2019 AMC 10B Solutions

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1. Alicia had two containers. The first was $\frac{5}{6}$ full of water and the second was empty. She poured all the water from the first container into the second container, at which point the second container was $\frac{3}{4}$ full of water. What is the ratio of the volume of the smaller container to the volume of the larger container?

A $\frac{5}{8}$

B $\frac{4}{5}$

C $\frac{7}{8}$

D $\frac{9}{10}$

E $\frac{11}{12}$

Solution:

Let the volumes of the first and second containers be F and S. The amount of water is both $\frac{5}{6}F$ and $\frac{3}{4}S$, so $\frac{5}{6}F = \frac{3}{4}S$.

Thus $\frac{F}{S} = \frac{\frac{3}{4}}{\frac{5}{6}} = \frac{9}{10}$. Since the first container is smaller, the ratio of the smaller container to the larger container is $\frac{9}{10}$. Thus, **D** is the correct answer.

2. Consider the statement, "If n is not prime, then $n-2$ is prime." Which of the following values of n is a counterexample to this statement?

A 11

B 15

C 19

D 21

E 27

Solution:

We need n to not be prime, so n can only be $15, 21, 27$. Then, $n-2$ must be not prime, leaving just 27.

Thus, the answer is **E**.

3. In a high school with 500 students, 40% of the seniors play a musical instrument, while 30% of the non-seniors do not play a musical instrument. In all, 46.8% of the students do not play a musical instrument. How many non-seniors play a musical instrument?

A 66

B 154

C 186

D 220

E 266

Solution:

Let the number of seniors be s. Then, $500 - s$ people aren't seniors. We know 60% of seniors don't play an instrument. Then, the number of students who don't play an instrument can be represented as

$$\begin{aligned} &0.3(500 - s) + 0.6(s) \\ &= 0.3s + 150 \end{aligned}$$

and

$$0.468 \cdot 500 = 234.$$

Thus,

$$0.3s + 150 = 234$$

$$s = 280.$$

This makes the number of non-seniors equal to 220. Since 70% of non-seniors play instruments, we have the total number as

$$220 \cdot 0.7 = 154.$$

Thus, the answer is **B**.

4. All lines with equation $ax + by = c$ such that a, b, c form an arithmetic progression pass through a common point. What are the coordinates of that point?

A $(-1, 2)$

B $(0, 1)$

C $(1, -2)$

D $(1, 0)$

E $(1, 2)$

Solution:

Let $d = b - a$.

Then, we have

$$(a, b, c) = (a, a + d, a + 2d).$$

Thus,

$$ax + (a + d)y = a + 2d.$$

If we match the parts of a and d, we get

$$ax + ay = a$$

and

$$dy = 2d$$

for all a, d. Therefore, we have

$$y = 2, x + y = 1$$

implying that

$$x = -1.$$

This makes the pair $(-1, 2)$.

Thus, the answer is **A** .

5. Triangle ABC lies in the first quadrant. Points A, B, and C are reflected across the line $y = x$ to points A', B', and C', respectively. Assume that none of the vertices of the triangle lie on the line $y = x$. Which of the following statements is <u>not</u> always true?

A Triangle $A'B'C'$ lies in the first quadrant.

B Triangles ABC and $A'B'C'$ have the same area.

C The slope of line AA' is -1.

D The slopes of lines AA' and CC' are the same.

E Lines AB and $A'B'$ are perpendicular to each other.

Solution:

Reflection across $y = x$ sends (x, y) to (y, x). It therefore preserves the first quadrant and preserves area, so A and B are always true.

The direction from (x, y) to its image (y, x) is $(y - x, x - y)$, whose slope is -1 because $x \neq y$. Thus C and D are always true as well.

For E, take $A = (2, 1)$, $B = (3, 2)$, and $C = (4, 1)$. These points satisfy all the conditions, but AB and $A'B'$ both have slope 1, so the two lines are parallel rather than perpendicular.

Thus, the answer is **E**.

6. A positive integer n satisfies the equation

$$(n+1)! + (n+2)! = n! \cdot 440.$$

What is the sum of the digits of n?

A 2

B 5

C 10

D 12

E 15

Solution:

We can rewrite the left side as

$$(n+1)n! + (n+2)(n+1)n!$$
$$= ((n+2)^2 - 1)n!,$$

so

$$((n+2)^2 - 1)n! = 440n!$$

Therefore,

$$(n+2)^2 = 441,$$

so $n = 19$. The sum of its digits is 10.

Thus, the answer is **C** .

7. Each piece of candy in a shop costs a whole number of cents. Casper has exactly enough money to buy either 12 pieces of red candy, 14 pieces of green candy, 15 pieces of blue candy, or n pieces of purple candy. A piece of purple candy costs 20 cents. What is the least possible value of n?

A 18

B 21

C 24

D 25

E 28

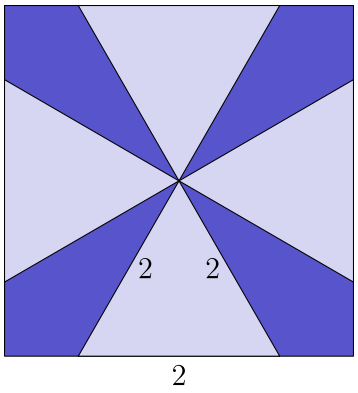
Solution:

Let the number of cents he has c. Then, c is a multiple of 12, 14, and 15. Thus, it must be a multiple of 420.

Let $c = 420k$ for some k. Also, $c = 20n$, so $420k = 20n$, making $n = 21k$. Since k is a whole number, the minimum possible value of n is 21.

Thus, the answer is **B**.

8. The figure below shows a square and four equilateral triangles, with each triangle having a side lying on a side of the square, such that each triangle has side length 2 and the third vertices of the triangles meet at the center of the square. The region inside the square but outside the triangles is shaded. What is the area of the shaded region?



A 4

B $12-4\sqrt{3}$

C $3\sqrt{3}$

D $4\sqrt{3}$

E $16-4\sqrt{3}$

Solution:

Each equilateral triangle has side length 2, so its altitude is $\sqrt{3}$. Because that altitude runs from a side of the square to its center, the square has side length $2\sqrt{3}$ and area 12 .

Each of the four equilateral triangles has area $\frac{\sqrt{3}}{4}(2^2)=\sqrt{3}$. Therefore the shaded area is

$$12-4\sqrt{3}.$$

Thus, the answer is **B** .

9. The function f is defined by

$$f(x) = \lfloor |x| \rfloor - |\lfloor x \rfloor|$$

for all real numbers x, where $\lfloor r \rfloor$ denotes the greatest integer less than or equal to the real number r. What is the range of f?

A $\{-1, 0\}$

B The set of nonpositive integers

C $\{-1, 0, 1\}$

D $\{0\}$

E The set of nonnegative integers

Solution:

If $x \geq 0$, then $\lfloor |x| \rfloor = \lfloor x \rfloor = |\lfloor x \rfloor|$, so $f(x) = 0$.

If x is a negative integer, both terms equal $|x|$, so again $f(x) = 0$.

If x is negative and not an integer, write $x = -k - t$, where k is a nonnegative integer and $0 < t < 1$. Then $\lfloor |x| \rfloor = k$, while $|\lfloor x \rfloor| = |-k - 1| = k + 1$, so $f(x) = -1$.

Therefore, the range is $\{-1, 0\}$.

Thus, the answer is **A**.

10. In a given plane, points A and B are 10 units apart. How many points C are there in the plane such that the perimeter of $\triangle ABC$ is 50 units and the area of $\triangle ABC$ is 100 square units?

A 0

B 2

C 4

D 8

E infinitely many

Solution:

The area condition fixes the distance from C to line AB. If that distance is h, then

$$\frac{10h}{2} = 100,$$

so $h = 20$. Thus C must lie on a line parallel to AB at distance 20.

Since the perpendicular distance from C to line AB is 20, both AC and BC are at least 20. They cannot both equal 20, because that would require the perpendicular from C to meet line AB at both distinct points A and B. Hence $AC + BC > 40$, contradicting the perimeter requirement $AC + BC = 40$. Therefore no point C works.

Thus, the answer is **A** .

11. Two jars each contain the same number of marbles, and every marble is either blue or green. In Jar 1 the ratio of blue to green marbles is $9:1$, and the ratio of blue to green marbles in Jar 2 is $8:1$. There are 95 green marbles in all. How many more blue marbles are in Jar 1 than in Jar 2?

A 5

B 10

C 25

D 45

E 50

Solution:

Let x and y be the numbers of green marbles in Jars 1 and 2, respectively. Then the total numbers of marbles in the jars are $10x$ and $9y$.

The jars contain the same number of marbles, so $10x = 9y$. Also $x + y = 95$. Solving gives $x = 45$ and $y = 50$.

Jar 1 has $9x = 405$ blue marbles, and Jar 2 has $8y = 400$ blue marbles. The difference is 5. Thus, **A** is the correct answer.

12. What is the greatest possible sum of the digits in the base-seven representation of a positive integer less than 2019?

A 11

B 14

C 22

D 23

E 27

Solution:

First, $2019 = 5613_7$. Any number with leading digit at most 4 has digit sum at most $4 + 6 + 6 + 6 = 22$, and $4666_7 < 5613_7$ attains that sum.

If the leading digit is 5, then a second digit at most 5 gives digit sum at most $5 + 5 + 6 + 6 = 22$. If the first two digits are 56, comparison with 5613_7 forces the last two digits to contribute at most $1 + 2$, giving an even smaller sum. Therefore, the largest digit sum is 22.

Thus, the answer is **C** .

13. What is the sum of all real numbers x for which the median of the numbers $4, 6, 8, 17,$ and x is equal to the mean of those five numbers?

A -5

B 0

C 5

D $\frac{15}{4}$

E $\frac{35}{4}$

Solution:

The mean is $7 + \frac{x}{5}$. If $x < 6$, the median is 6, so $7 + \frac{x}{5} = 6$ gives the valid value $x = -5$.

If $6 \leq x \leq 8$, the median is x, but $7 + \frac{x}{5} = x$ gives $x = \frac{35}{4}$, outside this interval. If $x > 8$, the median is 8, but $7 + \frac{x}{5} = 8$ gives $x = 5$, again outside the required range.

Therefore, the only possible x is -5, making the sum -5.

Thus, the answer is **A**.

14. The base-ten representation for $19!$ is $121,6T5,100,40M,832,H00$, where T, M, and H denote digits that are not given. What is $T+M+H$?

A 3

B 8

C 12

D 14

E 17

Solution:

Because $19!$ is divisible by 1000, its last three digits are zero, so $H=0$.

Since $19!$ is divisible by 9, its digit sum $33+T+M$ is divisible by 9. Hence $T+M \equiv 3 \pmod 9$, so $T+M$ is either 3 or 12.

Divisibility by 11 says the alternating digit sum $T-M-7$ is divisible by 11, so $T-M \equiv 7 \pmod{11}$. Checking the digit possibilities from these two congruences gives $T=4$ and $M=8$. Therefore $T+M+H=4+8+0=12$.

Thus, the answer is **C**.

15. Right triangles T_1 and T_2 have areas 1 and 2, respectively. A side of T_1 is congruent to a side of T_2, and a different side of T_1 is congruent to a different side of T_2. What is the square of the product of the lengths of the other (third) sides of T_1 and T_2?

A $\frac{28}{3}$

B 10

C $\frac{21}{2}$

D $\frac{32}{3}$

E 12

Solution:

Let the two shared side lengths be $a \leq b$. Because the triangles have different areas, the shared sides cannot play the same roles in both triangles. Thus the area-2 triangle has legs a and b, while the area-1 triangle has leg a, other leg $\sqrt{b^2 - a^2}$, and hypotenuse b.

The product of the two non-shared third sides is $\sqrt{a^2 + b^2}\sqrt{b^2 - a^2}$, whose square is $b^4 - a^4$.

Using the areas, $\frac{ab}{2} = 2$ and $\frac{a\sqrt{b^2 - a^2}}{2} = 1$. Hence $a^2b^2 = 16$ and $a^2(b^2 - a^2) = 4$, so $a^4 = 12$. Then $b^4 = \frac{256}{12} = \frac{64}{3}$, and $b^4 - a^4 = \frac{28}{3}$. Thus, **A** is the correct answer.

16. In $\triangle ABC$ with a right angle at C, point D lies in the interior of $\overline{AB}$ and point E lies in the interior of $\overline{BC}$ so that $AC = CD, DE = EB$, and the ratio $AC : DE = 4 : 3$. What is the ratio $AD : DB$?

A $2 : 3$

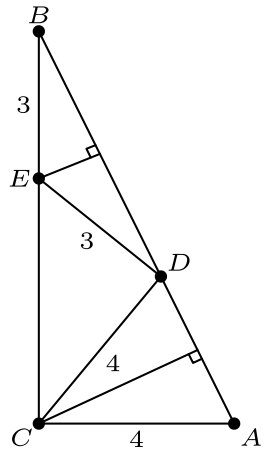
B $2 : \sqrt{5}$

C $1 : 1$

D $3 : \sqrt{5}$

E $3 : 2$

Solution:



Scale the figure so that $AC = CD = 4$ and $DE = EB = 3$. Let $A = \angle BAC$ and $B = \angle ABC$, so $A + B = 90^\circ$.

The isosceles triangles ACD and DEB give $\angle CDA = A$ and $\angle EDB = B$. Because DA and DB are opposite rays,

$$\angle CDE = 180^\circ - A - B = 90^\circ.$$

Therefore $CE = \sqrt{4^2 + 3^2} = 5$, so $BC = CE + EB = 8$ and $\tan A = \frac{BC}{AC} = 2$.

The bases of the two isosceles triangles have lengths $AD = 8\cos A$ and $BD = 6\cos B = 6\sin A$. Hence

$$\frac{AD}{BD} = \frac{8\cos A}{6\sin A} = \frac{4}{3\tan A} = \frac{2}{3}.$$

Thus, the answer is **A** .

17. A red ball and a green ball are randomly and independently tossed into bins numbered with the positive integers so that for each ball, the probability that it is tossed into bin k is 2^{-k} for $k = 1, 2, 3, \ldots$. What is the probability that the red ball is tossed into a higher-numbered bin than the green ball?

A $\frac{1}{4}$

B $\frac{2}{7}$

C $\frac{1}{3}$

D $\frac{3}{8}$

E $\frac{3}{7}$

Solution:

Given that the two balls were tossed into separate bins, the probability that the ball in the higher-numbered bin is red is $\frac{1}{2}$. Thus we must find

$$\frac{P(\text{Balls in different bins})}{2}$$

$$= \frac{1 - P(\text{Balls in same bins})}{2}$$

by complementary counting.

The probability that both balls are in bin k is

$$2^{-k} \cdot 2^{-k} = 4^{-k}.$$

The probability that they are both in the same bin is therefore

$$\sum_{k=1}^{\infty} 4^{-k}.$$

Using the geometric sequence formula, we get this to be

$$\frac{1}{4} \cdot \frac{1}{1-\frac{1}{4}} = \frac{1}{3}.$$

Therefore, our answer is

$$\frac{1-\frac{1}{3}}{2} = \frac{1}{3}.$$

Thus, the answer is **C** .

18. Henry decides one morning to do a workout, and he walks $\frac{3}{4}$ of the way from his home to his gym. The gym is 2 kilometers away from Henry's home. At that point, he changes his mind and walks $\frac{3}{4}$ of the way from where he is back toward home. When he reaches that point, he changes his mind again and walks $\frac{3}{4}$ of the distance from there back toward the gym. If Henry keeps changing his mind when he has walked $\frac{3}{4}$ of the distance toward either the gym or home from the point where he last changed his mind, he will get very close to walking back and forth between a point A kilometers from home and a point B kilometers from home. What is $|A - B|$?

A $\frac{2}{3}$

B 1

C $1\frac{1}{5}$

D $1\frac{1}{4}$

E $1\frac{1}{2}$

Solution:

Suppose a limiting homeward turnaround point is x kilometers from home. After Henry walks toward the gym, his position is

$$2 - \frac{2 - x}{4} = \frac{3}{2} + \frac{x}{4}.$$

After he turns back toward home, one quarter of that distance remains, so his next homeward turnaround point is

$$\frac{1}{4}\left(\frac{3}{2} + \frac{x}{4}\right) = \frac{3}{8} + \frac{x}{16}.$$

At the limiting point this two-step map leaves the position unchanged, so

$$x = \frac{3}{8} + \frac{x}{16},$$

which gives $x = \frac{2}{5}$.

The other limiting point is the position reached after walking toward the gym:

$$2 - \frac{2 - \frac{2}{5}}{4} = \frac{8}{5}.$$

Therefore

$$|A - B| = \frac{8}{5} - \frac{2}{5} = \frac{6}{5} = 1\frac{1}{5}.$$

Thus, the correct answer is **C** .

19. Let S be the set of all positive integer divisors of $100{,}000$. How many numbers are the product of two distinct elements of S?

A 98

B 100

C 117

D 119

E 121

Solution:

First, note that $100,000 = 2^5 \cdot 5^5$.

Therefore, any element of S must be of the form $2^a 5^b$ with $0 \leq a, b \leq 5$.

Suppose I have distinct $x, y \in S$ with

$$x = 2^a 5^b,$$

$$y = 2^c 5^d.$$

Then,

$$xy = 2^{a+c} 5^{b+d}.$$

Thus,

$$0 \leq a + c, b + d \leq 10.$$

This means that there are

$$(10 + 1)(10 + 1) = 121$$

possible exponent pairs for a product. However, some products can arise only when $x = y$, namely when $(a, b) = (c, d)$.

If $a + c = 0$, then $a = 0, c = 0$ must be true.

If $a + c = 10$, then $a = 5, c = 5$ must be true.

With any other value of $a+c$, we can have $a \neq c$.

Similar structure holds for $b+d$. Thus, if

$$a+c, b+d \in \{0, 10\},$$

then

$$(a, b) = (c, d),$$

thus making $x = y$.

This means we have to eliminate 4 choices, leaving

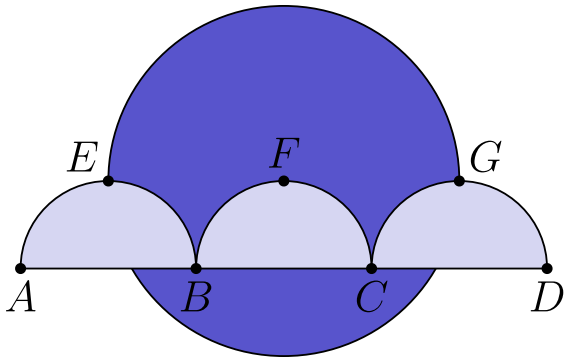
$$121 - 4 = 117.$$

Thus, the answer is **C** .

20. As shown in the figure, line segment $\overline{AD}$ is trisected by points B and C so that $AB = BC = CD = 2$. Three semicircles of radius 1, $\widehat{AEB}$, $\widehat{BFC}$, and $\widehat{CGD}$, have their diameters on $\overline{AD}$, lie in the same halfplane determined by line AD, and are tangent to line EG at E, F, and G, respectively. A circle of radius 2 has its center at F. The area of the region inside the circle but outside the three semicircles, shaded in the figure, can be expressed in the form

$$\frac{a}{b} \cdot \pi - \sqrt{c} + d,$$

where a, b, c, and d are positive integers and a and b are relatively prime. What is $a + b + c + d$?



A 13

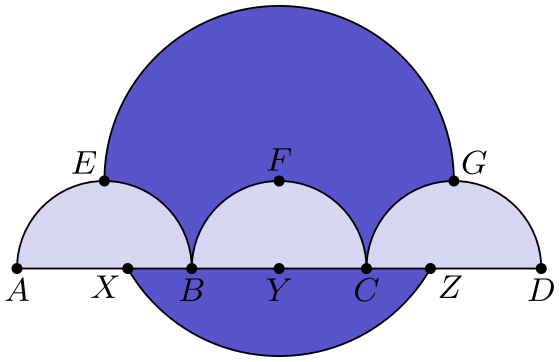
B 14

C 15

D 16

E 17

Solution:



Line EG passes through the center F of the radius-2 circle, so the shaded upper semicircle has area 2π.

The chord XZ lies on AD, one unit from F. Thus

$$\angle XFZ = 2\arccos\left(\frac{1}{2}\right) = \frac{2\pi}{3}.$$

The shaded circular segment below AD has area

$$\begin{aligned}&\frac{1}{2}(2^2)\left(\frac{2\pi}{3}\right)\\&\quad-\frac{1}{2}(2)(2)\sin\left(\frac{2\pi}{3}\right)\\&=\frac{4\pi}{3}-\sqrt{3}.\end{aligned}$$

The portion between EG and AD consists of four congruent pieces of the following form.

Each piece is a unit square with a quarter of a unit circle removed, so the four pieces have total area

$$4\left(1-\frac{\pi}{4}\right) = 4-\pi.$$

The total shaded area is therefore

$$\begin{aligned} &2\pi + \left(\frac{4\pi}{3} - \sqrt{3}\right) \\ &\quad + (4 - \pi) \\ &= \frac{7\pi}{3} - \sqrt{3} + 4. \end{aligned}$$

Hence $a = 7, b = 3, c = 3$, and $d = 4$, giving $a + b + c + d = 17$.

Thus, the answer is **E** .

21. Debra flips a fair coin repeatedly, keeping track of how many heads and how many tails she has seen in total, until she gets either two heads in a row or two tails in a row, at which point she stops flipping. What is the probability that she gets two heads in a row but she sees a second tail before she sees a second head?

A $\frac{1}{36}$

B $\frac{1}{24}$

C $\frac{1}{18}$

D $\frac{1}{12}$

E $\frac{1}{6}$

Solution:

Before the final repeated flip, the sequence must alternate. If it starts with H, the second head necessarily occurs before the second tail, so a successful sequence must start with T. To see a second tail before ending with HH, it must begin THT.

Thus the successful sequences are $THTHH, THTHTHH, \ldots$: exactly one sequence of each odd length at least 5. Their total probability is

$$\begin{aligned} &\frac{1}{2^5} + \frac{1}{2^7} + \cdots \\ &= \frac{1}{32} \cdot \frac{1}{1 - \frac{1}{4}} \\ &= \frac{1}{24}. \end{aligned}$$

Thus, the answer is **B**.

22. Raashan, Sylvia, and Ted play the following game. Each starts with $\$1$. A bell rings every 15 seconds, at which time each of the players who currently has money simultaneously chooses one of the other two players independently and at random and gives $\$1$ to that player. What is the probability that after the bell has rung 2019 times, each player will have $\$1$?

(For example, Raashan and Ted may each decide to give $\$1$ to Sylvia, and Sylvia may decide to give her dollar to Ted, at which point Raashan will have $\$0$, Sylvia will have $\$2$, and Ted will have $\$1$, and that is the end of the first round of play. In the second round Raashan has no money to give, but Sylvia and Ted might choose each other to give their $\$1$ to, and the holdings will be the same at the end of the second round.)

A $\frac{1}{7}$

B $\frac{1}{4}$

C $\frac{1}{3}$

D $\frac{1}{2}$

E $\frac{2}{3}$

Solution:

The only reachable money configurations up to order are $(1,1,1)$ and $(2,1,0)$. A player cannot finish a round with all 3 dollars: anyone who begins with money must give a dollar to someone else, and no one can give to themselves. From $(1,1,1)$, the next state is again $(1,1,1)$ exactly when all three players pass dollars in the same cyclic direction, which has probability $2\left(\frac{1}{2}\right)^3 = \frac{1}{4}$.

From $(2,1,0)$, label the players' holdings $A=2, B=1, C=0$. The next state is $(1,1,1)$ exactly when A gives to B and B gives to C, one of the four equally likely pairs of choices. This also has probability $\frac{1}{4}$.

Therefore, regardless of the state after 2018 rings, the probability that the state after the next ring is $(1,1,1)$ is $\frac{1}{4}$. Thus, **B** is the correct answer.

23. Points $A=(6,13)$ and $B=(12,11)$ lie on a circle ω in the plane. Suppose that the tangent lines to ω at A and B intersect at a point on the x-axis. What is the area of ω?

A $\frac{83\pi}{8}$

B $\frac{21\pi}{2}$

C $\frac{85\pi}{8}$

D $\frac{43\pi}{4}$

E $\frac{87\pi}{8}$

Solution:

Let P be the intersection point of the two tangents. Since tangent lengths from the same point are equal, $PA=PB$, so P lies on the perpendicular bisector of $\overline{AB}$.

The midpoint of $A(6,13)$ and $B(12,11)$ is $(9,12)$, and the slope of AB is $-\frac{1}{3}$, so the perpendicular bisector is $y=3x-15$. Its intersection with the x-axis is $P=(5,0)$.

The tangent line through P and A has slope 13, so the radius to A has slope $-\frac{1}{13}$.

Intersecting $y-13=-\frac{1}{13}(x-6)$ with $y=3x-15$ gives center $\left(\frac{37}{4},\frac{51}{4}\right)$.

Thus $r^2=\left(\frac{37}{4}-6\right)^2+\left(\frac{51}{4}-13\right)^2=\frac{85}{8}$, so the area is $\frac{85\pi}{8}$. Thus, **C** is the correct answer.

24. Define a sequence recursively by $x_0 = 5$ and

$$x_{n+1} = \frac{x_n^2 + 5x_n + 4}{x_n + 6}$$

for all nonnegative integers n. Let m be the least positive integer such that

$$x_m \leq 4 + \frac{1}{2^{20}}.$$

In which of the following intervals does m lie?

A $[9, 26]$

B $[27, 80]$

C $[81, 242]$

D $[243, 728]$

E $[729, \infty)$

Solution:

Let $a_n = x_n - 4$. Then $a_0 = 1$, and simplifying the recurrence gives

$$a_{n+1} = \frac{a_n(a_n + 9)}{a_n + 10}.$$

As long as $0 < a_n \leq 1$, this implies

$$\frac{9}{10}a_n \leq a_{n+1} \leq \frac{10}{11}a_n.$$

By induction, $\left(\frac{9}{10}\right)^n \leq a_n \leq \left(\frac{10}{11}\right)^n$. For $n = 80$, $\left(\frac{10}{9}\right)^{80} < 2^{20}$, so $\left(\frac{9}{10}\right)^{80} > 2^{-20}$, and therefore $m > 80$.

Also $\left(\frac{11}{10}\right)^8 > 2$, so $\left(\frac{10}{11}\right)^{160} < 2^{-20}$, which gives $m \leq 160$. Hence $81 \leq m \leq 160$, so m lies in $[81, 242]$. Thus, **C** is the correct answer.

25. How many sequences of 0s and 1s of length 19 are there that begin with a 0, end with a 0, contain no two consecutive 0s, and contain no three consecutive 1s?

A 55

B 60

C 65

D 70

E 75

Solution:

Our sequence starts with a 0 then has sequences of 110 and 10 in some order, where they each come after a 0.

Let the number of 110 be x and let the number of 10 be y. Then the number of terms in the sequence is

$$3x + 2y + 1 = 19,$$

making

$$3x + 2y = 18.$$

The possible ordered pairs are

$$(x, y) = (6, 0), (4, 3), (2, 6), (0, 9).$$

Then, the number of ways to order the $x + y$ blocks is

$$\binom{x+y}{x},$$

since we choose which x of the $x + y$ positions hold a 110 block.

Therefore, the total number of ways is

$$\begin{aligned}&\binom{6}{6}+\binom{7}{4}+\binom{8}{2}+\binom{9}{0}\\&=1+35+28+1\\&=65.\end{aligned}$$

Thus, the answer is **C** .

Problems: https://live.poshenloh.com/past-contests/amc10/2019B