

AMC 12 SOLUTION ARCHIVE

2025

12A and 12B full explanations

Contents

AMC 12A full solutions

30 pages

starts p. 2

AMC 12B full solutions

25 pages

starts p. 32

About these explanations

Each section contains a complete problem-by-problem solution booklet curated by LIVE by Po-Shen Loh. The archive states that the AMC problems are used with official legal permission from the Mathematical Association of America.

2025 AMC 12A, Problem 25

The source solution booklet discusses the flawed item while retaining the intended answer.

[Open AMC 12A solution archive](#)

[Open AMC 12B solution archive](#)

Solutions source: LIVE by Po-Shen Loh. AMC problems copyright MAA.

Compiled 2026-08-17

2025 AMC 12A Solutions

Typeset by: LIVE by Po-Shen Loh

<https://live.poshenloh.com/past-contests/amc12/2025A/solutions>



Problems © Mathematical Association of America. Reproduced with permission.

1. Andy and Betsy both live in Mathville. Andy leaves Mathville on his bicycle at 1:30, traveling due north at a steady 8 miles per hour. Betsy leaves on her bicycle from the same point at 2:30, traveling due east at a steady 12 miles per hour. At what time will they be exactly the same distance from their common starting point?

A 3:30

B 3:45

C 4:00

D 4:15

E 4:30

Solution:

Let t be the number of hours since 1:30. Andy has traveled $8t$ miles north, and Betsy, who started an hour later, has traveled $12(t - 1)$ miles east.

Setting the distances equal,

$$8t = 12(t - 1),$$

so $4t = 12$ and $t = 3$.

Three hours after 1:30 is 4:30.

Thus, the correct answer is **E**.

2. A box contains 10 pounds of a nut mix that is 50 percent peanuts, 20 percent cashews, and 30 percent almonds. A second nut mix containing 20 percent peanuts, 40 percent cashews, and 40 percent almonds is added to the box resulting in a new nut mix that is 40 percent peanuts. How many pounds of cashews are now in the box?

A 3.5

B 4

C 4.5

D 5

E 6

Solution:

The first box has 5 lb peanuts, 2 lb cashews, and 3 lb almonds. Adding x pounds of the second mix contributes $0.2x$ lb peanuts and $0.4x$ lb cashews.

The new peanut fraction is 40%, so

$$\frac{5 + 0.2x}{10 + x} = 0.4.$$

This gives $5 + 0.2x = 4 + 0.4x$, so $x = 5$.

The cashews now total $2 + 0.4(5) = 4$ pounds.

Thus, the correct answer is **B**.

3. A team of students is going to compete against a team of teachers in a trivia contest. The total number of students and teachers is 15. Ash, a cousin of one of the students, wants to join the contest. If Ash plays with the students, the average age on that team will increase from 12 to 14. If Ash plays with the teachers, the average age on that team will decrease from 55 to 52. How old is Ash?

A 28

B 29

C 30

D 32

E 33

Solution:

Let s be the number of students and a be Ash's age. The students' ages total $12s$, and adding Ash gives

$$\begin{aligned}12s + a &= 14(s + 1) \\ \implies a &= 2s + 14.\end{aligned}$$

There are $15 - s$ teachers with ages totaling $55(15 - s)$, and adding Ash gives

$$\begin{aligned}55(15 - s) + a &= 52(16 - s) \\ \implies a &= 3s + 7.\end{aligned}$$

Setting $2s + 14 = 3s + 7$ gives $s = 7$, so $a = 2(7) + 14 = 28$.

Thus, the correct answer is **A**.

4. Agnes writes the following four statements on a blank piece of paper.

- At least one of these statements is true.
- At least two of these statements are true.
- At least two of these statements are false.
- At least one of these statements is false.

Each statement is either true or false. How many false statements did Agnes write on the paper?

- ☐ A 0
- ☒ B 1
- ☐ C 2
- ☐ D 3
- ☐ E 4

Solution:

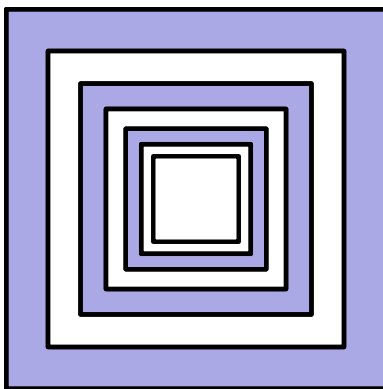
Let T be the number of true statements. The statements assert $T \geq 1$, $T \geq 2$, $T \leq 2$, and $T \leq 3$, respectively.

Testing $T = 3$: the conditions $T \geq 1$, $T \geq 2$, $T \leq 3$ hold (statements one, two, four) and $T \leq 2$ fails (statement three). Exactly 3 statements are true, matching $T = 3$.

For $T = 0, 1, 2, 3, 4$, the respective numbers of true conditions are 2, 3, 4, 3, 2, so no other value is self-consistent. Therefore exactly one statement is false.

Thus, the correct answer is **B**.

5. In the figure below, the outside square contains infinitely many squares, each of them with the same center and sides parallel to the outside square. The ratio of the side length of a square to the side length of the next inner square is k , where $0 < k < 1$. The spaces between squares are alternately shaded, as shown in the figure (which is not necessarily drawn to scale).



The area of the shaded portion of the figure is 64% of the area of the original square. What is k ?

- A $\frac{3}{5}$
- B $\frac{16}{25}$
- C $\frac{2}{3}$
- D $\frac{3}{4}$**
- E $\frac{4}{5}$

Solution:

Let the outer square have area 1. The nested squares have areas $1, k^2, k^4, \dots$, so the ring between the n th and $(n + 1)$ th squares has area $k^{2n}(1 - k^2)$.

The shaded rings are the alternate ones $n = 0, 2, 4, \dots$, with total area

$$\begin{aligned}\sum_{j=0}^{\infty} k^{4j}(1 - k^2) &= \frac{1 - k^2}{1 - k^4} \\ &= \frac{1}{1 + k^2}.\end{aligned}$$

Setting $\frac{1}{1 + k^2} = \frac{16}{25}$ gives $1 + k^2 = \frac{25}{16}$, so $k^2 = \frac{9}{16}$ and $k = \frac{3}{4}$.

Thus, the correct answer is **D**.

6. Six chairs are arranged around a round table. Two students and two teachers randomly select four of the chairs to sit in. What is the probability that the two students will sit in two adjacent chairs and the two teachers will also sit in two adjacent chairs?

A $\frac{1}{6}$

B $\frac{1}{5}$

C $\frac{2}{9}$

D $\frac{3}{13}$

E $\frac{1}{4}$

Solution:

Choosing 2 chairs for the students and 2 for the teachers gives $\binom{6}{2} \binom{4}{2} = 15 \cdot 6 = 90$ equally likely outcomes.

A round table has 6 adjacent pairs of chairs. Give the students any adjacent pair; among the remaining 4 chairs there are exactly 3 adjacent pairs for the teachers. That is $6 \cdot 3 = 18$ favorable outcomes.

The probability is $\frac{18}{90} = \frac{1}{5}$.

Thus, the correct answer is **B**.

7. In a certain alien world, the maximum running speed v of an organism is dependent on its number of toes n and number of eyes m . The relationship can be expressed as $v = kn^am^b$ centimeters per hour, where k , a , and b are integer constants. In a population where all organisms have 5 toes, $\log v = 4 + 2 \log m$; and in a population where all organisms have 25 eyes, $\log v = 4 + 4 \log n$, where the logarithms are base 10. What is $k + a + b$?

A 20

B 21

C 22

D 23

E 24

Solution:

Taking logarithms, $\log v = \log k + a \log n + b \log m$.

With $n = 5$, this reads $\log v = (\log k + a \log 5) + b \log m$, matching $4 + 2 \log m$, so $b = 2$ and $\log k + a \log 5 = 4$.

With $m = 25$, it reads $\log v = (\log k + b \log 25) + a \log n$, matching $4 + 4 \log n$, so $a = 4$ and $\log k + 2 \log 25 = 4$.

Then $\log k = 4 - \log 625 = \log \frac{10000}{625} = \log 16$, so $k = 16$. Hence $k + a + b = 16 + 4 + 2 = 22$.

Thus, the correct answer is **C**.

8. Pentagon $ABCDE$ is inscribed in a circle, and $\angle BEC = \angle CED = 30^\circ$. Let AC and BD intersect at point F , and suppose that $AB = 9$ and $AD = 24$. What is BF ?

A $\frac{57}{11}$

B $\frac{59}{11}$

C $\frac{60}{11}$

D $\frac{61}{11}$

E $\frac{63}{11}$

Solution:

The inscribed angle $\angle BEC = 30^\circ$ subtends arc $BC = 60^\circ$, so $\angle BAC$, which also subtends arc BC , equals 30° . Likewise $\angle CAD = 30^\circ$.

Thus AC bisects $\angle BAD = 60^\circ$. In $\triangle ABD$,

$$\begin{aligned} BD^2 &= 9^2 + 24^2 \\ &\quad - 2(9)(24) \cos 60^\circ \\ &= 657 - 216 = 441, \end{aligned}$$

so $BD = 21$.

Since AF (along AC) bisects $\angle BAD$, the Angle Bisector Theorem gives $\frac{BF}{FD} = \frac{AB}{AD} = \frac{9}{24} = \frac{3}{8}$. Hence $BF = \frac{3}{11} \cdot 21 = \frac{63}{11}$.

Thus, the correct answer is **E**.

9. Let w be the complex number $2 + i$, where $i = \sqrt{-1}$. What real number r has the property that r , w , and w^2 are three collinear points in the complex plane?

A $\frac{3}{4}$

B 1

C $\frac{7}{5}$

D $\frac{3}{2}$

E $\frac{5}{3}$

Solution:

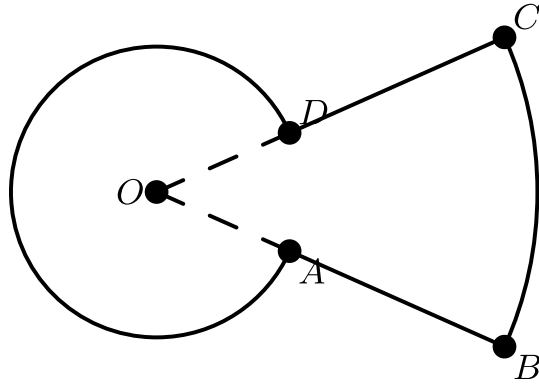
Compute $w^2 = (2 + i)^2 = 3 + 4i$, so the points are $(2, 1)$ and $(3, 4)$.

The line through them has slope $\frac{4 - 1}{3 - 2} = 3$, giving $y = 3x - 5$. Setting $y = 0$ yields $x = \frac{5}{3}$.

So $r = \frac{5}{3}$.

Thus, the correct answer is **E**.

10. In the figure shown below, major arc AD and minor arc BC have the same center, O . Also, A lies between O and B , and D lies between O and C . Major arc AD , minor arc BC , and each of the two segments AB and CD have length 2π .



What is the distance from O to A ?

- ☐ A 1
- ☒ B $1 - \pi + \sqrt{1 + \pi^2}$
- ☐ C $\frac{1}{2}\pi$
- ☐ D $\frac{1}{2}\sqrt{1 + \pi^2}$
- ☐ E 2

Solution:

Let $R_1 = OA = OD$ and $R_2 = OB = OC$, and let $\alpha = \angle AOD = \angle BOC$ (the rays coincide). The minor arc BC has length $R_2\alpha = 2\pi$, and the major arc AD is the reflex arc, so $R_1(2\pi - \alpha) = 2\pi$.

Each segment $AB = CD = R_2 - R_1 = 2\pi$.

From the first two equations, $R_2 = \frac{2\pi}{\alpha}$ and $R_1 = \frac{2\pi}{2\pi - \alpha}$. Substituting into $R_2 - R_1 = 2\pi$ and dividing by 2π gives

$$\frac{1}{\alpha} - \frac{1}{2\pi - \alpha} = 1,$$

which simplifies to $\alpha^2 - 2(1 + \pi)\alpha + 2\pi = 0$.

The larger root is greater than 2π , so it cannot be an angle of this minor arc. Thus $\alpha = (1 + \pi) - \sqrt{1 + \pi^2}$. Then

$$\begin{aligned} R_1 &= \frac{2\pi}{2\pi - \alpha} \\ &= \frac{2\pi}{\pi - 1 + \sqrt{1 + \pi^2}} \\ &= 1 - \pi + \sqrt{1 + \pi^2}, \end{aligned}$$

after rationalizing (the denominator times $\sqrt{1 + \pi^2} - (\pi - 1)$ equals 2π).

Thus, the correct answer is **B**.

11. The orthocenter of a triangle is the concurrent intersection of the three (possibly extended) altitudes. What is the sum of the coordinates of the orthocenter of the triangle whose vertices are $A(2, 31)$, $B(8, 27)$, and $C(18, 27)$?

A 5

B 17

C $10 + 4\sqrt{17} + 2\sqrt{13}$

D $\frac{113}{3}$

E 54

Solution:

Since B and C both have $y = 27$, side BC is horizontal and the altitude from A is the vertical line $x = 2$.

Side AC has slope $\frac{27 - 31}{18 - 2} = -\frac{1}{4}$, so the altitude from B has slope 4:

$$y - 27 = 4(x - 8).$$

At $x = 2$, $y = 27 + 4(2 - 8) = 3$. The orthocenter is $(2, 3)$, with coordinate sum 5.

Thus, the correct answer is **A**.

12. The harmonic mean of a collection of numbers is the reciprocal of the arithmetic mean of the reciprocals of the numbers in the collection. For example, the harmonic mean of 4, 4, and 5 is

$$\frac{1}{\frac{1}{3} \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{5} \right)} = \frac{30}{7}.$$

What is the harmonic mean of all the real roots of the 4050th degree polynomial

$$\begin{aligned} & \prod_{k=1}^{2025} (kx^2 - 4x - 3) \\ &= (x^2 - 4x - 3) \\ & \quad \cdot (2x^2 - 4x - 3) \\ & \quad \cdot (3x^2 - 4x - 3) \cdots \\ & \quad (2025x^2 - 4x - 3)? \end{aligned}$$

- A $-\frac{5}{3}$
- B $-\frac{3}{2}$**
- C $-\frac{6}{5}$
- D $-\frac{5}{6}$
- E $-\frac{2}{3}$

Solution:

Each factor $kx^2 - 4x - 3$ has discriminant $16 + 12k > 0$, so it has two real roots; there are 4050 roots in all.

For the roots of $kx^2 - 4x - 3$, the sum of reciprocals is $\frac{\text{sum}}{\text{product}} = \frac{\frac{4}{k}}{-\frac{3}{k}} = -\frac{4}{3}$, independent of k .

Summing over all 2025 factors, $\sum \frac{1}{r} = 2025 \left(-\frac{4}{3} \right) = -2700$. The harmonic mean is

$$\frac{4050}{-2700} = -\frac{3}{2}.$$

Thus, the correct answer is **B**.

13. Let $C = \{1, 2, 3, \dots, 13\}$. Let N be the greatest integer such that there exists a subset of C with N elements that does not contain five consecutive integers. Suppose N integers are chosen at random from C without replacement. What is the probability that the chosen elements do not include five consecutive integers?

A $\frac{3}{130}$

B $\frac{3}{143}$

C $\frac{5}{143}$

D $\frac{1}{26}$

E $\frac{5}{78}$

Solution:

To avoid five consecutive integers, it suffices to remove two elements (for example 5 and 10), and no single removal can hit both disjoint blocks $\{1, \dots, 5\}$ and $\{9, \dots, 13\}$. Thus $N = 11$.

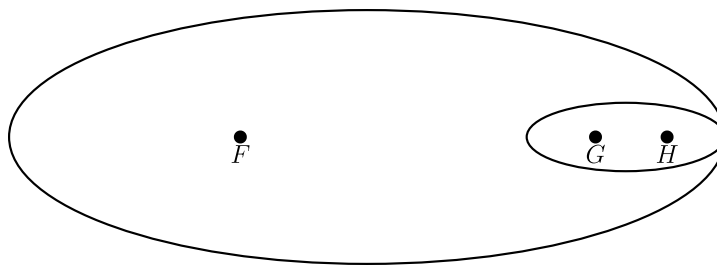
Choosing 11 of 13 elements is the same as removing 2, which can be done in $\binom{13}{2} = 78$ ways. The chosen set avoids five consecutive integers exactly when the two removed elements together intersect every window $\{t, t + 1, t + 2, t + 3, t + 4\}$ for $t = 1, \dots, 9$.

This forces one removed element in $\{1, \dots, 5\}$, the other in $\{9, \dots, 13\}$, and the two within 5 of each other. The valid removals are $\{4, 9\}$, $\{5, 9\}$, and $\{5, 10\}$, giving 3 of them.

The probability is $\frac{3}{78} = \frac{1}{26}$.

Thus, the correct answer is **D**.

14. Points F , G , and H are collinear with G between F and H . The ellipse with foci at G and H is internally tangent to the ellipse with foci at F and G , as shown below.



The two ellipses have the same eccentricity e , and the ratio of their areas is 2025. (Recall that the eccentricity of an ellipse is $e = \frac{c}{a}$, where c is the distance from the center to a focus, and $2a$ is the length of the major axis.) What is e ?

- A $\frac{3}{5}$
- B $\frac{16}{25}$
- C $\frac{4}{5}$
- D $\frac{22}{23}$**
- E $\frac{44}{45}$

Solution:

With the same eccentricity, $b = a\sqrt{1 - e^2}$, so the area $\pi ab \propto a^2$. The area ratio 2025 gives $\frac{a_1}{a_2} = \sqrt{2025} = 45$, where a_1, a_2 are the semi-major axes.

Both ellipses share focus G . On the large ellipse G is the right focus, so its right vertex lies $a_1 - c_1$ to the right of G . On the small ellipse G is the left focus, so its right vertex lies $a_2 + c_2$ to the right of G . Internal tangency makes these coincide:

$$a_1 - c_1 = a_2 + c_2.$$

Using $c = ea$, $a_1(1 - e) = a_2(1 + e)$, so $45(1 - e) = 1 + e$, giving $46e = 44$ and $e = \frac{22}{23}$.

Thus, the correct answer is **D**.

15. A set of numbers is called sum-free if whenever x and y are (not necessarily distinct) elements of the set, $x + y$ is not an element of the set. For example, $\{1, 4, 6\}$ and the empty set are sum-free, but $\{2, 4, 5\}$ is not. What is the greatest possible number of elements in a sum-free subset of $\{1, 2, 3, \dots, 20\}$?

- ☐ A 8
- ☐ B 9
- ☒ C 10
- ☐ D 11
- ☐ E 12

Solution:

The set $\{11, 12, \dots, 20\}$ has 10 elements and is sum-free, since any two elements sum to at least $22 > 20$.

For the upper bound, let $a_1 < a_2 < \dots < a_k$ be a sum-free subset. Each difference $a_k - a_i$ for $i < k$ cannot lie in S , because $(a_k - a_i) + a_i = a_k \in S$ would violate sum-freeness.

These $k - 1$ differences are distinct, lie in $\{1, \dots, 19\}$, and are disjoint from the k elements of S . So $k + (k - 1) \leq 20$, giving $k \leq 10$.

Thus, the correct answer is **C**.

16. Triangle $\triangle ABC$ has side lengths $AB = 80$, $BC = 45$, and $AC = 75$. The bisector of $\angle B$ and the altitude to side AB intersect at point P . What is BP ?

A 18

B 19

C 20

D 21

E 22

Solution:

By the Law of Cosines,

$$\begin{aligned}\cos B &= \frac{80^2 + 45^2 - 75^2}{2 \cdot 80 \cdot 45} \\ &= \frac{2800}{7200} = \frac{7}{18}.\end{aligned}$$

The altitude to AB is drawn from C , and its foot is at distance $BC \cos B = 45 \cdot \frac{7}{18} = 17.5$ from B along AB .

Along the bisector from B , the component parallel to AB is $BP \cos \frac{B}{2}$, which must reach the altitude's foot: $BP \cos \frac{B}{2} = 17.5$.

Since $\cos \frac{B}{2} = \sqrt{\frac{1 + \frac{7}{18}}{2}} = \sqrt{\frac{25}{36}} = \frac{5}{6}$, we get $BP = \frac{17.5}{\frac{5}{6}} = 21$.

Thus, the correct answer is **D**.

17. The polynomial $(z + i)(z + 2i)(z + 3i) + 10$ has three roots in the complex plane, where $i = \sqrt{-1}$. What is the area of the triangle formed by these roots?

- ☒ A 6
- ☐ B 8
- ☐ C 10
- ☐ D 12
- ☐ E 14

Solution:

The sum of the roots is $-6i$, so the centroid is $-2i$. Substituting $z = u - 2i$,

$$\begin{aligned}(u - i)(u)(u + i) + 10 \\&= u(u^2 + 1) + 10 \\&= u^3 + u + 10.\end{aligned}$$

Since $u = -2$ is a root, $u^3 + u + 10 = (u + 2)(u^2 - 2u + 5)$, giving roots $u = -2$ and $u = 1 \pm 2i$.

These are the points $(-2, 0)$, $(1, 2)$, $(1, -2)$. The base between $(1, 2)$ and $(1, -2)$ has length 4, at horizontal distance 3 from $(-2, 0)$, so the area is $\frac{1}{2}(4)(3) = 6$. Translation does not change the area.

Thus, the correct answer is **A**.

18. How many ordered triples (x, y, z) of distinct nonnegative integers less than or equal to 8 satisfy $xy > z$, $zx > y$, and $yz > x$?

- A

 36
- B

 84
- C

 186
- D

 336
- E

 486

Solution:

If any variable is 0, say $z = 0$, then $zx = 0 > y$ is impossible. So $x, y, z \in \{1, \dots, 8\}$ are distinct positive integers.

The conditions are symmetric. For distinct values $a < b < c$, we have $ac > b$ and $bc > a$ automatically, so the only real constraint is $ab > c$. When it holds, all 6 orderings work.

For each possible smallest value a , the numbers of pairs $a < b < c \leq 8$ satisfying $ab > c$ are

a	1	2	3	4	5	6
count	0	11	10	6	3	1.

For fixed a, b , this count comes from choosing c between $b + 1$ and $\min(8, ab - 1)$. Thus there are 31 unordered triples, and all 6 orderings of each work. The answer is $6 \cdot 31 = 186$.

Thus, the correct answer is **C**.

19. Let a , b , and c be the roots of the polynomial $x^3 + kx + 1$. What is the sum

$$a^3b^2 + a^2b^3 + b^3c^2 + b^2c^3 + c^3a^2 + c^2a^3?$$

A $-k$

B $-k + 1$

C 1

D $k - 1$

E k

Solution:

By Vieta's formulas, $a + b + c = 0$, $ab + bc + ca = k$, and $abc = -1$.

Group the sum as

$$a^2b^2(a + b) + b^2c^2(b + c) + c^2a^2(c + a).$$

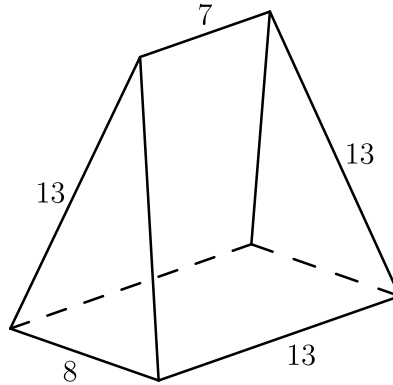
Since $a + b + c = 0$, we have $a + b = -c$, $b + c = -a$, $c + a = -b$.

So the sum equals

$$\begin{aligned} & -a^2b^2c - ab^2c^2 - a^2bc^2 \\ & = -abc(ab + bc + ca) \\ & = -(-1)(k) = k. \end{aligned}$$

Thus, the correct answer is **E**.

20. The base of the pentahedron shown below is a 13×8 rectangle, and its lateral faces are two isosceles triangles with base of length 8 and congruent sides of length 13, and two isosceles trapezoids with bases of lengths 7 and 13 and nonparallel sides of length 13.



What is the volume of the pentahedron?

- ☐ A 416
- ☐ B 520
- ☒ C 528
- ☐ D 676
- ☐ E 832

Solution:

The top is a ridge of length 7, centered above the base at some height h . Its endpoints sit above $(3, 4)$ and $(10, 4)$ of the 13×8 base. A slant edge to a base corner has length $\sqrt{3^2 + 4^2 + h^2} = 13$, so $h = 12$.

At height z , the horizontal cross-section is a rectangle measuring $\left(13 - \frac{z}{2}\right)$ by $\left(8 - \frac{2z}{3}\right)$. At $z = 0$ its area is 104; at $z = 6$ it is $10 \cdot 4 = 40$; at $z = 12$ the ridge has area 0.

By the prismatoid formula,

$$\begin{aligned} V &= \frac{12}{6} (104 + 4 \cdot 40 + 0) \\ &= 2(264) = 528. \end{aligned}$$

Thus, the correct answer is **C**.

21. There is a unique ordered triple (a, k, m) of nonnegative integers such that

$$\frac{4^a + 4^{a+k} + 4^{a+2k} + \dots + 4^{a+mk}}{2^a + 2^{a+k} + 2^{a+2k} + \dots + 2^{a+mk}} = 964.$$

What is $a + k + m$?

- ☒ A 8
- ☐ B 9
- ☐ C 10
- ☐ D 11
- ☐ E 12

Solution:

If $k = 0$ or $m = 0$, the original ratio is just 2^a , which cannot equal 964. Hence $k, m > 0$.

Summing the geometric series, the numerator is $4^a \frac{4^{k(m+1)} - 1}{4^k - 1}$ and the denominator is $2^a \frac{2^{k(m+1)} - 1}{2^k - 1}$. Using $4^N - 1 = (2^N - 1)(2^N + 1)$, the ratio simplifies to

$$2^a \cdot \frac{2^{k(m+1)} + 1}{2^k + 1} = 964.$$

The fraction on the left is odd over odd, so its power of 2 is 0. Since $964 = 2^2 \cdot 241$, we must have $a = 2$, and

$$2^{k(m+1)} + 1 = 241(2^k + 1).$$

Reducing this equation modulo 2^k shows that $2^k \mid 240$, so $k \leq 4$. For $k = 1, 2, 3$, the required powers of 2 would be 722, 1204, 2168, none of which is a power of 2. For $k = 4$, however, $241(2^4 + 1) = 4097 = 2^{12} + 1$, so $k(m + 1) = 12$ and $m = 2$. This also proves uniqueness.

Then $a + k + m = 2 + 4 + 2 = 8$.

Thus, the correct answer is **A**.

22. Three real numbers are chosen independently and uniformly at random between 0 and 1. What is the probability that the greatest of these three numbers is greater than 2 times each of the other two numbers? (In other words, if the chosen numbers are $a \geq b \geq c$, then $a > 2b$.)

A $\frac{1}{12}$

B $\frac{1}{9}$

C $\frac{1}{8}$

D $\frac{1}{6}$

E $\frac{1}{4}$

Solution:

Order the values as $x_1 > x_2 > x_3$; the joint density of the order statistics is 6 on this region. The event is $x_1 > 2x_2$.

Integrating x_3 from 0 to x_2 contributes a factor of x_2 . Then

$$\begin{aligned} P &= 6 \int_0^{\frac{1}{2}} x_2 \int_{2x_2}^1 dx_1 dx_2 \\ &= 6 \int_0^{\frac{1}{2}} x_2(1 - 2x_2) dx_2. \end{aligned}$$

This equals $6 \left(\frac{1}{8} - \frac{1}{12} \right) = 6 \cdot \frac{1}{24} = \frac{1}{4}$.

Thus, the correct answer is **E**.

23. Call a positive integer fair if no digit is used more than once, it has no 0s, and no digit is adjacent to two greater digits. For example, 196, 23, and 12463 are fair, but 1546, 320, and 34321 are not fair. How many fair positive integers are there?

- A 511
- B 2584
- C 9841**
- D 17711
- E 19682

Solution:

The digits are distinct and drawn from $\{1, \dots, 9\}$, and “no digit adjacent to two greater digits” means no interior digit is smaller than both neighbors.

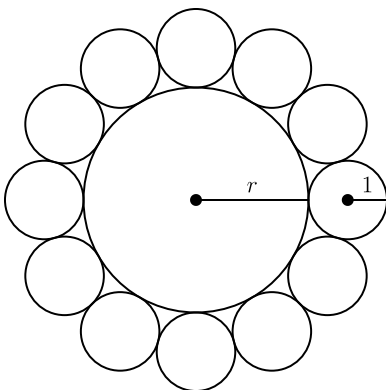
For a fixed set of n digits, build the arrangement by inserting digits from largest to smallest; each new (smaller) digit must go to one of the two ends, giving 2^{n-1} valid arrangements.

Summing over all nonempty digit subsets,

$$\begin{aligned} & \sum_{n=1}^9 \binom{9}{n} 2^{n-1} \\ &= \frac{1}{2} \sum_{n=1}^9 \binom{9}{n} 2^n \\ &= \frac{3^9 - 1}{2} = \frac{19682}{2} = 9841. \end{aligned}$$

Thus, the correct answer is **C**.

24. A circle of radius r is surrounded by 12 circles of radius 1, externally tangent to the central circle and sequentially tangent to each other, as shown. Then r can be written as $\sqrt{a} + \sqrt{b} + c$, where a , b , and c are integers. What is $a + b + c$?



- A 3
- B 5
- C 7**
- D 9
- E 11

Solution:

The centers of the 12 outer circles lie on a circle of radius $r + 1$, forming a regular 12-gon. Adjacent centers are 2 apart (both circles have radius 1), and the central angle between them is 30° .

Thus $2(r + 1) \sin 15^\circ = 2$, so $r + 1 = \frac{1}{\sin 15^\circ}$. Since $\sin 15^\circ = \frac{\sqrt{6} - \sqrt{2}}{4}$,

$$r + 1 = \frac{4}{\sqrt{6} - \sqrt{2}} = \sqrt{6} + \sqrt{2}.$$

Then $r = \sqrt{6} + \sqrt{2} - 1$, so $a + b + c = 6 + 2 - 1 = 7$.

Thus, the correct answer is **C**.

25. Polynomials $P(x)$ and $Q(x)$ each have degree 3 and leading coefficient 1, and their roots are all elements of $\{1, 2, 3, 4, 5\}$. The function $f(x) = \frac{P(x)}{Q(x)}$ has the property that there exist real numbers $a < b < c < d$ such that the set of all real numbers x such that $f(x) \leq 0$ consists of the closed interval $[a, b]$ together with the open interval (c, d) . How many functions $f(x)$ are possible?

- A 7
- B 9
- C 11
- D 12
- E 13**

Solution:

This problem was voided: as written, its answer is not among the choices. Here is the count.

For $\{f \leq 0\} = [a, b] \cup (c, d)$, the endpoints a, b of the closed interval must be zeros of P at which $Q \neq 0$, while c, d must be poles. The required sign pattern is therefore that of

$$g(x) = \frac{(x-a)(x-b)}{(x-c)(x-d)}.$$

The unused third factor of P must match the unused third factor of Q ; otherwise there would be an extra zero, pole, or sign change.

If the common factor is the fifth value not among a, b, c, d , it cannot make a hole inside either interval. In the ordered list of five values it may therefore occur before a , between b and c , or after d , giving 3 functions.

Alternatively, the common factor can equal c or d . For each of the $\binom{5}{4} = 5$ choices of a, b, c, d , these two polynomial pairs simplify to the same formula g and have the same domain (both omit c, d), so they define only one function. This gives 5 more functions, for a literal total of $3 + 5 = 8$.

Counting the two polynomial pairs separately in each of the last five cases gives $3 + 2 \cdot 5 = 13$, the provisional answer (E), but that does not answer the stated question about functions. Thus none of the printed choices is correct.

Problems: <https://live.poshenloh.com/past-contests/amc12/2025A>



2025 AMC 12B Solutions

Typeset by: LIVE by Po-Shen Loh

<https://live.poshenloh.com/past-contests/amc12/2025B/solutions>



Problems © Mathematical Association of America. Reproduced with permission.

1. The instructions on a 350-gram bag of coffee beans say that proper brewing of a large mug of pour-over coffee requires 20 grams of coffee beans. What is the greatest number of properly brewed large mugs of coffee that can be made from the coffee beans in that bag?

A 16

B 17

C 18

D 19

E 20

Solution:

Each mug uses 20 grams, and $\frac{350}{20} = 17.5$. Only complete mugs can be brewed, so the greatest number is 17.

Thus, the correct answer is **B**.

2. Jerry wrote down the ones digit of each of the first 2025 positive squares: 1, 4, 9, 6, 5, 6, What is the sum of all the numbers Jerry wrote down?

- ☐ A 9025
- ☐ B 9070
- ☐ C 9090
- ☒ D 9115
- ☐ E 9160

Solution:

The ones digits of $1^2, 2^2, \dots, 10^2$ are 1, 4, 9, 6, 5, 6, 9, 4, 1, 0, which sum to 45. The 2025 terms contain 202 full blocks (2020 terms) plus 5 more with digits 1, 4, 9, 6, 5 summing to 25. The total is $202 \cdot 45 + 25 = 9090 + 25 = 9115$.

Thus, the correct answer is **D**.

3. What is the value of $i(i - 1)(i - 2)(i - 3)$, where $i = \sqrt{-1}$?

A $6 - 5i$

B $-10i$

C $10i$

D -10

E 10

Solution:

$i(i - 1) = i^2 - i = -1 - i$, and $(i - 2)(i - 3) = i^2 - 5i + 6 = 5 - 5i$. Then

$$\begin{aligned}(-1 - i)(5 - 5i) &= -5 \\ &\quad + 5i - 5i \\ &\quad + 5i^2 \\ &= -5 - 5 = -10.\end{aligned}$$

Thus, the correct answer is **D**.

4. The value of the two-digit number $\underline{a}\underline{b}$ in base seven equals the value of the two-digit number $\underline{b}\underline{a}$ in base nine. What is $a + b$?

- ☒ A 7
- ☐ B 9
- ☐ C 10
- ☐ D 11
- ☐ E 14

Solution:

Setting $7a + b = 9b + a$ gives $6a = 8b$, so $3a = 4b$. Thus $a = 4t$ and $b = 3t$.

Because both are base-seven digits and $a \neq 0$, only $t = 1$ works. Hence $a = 4$, $b = 3$, and indeed $43_7 = 31 = 34_9$. Therefore $a + b = 7$.

Thus, the correct answer is **A**.

5. Positive integers x and y satisfy the equation $57x + 22y = 400$. What is the least possible value of $x + y$?

A 10

B 11

C 13

D 14

E 15

Solution:

Modulo 22, the equation gives $13x \equiv 4$, so $x \equiv 2 \pmod{22}$. With $57x < 400$, the only option is $x = 2$, which gives $22y = 286$, so $y = 13$. Then $x + y = 15$.

Thus, the correct answer is **E**.

6. Emmy says to Max, "I ordered 36 math club sweatshirts today." Max asks, "How much did each shirt cost?" Emmy responds, "I'll give you a hint. The total cost was $\$ \underline{A} \underline{B} \underline{B} . \underline{B} \underline{A}$, where A and B are digits and $A \neq 0$." After a pause, Max says, "That was a good price." What is $A + B$?

A 7

B 8

C 11

D 14

E 15

Solution:

The total in cents is $10000A + 1110B + A = 10001A + 1110B$, which must be a multiple of 36. Since $10001 \equiv 29$ and $1110 \equiv 30 \pmod{36}$, the condition is $29A + 30B \equiv 0$, i.e. $7A + 6B \equiv 0 \pmod{36}$. Reducing once more modulo 6 gives $A \equiv 0 \pmod{6}$, so the only possible nonzero digit is $A = 6$. Then $42 + 6B$ is divisible by 36 only for the digit $B = 5$. Indeed $\$655.56 = 36 \times \18.21 , so $A + B = 11$.

Thus, the correct answer is **C**.

7. What is the value of

$$\sum_{n=2}^{255} \frac{\log_2 \left(1 + \frac{1}{n}\right)}{(\log_2 n)(\log_2(n+1))}?$$

A $\frac{3}{4}$

B $1 - \frac{1}{\log_2 255}$

C $\frac{7}{8}$

D $\frac{15}{16}$

E 1

Solution:

Let $a_n = \log_2 n$. The numerator equals $a_{n+1} - a_n$, so each term is $\frac{a_{n+1} - a_n}{a_n a_{n+1}} = \frac{1}{a_n} - \frac{1}{a_{n+1}}$. Telescoping from $n = 2$ to 255 leaves $\frac{1}{\log_2 2} - \frac{1}{\log_2 256} = 1 - \frac{1}{8} = \frac{7}{8}$.

Thus, the correct answer is **C**.

8. There are integers a and b such that the polynomial $x^3 - 5x^2 + ax + b$ has $4 + \sqrt{5}$ as a root. What is $a + b$?

A 13

B 17

C 20

D 30

E 68

Solution:

The conjugate $4 - \sqrt{5}$ is also a root, and these two are the roots of $x^2 - 8x + 11$. The third root r satisfies $8 + r = 5$, so $r = -3$. Then $(x^2 - 8x + 11)(x + 3) = x^3 - 5x^2 - 13x + 33$, giving $a = -13$ and $b = 33$, so $a + b = 20$.

Thus, the correct answer is **C**.

9. What is the tens digit of 6^{6^6} ?

- ☐ A 1
- ☐ B 3
- ☒ C 5
- ☐ D 7
- ☐ E 9

Solution:

Here $6^6 = 46656$. For $n \geq 2$, the last two digits of 6^n cycle with period 5 through 36, 16, 96, 76, 56. Since $46656 \equiv 1 \pmod{5}$, 6^{46656} ends in 56, so the tens digit is 5.

Thus, the correct answer is **C**.

10. The altitude to the hypotenuse of a 30-60-90 right triangle is divided into two segments of lengths $x < y$ by the median to the shortest side of the triangle. What is the ratio $\frac{x}{x+y}$?

☒ A $\frac{3}{7}$

☐ B $\frac{\sqrt{3}}{4}$

☐ C $\frac{4}{9}$

☐ D $\frac{5}{11}$

☐ E $\frac{4\sqrt{3}}{15}$

Solution:

Take $C = (0, 0)$, $A = (\sqrt{3}, 0)$, $B = (0, 1)$, so AB is the hypotenuse and BC is the shortest side. The altitude from C meets AB at $H = \left(\frac{\sqrt{3}}{4}, \frac{3}{4}\right)$. The median from A to $M = \left(0, \frac{1}{2}\right)$ crosses the altitude at $P = \left(\frac{\sqrt{3}}{7}, \frac{3}{7}\right)$. This splits the altitude into $CP = \frac{2\sqrt{3}}{7} = \frac{4\sqrt{3}}{14}$ and $PH = \frac{3\sqrt{3}}{14}$, so $x = \frac{3\sqrt{3}}{14}$ and $\frac{x}{x+y} = \frac{3}{7}$.

Thus, the correct answer is **A**.

11. Nine athletes, no two of whom are the same height, try out for the basketball team. One at a time, they draw a wristband at random, without replacement, from a bag containing 3 blue bands, 3 red bands, and 3 green bands. They are divided into a blue group, a red group, and a green group. The tallest member of each group is named the group captain. What is the probability that the group captains are the three tallest athletes?

A $\frac{2}{9}$

B $\frac{2}{7}$

C $\frac{9}{28}$

D $\frac{1}{3}$

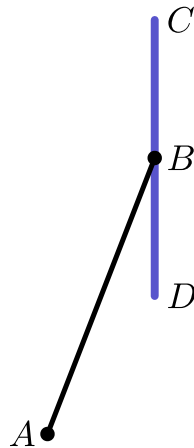
E $\frac{3}{8}$

Solution:

Each group has 3 slots. The three tallest athletes are the captains precisely when they fall into three different groups. Placing them one at a time into the 9 slots, the second must avoid the first's group (6 of the remaining 8 slots) and the third must avoid both used groups (3 of the remaining 7 slots). The probability is $\frac{6}{8} \cdot \frac{3}{7} = \frac{9}{28}$.

Thus, the correct answer is **C**.

12. The windshield wiper on the driver's side of a large bus is depicted below.



Arm $\overline{AB}$ pivots back and forth around point A , sweeping out an arc of 60° , symmetric about the vertical line through A . The wiper blade $\overline{CD}$ is attached to B at its midpoint and stays vertical as the arm moves. The arm is 3 feet long, and the wiper blade is 3.5 feet tall. What is the area of the windshield cleaned by the wiper, in square feet, to the nearest hundredth? (Assume that the windshield is a flat vertical surface.)

A 9.68

B 10.14

C 10.50

D 11.32

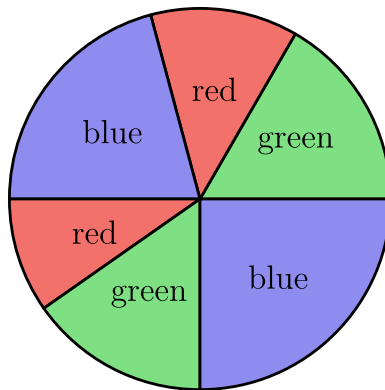
E 12.00

Solution:

Put A at the origin. Then $B = (3 \sin \theta, 3 \cos \theta)$ for $\theta \in [-30^\circ, 30^\circ]$, so the horizontal coordinate of B ranges over $[-1.5, 1.5]$, a width of 3. At each horizontal position exactly one vertical blade of height 3.5 passes through, so by Cavalieri's principle the cleaned area is $3.5 \times 3 = 10.5$ square feet.

Thus, the correct answer is **C**.

13. A circle has been divided into 6 sectors of different sizes. Then 2 of the sectors are painted red, 2 painted green, and 2 painted blue so that no two neighboring sectors are painted the same color. One such coloring is shown below.



How many different colorings are possible?

- ☐ A 12
- ☐ B 16
- ☐ C 18
- ☒ D 24
- ☐ E 28

Solution:

The two sectors of each color must be a non-adjacent pair, so a coloring is a way to split the 6 cyclic sectors into three non-adjacent pairs together with an assignment of the three colors. The non-adjacent pairs are the edges of the complement of the 6-cycle, the triangular prism, which has 4 perfect matchings. Assigning the three colors in $3!$ ways gives $4 \times 6 = 24$ colorings.

Thus, the correct answer is **D**.

14. Consider a decreasing sequence of n positive integers $x_1 > x_2 > \cdots > x_n$ that satisfies the following two conditions:

- The average (arithmetic mean) of the first 3 terms in the sequence is 2025.
- For all $4 \leq k \leq n$, the average of the first k terms in the sequence is 1 less than the average of the first $k - 1$ terms in the sequence.

What is the greatest possible value of n ?

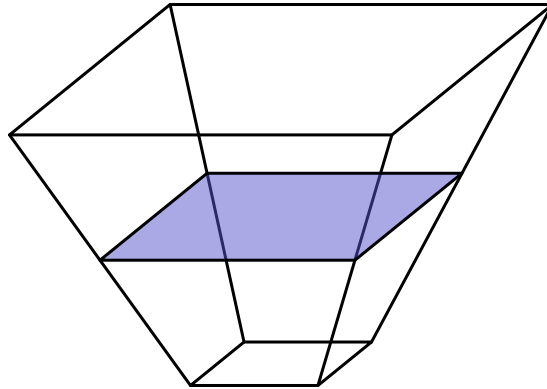
- A 1013
- B 1014**
- C 1016
- D 2016
- E 2025

Solution:

The average of the first k terms is $A_k = 2028 - k$ for $k \geq 3$, so the partial sum is $S_k = k(2028 - k)$. For $k \geq 4$, $x_k = S_k - S_{k-1} = 2029 - 2k$, which is positive exactly when $k \leq 1014$. A valid start such as $x_1, x_2, x_3 = 2030, 2023, 2022$ keeps the whole sequence strictly decreasing, so the greatest possible n is 1014.

Thus, the correct answer is **B**.

15. A container has a 1×1 square bottom, a 3×3 open square top, and four congruent trapezoidal sides, as shown. Starting when the container is empty, a hose that runs water at a constant rate takes 35 minutes to fill the container up to the midline of the trapezoids.



How many more minutes will it take to fill the remainder of the container?

- ☐ A 70
- ☐ B 85
- ☐ C 90
- ☒ D 95
- ☐ E 105

Solution:

At height fraction t the square cross-section has side $1 + 2t$, so the volume filled up to height t is $\int_0^t (1 + 2u)^2 du$. Up to the midline ($t = \frac{1}{2}$) this is $\frac{7}{6}$, and the full volume is $\frac{13}{3}$. The remaining volume is $\frac{13}{3} - \frac{7}{6} = \frac{19}{6}$, which is $\frac{19}{7}$ times the first part. So the remainder takes $35 \cdot \frac{19}{7} = 95$ more minutes.

Thus, the correct answer is **D**.

16. An analog clock starts at midnight and runs for 2025 minutes before stopping. What is the tangent of the acute angle between the hour hand and the minute hand when the clock stops?

A 0

B $\sqrt{2} - 1$

C $2 - \sqrt{2}$

D $\frac{\sqrt{2}}{2}$

E $3 - \sqrt{2}$

Solution:

2025 minutes is 33 hours 45 minutes, which modulo 12 hours reads 9:45. The minute hand points at 270° and the hour hand at $9.75 \times 30^\circ = 292.5^\circ$, so the acute angle between them is 22.5° . Using the half-angle value, $\tan 22.5^\circ = \sqrt{2} - 1$.

Thus, the correct answer is **B**.

17. Each of the 9 squares in a 3×3 grid is to be colored red, blue, or yellow in such a way that each red square shares an edge with at least one blue square, each blue square shares an edge with at least one yellow square, and each yellow square shares an edge with at least one red square. Colorings that can be obtained from one another by rotations and/or reflections are to be considered the same. How many different colorings are possible?

- A 3
- B 9
- C 12**
- D 18
- E 27

Solution:

First count colorings of the grid with its positions labeled. Checking the 27 possible rows in succession and rejecting a row as soon as a square whose neighbors are now known lacks its required next color gives the following complete count by the numbers of red, blue, and yellow squares:

$(\#R, \#B, \#Y)$	valid colorings
(2, 4, 3)	16
(3, 2, 4)	16
(3, 3, 3)	36
(4, 3, 2)	16

Thus the identity symmetry fixes $16 + 16 + 36 + 16 = 84$ colorings.

For the other symmetries, the fixed-coloring counts are 0 for each nontrivial rotation, 6 for each reflection across a horizontal or vertical axis, and 0 for each diagonal reflection. (For an axis reflection, a direct check of the three palindromic rows gives the 6 possibilities.) Therefore Burnside's lemma gives

$$\frac{84 + 2 \cdot 6}{8} = 12.$$

Thus, the correct answer is **C**.

18. Awnik repeatedly plays a game that has a probability of winning of $\frac{1}{3}$. The outcomes of the games are independent. What is the expected value of the number of games he will play until he has both won and lost at least once?

- A $\frac{5}{2}$
- B 3
- C $\frac{16}{5}$
- D $\frac{7}{2}$
- E $\frac{15}{4}$

Solution:

The first game produces one outcome. If it was a win (probability $\frac{1}{3}$), the expected wait for a loss is $\frac{1}{\frac{2}{3}} = \frac{3}{2}$; if it was a loss (probability $\frac{2}{3}$), the expected wait for a win is $\frac{1}{\frac{1}{3}} = 3$.

So the expected total is $1 + \frac{1}{3} \cdot \frac{3}{2} + \frac{2}{3} \cdot 3 = 1 + \frac{1}{2} + 2 = \frac{7}{2}$.

Thus, the correct answer is **D**.

19. A rectangular grid of squares has 141 rows and 91 columns. Each square has room for two numbers. Horace and Vera each fill in the grid by putting the numbers from 1 through $141 \times 91 = 12,831$ into the squares. Horace fills the grid horizontally: he puts 1 through 91 in order from left to right into row 1, puts 92 through 182 into row 2 in order from left to right, and continues similarly through row 141. Vera fills the grid vertically: she puts 1 through 141 in order from top to bottom into column 1, then 142 through 282 into column 2 in order from top to bottom, and continues similarly through column 91. How many squares get two copies of the same number?

A 7

B 10

C 11

D 12

E 19

Solution:

In row r , column c , Horace writes $(r - 1) \cdot 91 + c$ and Vera writes $(c - 1) \cdot 141 + r$. Setting these equal gives $90r - 140c = -50$, i.e. $9r = 14c - 5$. This requires $c \equiv 1 \pmod{9}$, so $c = 1, 10, 19, \dots, 91$ — that is 11 values, and each yields a valid r between 1 and 141. So 11 squares match.

Thus, the correct answer is **C**.

20. A frog hops along the number line according to the following rules.

- It starts at 0.
- If it is at 0, then it moves to 1 with probability $\frac{1}{2}$ and it disappears with probability $\frac{1}{2}$.
- For $n = 1, 2$, or 3 , if it is at n , then it moves to $n + 1$ with probability $\frac{1}{4}$, it moves to $n - 1$ with probability $\frac{1}{4}$, and it disappears with probability $\frac{1}{2}$.

What is the probability that the frog reaches 4?

A $\frac{1}{101}$

B $\frac{1}{100}$

C $\frac{1}{99}$

D $\frac{1}{98}$

E $\frac{1}{97}$

Solution:

Let $f(n)$ be the probability of reaching 4 from n . Then $f(0) = \frac{1}{2}f(1)$, and for $n = 1, 2, 3$, $f(n) = \frac{1}{4}f(n+1) + \frac{1}{4}f(n-1)$, with $f(4) = 1$. Solving upward gives $f(2) = \frac{7}{2}f(1)$ and $f(3) = 13f(1)$; then $4 \cdot 13f(1) = 1 + \frac{7}{2}f(1)$ yields $f(1) = \frac{2}{97}$, so $f(0) = \frac{1}{97}$.

Thus, the correct answer is **E**.

21. Two non-congruent triangles have the same area. Each triangle has sides of length 8 and 9, and the third side of each triangle has integer length. What is the sum of the lengths of the third sides?

A 20

B 22

C 24

D 26

E 28

Solution:

The area with included angle θ is $36 \sin \theta$, so two triangles of equal area use angles θ and $180^\circ - \theta$, with cosines $\pm \cos \theta$. By the law of cosines the third sides satisfy $t^2 = 145 \mp 144 \cos \theta$, hence $t_1^2 + t_2^2 = 290$. Since $290 \equiv 2 \pmod{8}$, both integer sides must be odd. Checking the odd squares in the triangle-inequality range $1 < t < 17$ leaves only $11^2 + 13^2 = 121 + 169 = 290$. Therefore the sum is 24.

Thus, the correct answer is **C**.

22. What is the greatest possible area of the triangle in the complex plane with vertices $2z$, $(1 + i)z$, and $(1 - i)z$, where z is a complex number satisfying $|4z - 2| = 1$?

- A $\frac{1}{4}$
- B $\frac{1}{2}$
- C $\frac{9}{16}$**
- D $\frac{3}{4}$
- E 1

Solution:

The vertices are $z \cdot 2$, $z(1 + i)$, and $z(1 - i)$, so the triangle is the fixed triangle with vertices 2 , $1 + i$, $1 - i$ — which has area 1 — scaled by $|z|$, giving area $|z|^2$. The condition $|4z - 2| = 1$ is the circle $|z - \frac{1}{2}| = \frac{1}{4}$, on which $|z|$ is at most $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$. So the greatest area is $(\frac{3}{4})^2 = \frac{9}{16}$.

Thus, the correct answer is **C**.

23. Let S be the set of all integers $z > 1$ such that for all pairs of nonnegative integers (x, y) with $x < y < z$, the remainder when $2025x$ is divided by z is less than the remainder when $2025y$ is divided by z . What is the sum of the elements of S ?

A 3041

B 3542

C 3750

D 4044

E 4319

Solution:

The condition requires $k \mapsto 2025k \bmod z$ to be strictly increasing on $\{0, 1, \dots, z - 1\}$. A strictly increasing list of z distinct values in $[0, z - 1]$ must be $0, 1, \dots, z - 1$, so $2025 \equiv 1 \pmod{z}$, i.e. $z \mid 2024$. Since $2024 = 2^3 \cdot 11 \cdot 23$, the sum of all its divisors is $(1 + 2 + 4 + 8)(1 + 11)(1 + 23) = 15 \cdot 12 \cdot 24 = 4320$. Excluding $z = 1$ leaves 4319.

Thus, the correct answer is **E**.

24. How many real numbers satisfy the equation $\sin(20\pi x) = \log_{20}(x)$?

A 199

B 200

C 398

D 399

E 400

Solution:

Since $|\sin(20\pi x)| \leq 1$, every solution lies in $[\frac{1}{20}, 20]$. For $x < 1$ the logarithm is negative, so only the 10 negative sine lobes in $[\frac{1}{20}, 1]$ can contribute. Each has one crossing on each side of its minimum; on the last lobe, the second crossing is the endpoint $x = 1$. Hence this part contributes 20 solutions.

For $x > 1$, only positive lobes contribute. There are 190 of them: one in each interval from $1 + \frac{k}{10}$ to $1 + \frac{k}{10} + \frac{1}{20}$ for $0 \leq k \leq 189$. Every one after the first has one crossing on each side of its maximum. The first has only one new crossing because its left endpoint is the already-counted solution $x = 1$. Thus $x > 1$ contributes $1 + 2 \cdot 189 = 379$ more solutions.

For completeness, the claimed uniqueness on each half-lobe follows by differentiating the difference of the two sides. On a falling positive half-lobe it is strictly decreasing. On a rising positive half-lobe its derivative increases and then decreases once, so the difference crosses from negative to positive only once. Applying the same argument to $-\sin(20\pi x) + \log_{20} x$ handles a negative lobe. The total is $20 + 379 = 399$.

Thus, the correct answer is **D**.

25. Three concentric circles have radii 1, 2, 3. An equilateral triangle with side length s has one vertex on each circle. What is s^2 ?

A 6

B $\frac{25}{4}$

C $\frac{13}{2}$

D $\frac{27}{4}$

E 7

Solution:

For the common center at distances 1, 2, 3 from the vertices of an equilateral triangle of side s , the identity $3(1 + 16 + 81 + s^4) = (1 + 4 + 9 + s^2)^2$ holds. This simplifies to $2s^4 - 28s^2 + 98 = 0$, i.e. $(s^2 - 7)^2 = 0$, so $s^2 = 7$.

Thus, the correct answer is **E**.

Problems: <https://live.poshenloh.com/past-contests/amc12/2025B>

