(D) 10

(E) 26

Select one:

☐ A

☐ B

☐ C

☐ D

☐ E

☐ Leave blank (1.5 points)

Question 18

Not yet answered

Points out of 6

A sphere with center O has radius 6. A triangle with sides of length 15, 15, and 24 is situated in space so that each of its sides is tangent to the sphere. What is the distance between O and the plane determined by the triangle?

(A) $2\sqrt{3}$

(B) 4

(C) $3\sqrt{2}$

(D) $2\sqrt{5}$

(E) 5

Select one:

☐ A

☐ B

☐ C

☐ D

☐ E

☐ Leave blank (1.5 points)

Question 19

Not yet answered

Points out of 6

In $\triangle ABC$ with integer side lengths,

$$\cos A = \frac{11}{16}, \quad \cos B = \frac{7}{8}, \quad \text{and} \quad \cos C = -\frac{1}{4}.$$

What is the least possible perimeter for $\triangle ABC$?

- (A) 9
- (B) 12
- (C) 23
- (D) 27
- (E) 44

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 20

Not yet answered

Points out of 6

Real numbers between 0 and 1, inclusive, are chosen in the following manner. A fair coin is flipped. If it lands heads, then it is flipped again and the chosen number is 0 if the second flip is heads and 1 if the second flip is tails. On the other hand, if the first coin flip is tails, then the number is chosen uniformly at random from the closed interval $[0, 1]$. Two random numbers x and y are chosen independently in this manner. What is the probability that

$$|x - y| > \frac{1}{2}?$$

(A) $\frac{1}{3}$

(B) $\frac{7}{16}$

(C) $\frac{1}{2}$

(D) $\frac{9}{16}$

(E) $\frac{2}{3}$

Select one:

☐ A

☐ B

☐ C

☐ D

☐ E

☐ Leave blank (1.5 points)

Question 21

Not yet answered

Points out of 6

Let

$$z = \frac{1+i}{\sqrt{2}}.$$

What is

$$(z^{1^2} + z^{2^2} + z^{3^2} + \dots + z^{12^2}) \cdot \left(\frac{1}{z^{1^2}} + \frac{1}{z^{2^2}} + \frac{1}{z^{3^2}} + \dots + \frac{1}{z^{12^2}} \right)?$$

(A) 18**(B)** $71 - 36\sqrt{2}$ **(C)** 36**(D)** 72**(E)** $72 + 36\sqrt{2}$

Select one:

☐ **A**☐ **B**☐ **C**☐ **D**☐ **E**☐ **Leave blank (1.5 points)**

Question 22

Not yet answered

Points out of 6

Circles ω and γ , both centered at O , have radii 20 and 17, respectively. Equilateral triangle ABC , whose interior lies in the interior of ω but in the exterior of γ , has vertex A on ω , and the line containing side $\overline{BC}$ is tangent to γ . Segments $\overline{AO}$ and $\overline{BC}$ intersect at P , and $\frac{BP}{CP} = 3$. Then AB can be written in the form $\frac{m}{\sqrt{n}} - \frac{p}{\sqrt{q}}$ for positive integers m, n, p, q with $\gcd(m, n) = \gcd(p, q) = 1$. What is $m + n + p + q$?

- (A) 42
(B) 86
(C) 92
(D) 114
(E) 130

Select one:

- ☐ A
☐ B
☐ C
☐ D
☐ E
☐ Leave blank (1.5 points)

Question 23

Not yet answered

Points out of 6

Define binary operations $\diamond$ and $\heartsuit$ by

$$a \diamond b = a^{\log_7(b)} \quad \text{and} \quad a \heartsuit b = a^{\frac{1}{\log_7(b)}}$$

for all real numbers a and b for which these expressions are defined. The sequence (a_n) is defined recursively by $a_3 = 3 \heartsuit 2$ and

$$a_n = (n \heartsuit (n - 1)) \diamond a_{n-1}$$

for all integers $n \geq 4$. To the nearest integer, what is $\log_7(a_{2019})$?

- (A) 8
- (B) 9
- (C) 10
- (D) 11
- (E) 12

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 24

Not yet answered

Points out of 6

For how many integers n between 1 and 50, inclusive, is

$$\frac{(n^2 - 1)!}{(n!)^n}$$

an integer? (Recall that $0! = 1$.)

- (A) 31
- (B) 32
- (C) 33
- (D) 34
- (E) 35

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 25

Not yet answered

Points out of 6

Let $\triangle A_0 B_0 C_0$ be a triangle whose angle measures are exactly 59.999° , 60° , and 60.001° . For each positive integer n define A_n to be the foot of the altitude from A_{n-1} to line $B_{n-1} C_{n-1}$. Likewise, define B_n to be the foot of the altitude from B_{n-1} to line $A_{n-1} C_{n-1}$, and C_n to be the foot of the altitude from C_{n-1} to line $A_{n-1} B_{n-1}$. What is the least positive integer n for which $\triangle A_n B_n C_n$ is obtuse?

- (A) 10
- (B) 11
- (C) 13
- (D) 14
- (E) 15

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

2019 AMC 12A

Answer key

1 E	2 D	3 B	4 D	5 C
6 C	7 E	8 D	9 E	10 A
11 D	12 B	13 E	14 E	15 D
16 B	17 D	18 D	19 A	20 B
21 C	22 E	23 D	24 D	25 E



2019 AMC 12B

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Question 1

Not yet answered

Points out of 6

Alicia had two containers. The first was $\frac{5}{6}$ full of water and the second was empty. She poured all the water from the first container into the second container, at which point the second container was $\frac{3}{4}$ full of water. What is the ratio of the volume of the smaller container to the volume of the larger container?

(A) $\frac{5}{8}$

(B) $\frac{4}{5}$

(C) $\frac{7}{8}$

(D) $\frac{9}{10}$

(E) $\frac{11}{12}$

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 2

Not yet answered

Points out of 6

Consider the statement, "If n is not prime, then $n - 2$ is prime." Which of the following values of n is a counterexample to this statement?

(A) 11

(B) 15

(C) 19

(D) 21

(E) 27

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 3

Not yet answered

Points out of 6

Which one of the following rigid transformations (isometries) maps the line segment $\overline{AB}$ onto the line segment $\overline{A'B'}$ so that the image of $A(-2, 1)$ is $A'(2, -1)$ and the image of $B(-1, 4)$ is $B'(1, -4)$?

- (A) reflection in the y -axis
- (B) counterclockwise rotation around the origin by 90°
- (C) translation by 3 units to the right and 5 units down
- (D) reflection in the x -axis
- (E) clockwise rotation about the origin by 180°

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 4

Not yet answered

Points out of 6

A positive integer n satisfies the equation $(n + 1)! + (n + 2)! = 440 \cdot n!$. What is the sum of the digits of n ?

- (A) 2
- (B) 5
- (C) 10
- (D) 12
- (E) 15

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 5

Not yet answered

Points out of 6

Each piece of candy in a store costs a whole number of cents. Casper has exactly enough money to buy either 12 pieces of red candy, 14 pieces of green candy, 15 pieces of blue candy, or n pieces of purple candy. A piece of purple candy costs 20 cents. What is the smallest possible value of n ?

- (A) 18
- (B) 21
- (C) 24
- (D) 25
- (E) 28

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 6

Not yet answered

Points out of 6

In a given plane, points A and B are 10 units apart. How many points C are there in the plane such that the perimeter of $\triangle ABC$ is 50 units and the area of $\triangle ABC$ is 100 square units?

- (A) 0
- (B) 2
- (C) 4
- (D) 8
- (E) infinitely many

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 7

Not yet answered

Points out of 6

What is the sum of all real numbers x for which the median of the numbers 4, 6, 8, 17, and x is equal to the mean of those five numbers?

- (A) -5
- (B) 0
- (C) 5
- (D) $\frac{15}{4}$
- (E) $\frac{35}{4}$

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 8

Not yet answered

Points out of 6

Let $f(x) = x^2(1 - x)^2$. What is the value of the sum

$$f\left(\frac{1}{2019}\right) - f\left(\frac{2}{2019}\right) + f\left(\frac{3}{2019}\right) - f\left(\frac{4}{2019}\right) + \cdots \\ + f\left(\frac{2017}{2019}\right) - f\left(\frac{2018}{2019}\right)?$$

- (A) 0
- (B) $\frac{1}{2019^4}$
- (C) $\frac{2018^2}{2019^4}$
- (D) $\frac{2020^2}{2019^4}$
- (E) 1

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 9

Not yet answered

Points out of 6

For how many integral values of x can a triangle of positive area be formed having side lengths $\log_2 x$, $\log_4 x$, and 3?

(A) 57

(B) 59

(C) 61

(D) 62

(E) 63

Select one:

☐ A

☐ B

☐ C

☐ D

☐ E

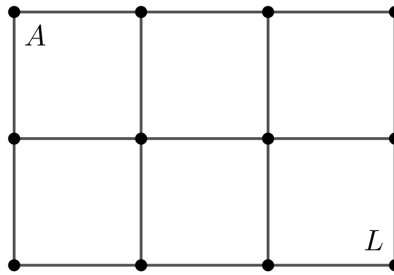
☐ Leave blank (1.5 points)

Question 10

Not yet answered

Points out of 6

The figure below is a map showing 12 cities and 17 roads connecting certain pairs of cities. Paula wishes to travel along exactly 13 of those roads, starting at city A and ending at city L , without traveling along any portion of a road more than once. (Paula is allowed to visit a city more than once.)



How many different routes can Paula take?

- (A) 0
- (B) 1
- (C) 2
- (D) 3
- (E) 4

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 11

Not yet answered

Points out of 6

How many unordered pairs of edges of a given cube determine a plane?

- (A) 21
- (B) 28
- (C) 36
- (D) 42
- (E) 66

Select one:

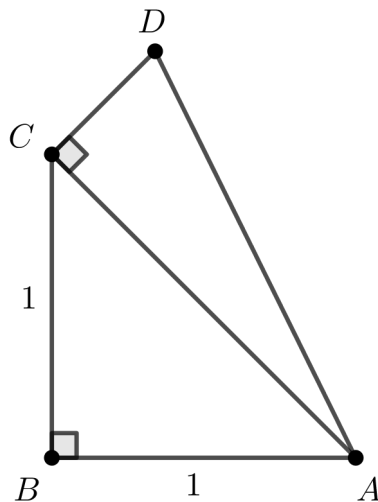
- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 12

Not yet answered

Points out of 6

Right triangle ACD with right angle at C is constructed outwards on the hypotenuse $\overline{AC}$ of isosceles right triangle ABC with leg length 1, as shown, so that the two triangles have equal perimeters. What is $\sin(2\angle BAD)$?



- (A) $\frac{1}{3}$
(B) $\frac{\sqrt{2}}{2}$
(C) $\frac{3}{4}$
(D) $\frac{7}{9}$
(E) $\frac{\sqrt{3}}{2}$

Select one:

- ☐ A
☐ B
☐ C
☐ D
☐ E
☐ Leave blank (1.5 points)

Question 13

Not yet answered

Points out of 6

A red ball and a green ball are randomly and independently tossed into bins numbered with the positive integers so that for each ball, the probability that it is tossed into bin k is 2^{-k} for $k = 1, 2, 3, \dots$. What is the probability that the red ball is tossed into a higher-numbered bin than the green ball?

(A) $\frac{1}{4}$

(B) $\frac{2}{7}$

(C) $\frac{1}{3}$

(D) $\frac{3}{8}$

(E) $\frac{3}{7}$

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 14

Not yet answered

Points out of 6

Let S be the set of all positive integer divisors of 100,000. How many numbers are the product of two distinct elements of S ?

(A) 98

(B) 100

(C) 117

(D) 119

(E) 121

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 15

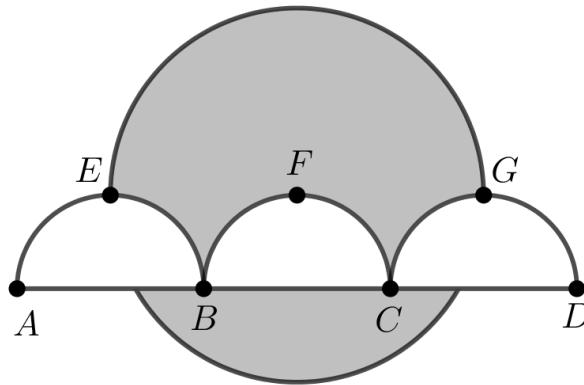
Not yet answered

Points out of 6

As shown in the figure, line segment $\overline{AD}$ is trisected by points B and C so that $AB = BC = CD = 2$. Three semicircles of radius 1, $\widehat{AEB}$, $\widehat{BFC}$, and $\widehat{CGD}$, have their diameters on $\overline{AD}$, lie in the same halfplane determined by line AD , and are tangent to line EG at E , F , and G , respectively. A circle of radius 2 has its center at F . The area of the region inside the circle but outside the three semicircles, shaded in the figure, can be expressed in the form

$$\frac{a}{b} \cdot \pi - \sqrt{c} + d,$$

where a , b , c , and d are positive integers and a and b are relatively prime. What is $a + b + c + d$?



- (A) 13
 (B) 14
 (C) 15
 (D) 16
 (E) 17

Select one:

- ☐ A
☐ B
☐ C
☐ D
☐ E
☐ Leave blank (1.5 points)

Question 16

Not yet answered

Points out of 6

Lily pads numbered from 0 to 11 lie in a row on a pond. Fiona the frog sits on pad 0, a morsel of food sits on pad 10, and predators sit on pads 3 and 6. At each unit of time the frog jumps either to the next higher numbered pad or to the pad after that, each with probability $\frac{1}{2}$, independently from previous jumps. What is the probability that Fiona skips over pads 3 and 6 and lands on pad 10?

(A) $\frac{15}{256}$

(B) $\frac{1}{16}$

(C) $\frac{15}{128}$

(D) $\frac{1}{8}$

(E) $\frac{1}{4}$

Select one:

☐ A☐ B☐ C☐ D☐ E☐ Leave blank (1.5 points)**Question 17**

Not yet answered

Points out of 6

How many nonzero complex numbers z have the property that 0 , z , and z^3 , when represented by points in the complex plane, are the three distinct vertices of an equilateral triangle?

(A) 0

(B) 1

(C) 2

(D) 4

(E) infinitely many

Select one:

☐ A☐ B☐ C☐ D☐ E☐ Leave blank (1.5 points)

Question 18

Not yet answered

Points out of 6

Square pyramid $ABCDE$ has base $ABCD$, which measures 3 cm on a side, and altitude $\overline{AE}$ perpendicular to the base, which measures 6 cm. Point P lies on $\overline{BE}$, one third of the way from B to E ; point Q lies on $\overline{DE}$, one third of the way from D to E ; and point R lies on $\overline{CE}$, two thirds of the way from C to E . What is the area, in square centimeters, of $\triangle PQR$?

(A) $\frac{3\sqrt{2}}{2}$

(B) $\frac{3\sqrt{3}}{2}$

(C) $2\sqrt{2}$

(D) $2\sqrt{3}$

(E) $3\sqrt{2}$

Select one:

☐ A

☐ B

☐ C

☐ D

☐ E

☐ Leave blank (1.5 points)

Question 19

Not yet answered

Points out of 6

Raashan, Sylvia, and Ted play the following game. Each starts with \$1. A bell rings every 15 seconds, at which time each of the players who currently have money simultaneously chooses one of the other two players independently and at random and gives \$1 to that player. What is the probability that after the bell has rung 2019 times, each player will have \$1? (For example, Raashan and Ted may each decide to give \$1 to Sylvia, and Sylvia may decide to give her dollar to Ted, at which point Raashan will have \$0, Sylvia would have \$2, and Ted would have \$1, and that is the end of the first round of play. In the second round Raashan has no money to give, but Sylvia and Ted might choose each other to give their \$1 to, and the holdings will be the same at the end of the second round.)

(A) $\frac{1}{7}$

(B) $\frac{1}{4}$

(C) $\frac{1}{3}$

(D) $\frac{1}{2}$

(E) $\frac{2}{3}$

Select one:

☐ A

☐ B

☐ C

☐ D

☐ E

☐ Leave blank (1.5 points)

Question 20

Not yet answered

Points out of 6

Points $A(6, 13)$ and $B(12, 11)$ lie on circle ω in the plane. Suppose that the tangent lines to ω at A and B intersect at a point on the x -axis. What is the area of ω ?

(A) $\frac{83\pi}{8}$

(B) $\frac{21\pi}{2}$

(C) $\frac{85\pi}{8}$

(D) $\frac{43\pi}{4}$

(E) $\frac{87\pi}{8}$

Select one:

☐ A

☐ B

☐ C

☐ D

☐ E

☐ Leave blank (1.5 points)

Question 21

Not yet answered

Points out of 6

How many quadratic polynomials with real coefficients are there such that the set of roots equals the set of coefficients? (For clarification: If the polynomial is $ax^2 + bx + c$, $a \neq 0$, and the roots are r and s , then the requirement is that $\{a, b, c\} = \{r, s\}$.)

(A) 3

(B) 4

(C) 5

(D) 6

(E) infinitely many

Select one:

☐ A

☐ B

☐ C

☐ D

☐ E

☐ Leave blank (1.5 points)

Question 22

Not yet answered

Points out of 6

Define a sequence recursively by $x_0 = 5$ and

$$x_{n+1} = \frac{x_n^2 + 5x_n + 4}{x_n + 6}$$

for all nonnegative integers n . Let m be the least positive integer such that

$$x_m \leq 4 + \frac{1}{2^{20}}.$$

In which of the following intervals does m lie?

- (A) $[9, 26]$
- (B) $[27, 80]$
- (C) $[81, 242]$
- (D) $[243, 728]$
- (E) $[729, \infty)$

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 23

Not yet answered

Points out of 6

How many sequences of 0s and 1s of length 19 are there that begin with a 0, end with a 0, contain no two consecutive 0s, and contain no three consecutive 1s?

- (A) 55
- (B) 60
- (C) 65
- (D) 70
- (E) 75

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 24

Not yet answered

Points out of 6

Let $\omega = -\frac{1}{2} + \frac{1}{2}i\sqrt{3}$. Let S denote all points in the complex plane of the form $a + b\omega + c\omega^2$, where $0 \leq a \leq 1$, $0 \leq b \leq 1$, and $0 \leq c \leq 1$. What is the area of S ?

- (A) $\frac{1}{2}\sqrt{3}$
- (B) $\frac{3}{4}\sqrt{3}$
- (C) $\frac{3}{2}\sqrt{3}$
- (D) $\frac{1}{2}\pi\sqrt{3}$
- (E) π

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

Question 25

Not yet answered

Points out of 6

Let $ABCD$ be a convex quadrilateral with $BC = 2$ and $CD = 6$. Suppose that the centroids of $\triangle ABC$, $\triangle BCD$, and $\triangle ACD$ form the vertices of an equilateral triangle. What is the maximum possible value of $ABCD$?

- (A) 27
- (B) $16\sqrt{3}$
- (C) $12 + 10\sqrt{3}$
- (D) $9 + 12\sqrt{3}$
- (E) 30

Select one:

- ☐ A
- ☐ B
- ☐ C
- ☐ D
- ☐ E
- ☐ Leave blank (1.5 points)

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Answer key

1 D	2 E	3 E	4 C	5 B
6 A	7 A	8 A	9 B	10 E
11 D	12 D	13 C	14 C	15 E
16 A	17 D	18 C	19 B	20 C
21 B	22 C	23 C	24 C	25 C